MATH 206 HW8

1) Find the residues at each isolated singularity of the following functions:

a) $\dfrac{\tan z}{z^2+z+1}$ **b)** $\dfrac{\sqrt{z}}{z^3-4z^2+4z}$ where the principal branch is used for $\sqrt{z}$

c) $\dfrac{1}{10^z-e^z}$ **d)** $\dfrac{\cos\left(\dfrac{1}{z}\right)}{\sin z}$

2) Use the path given in Figure 1 ,where $R>1$, to find the value of the integral $\displaystyle\int_0^\infty \frac{dx}{(x^3+1)^2}$.

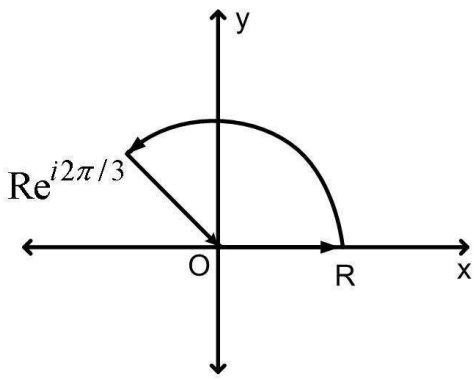

Figure 1

3) P.V $\displaystyle\int_{-\infty}^{\infty} \frac{(x+1)\cos x}{x^2+4x+5}\,dx = ?$ **4)** $\displaystyle\int_0^{\pi} (\sin\theta)^{2n}\,d\theta = ?$ $(n=1,2,\ldots)$.

5) Use the function

$$f(z)=\frac{z^{1/3}\log z}{z^2+1}=\frac{e^{(1/3)\log z}\log z}{z^2+1} \qquad \left(|z|>0, -\frac{\pi}{2}<\arg z<\frac{3\pi}{2}\right)$$

to derive this pair of integration formulas:

$$\int_0^\infty \frac{\sqrt[3]{x}\,\ln x}{x^2+1}\,dx=\frac{\pi^2}{6}, \qquad \int_0^\infty \frac{\sqrt[3]{x}}{x^2+1}\,dx=\frac{\pi}{\sqrt{3}} \quad .$$