Date: May 17, 2006, Wednesday NAME:.....
Time: 12:15-14:15

STUDENT NO:.....

Math 206 Complex Calculus – Final Exam

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

PLEASE READ:
Check that there are 4 questions on your exam booklet.
No correct answer without a satisfying reasoning is accepted. Show your work in detail.
Write your name on the top of every page.

Q-1) Consider the difference equation

$$y(n+2) - 4y(n+1) + 4y(n) = 2^n,\ y(0) = 1,\ y(1) = -1.$$

Solution: Example 3-14 of the notes.

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Q-2) Solve the initial value problem

$$f''(t) + f(t) = H(t-2)sin(3t-6), f(0) = 1,\ f'(t) = 0.$$

Answer: $f(t) = [\frac{3}{8}sin(t-2) - \frac{1}{8}sin(3t-6)]H(t-2) + cos(t)H(t).$

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Q-3) Determine the value of the improper integral

$$\int_0^\infty \frac{(cos\, ax - cos\, bx)dx}{x^2},\ a > 0,\ b > 0,$$

by contour integration for a suitably chosen simple closed contour in z-plane. If you evaluate certain limits in your derivation, show all steps of your evaluation clearly.
Answer: $\pi(b - a)$

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Q-4) (i) (10 pts.) Find a linear fractional transformation $w = T(z)$ that maps the three points $z_1 = \infty,\ z_2 = 1,\ z_3 = -i$ onto $w_1 = 1,\ w_2 = i,\ w_3 = -i$, respectively.
(ii) (10 pts.) What is the image of the region $Re\,(z-1) > Im\,(z)$ under this transformation?
(iii) (5pts.) If $Z = \frac{1}{w} - 2 = \frac{1}{T(z)} - 2$, draw and indicate the image of $Re\,(z-1) > Im\,(z)$ in the Z-plane?
Answer: (i)$w = T(z) = \frac{z-1+i}{z}$.
(ii) $|w| < 1$.
(iii)$|Z + 2| > 1$.