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Math 302 Complex Analysis II - Homework 4

| 1 | 2 | TOTAL |
| :---: | :---: | :---: |
|  |  |  |
| 10 | 10 | 20 |

Please do not write anything inside the above boxes!
Check that there are 2 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

Q-1) Let $R$ be the complex plane with the non-positive real axis taken out. Find explicitly a conformal mapping $f$ of $R$ onto the unit disc $U$ such that $f(1)=0$ and $f^{\prime}(1)>0$.

## Solution:

Q-2) Let $S$ be the Archimedean spiral given parametrically as

$$
x(t)=t \cos t, y(t)=t \sin t, \quad t \in[0, \infty)
$$

Let $R$ be the complement of $S$ in $\mathbb{C}$.
Can you define a branch of $\log$ function on $R$ ? If yes, construct this branch. If no, explain why.
Is $R$ still uniformly conformal to the open unit disc?

## Solution:

