Math 302 Complex Calculus II – Homework

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Please do not write anything inside the above boxes!

C Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

Q-5) Here we want to prove a finer version of a result in the book.

Let R be an open, simply connected, non-empty proper subset of the complex plane. Fix a point $z_0 \in R$. Define a collection of functions on R into the unit disk U as

$$\mathcal{F} = \{f : R \to U \mid f \text{ is analytic and one-to-one}, \ f(z_0) = 0 \ \text{and} \ f'(z_0) > 0 \}.$$

Show by a direct proof, i.e. not using the non-emptiness of a similar set of the textbook, that $\mathcal{F}$ is not empty.

Solution: