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Math 302 Complex Analysis II - Homework 3

| 1 | 2 | TOTAL |
| :---: | :---: | :---: |
|  |  |  |
| 10 | 10 | 20 |

Please do not write anything inside the above boxes!
Check that there are 2 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

Q-1) Classify all invertible meromorphic functions from $\mathbb{C} \cup\{\infty\}$ to $\mathbb{C} \cup\{\infty\}$.
Solution:

Q-2) Let $\left(z_{1}, z_{2}, z_{3}, z_{4}\right)$ and $\left(z_{1}^{\prime}, z_{2}^{\prime}, z_{3}^{\prime}, z_{4}^{\prime}\right)$ be two four-tuples of distinct points with cross-ratios of $\lambda$ and $\lambda^{\prime}$ respectively. Show that a Mobius transformation $T$ exists with $T\left(z_{i}\right)=z_{i}^{\prime}, i=1, \ldots, 4$, if and only if $j(\lambda)=j\left(\lambda^{\prime}\right)$, where

$$
j(\lambda)=256 \frac{\left(\lambda^{2}-\lambda+1\right)^{3}}{\lambda^{2}(\lambda-1)^{2}}
$$

## Solution:

