Due Date: 20 May 2015, Wednesday
Due Time: 15:30
Instructor: Ali Sinan Sertöz

NAME:..

STUDENT NO:.......................................

Math 430 / Math 505 Introduction to Complex Geometry – Final Exam

1	2	3	4	5	TOTAL
50	50	0	0	0	100

Please do not write anything inside the above boxes!

Check that there are **2** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit. **Submit your solutions on this booklet only. Use extra pages if necessary.**

Rules for Homework and Take-Home Exams

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you write your answers alone. Any similarity with your written words with any other solution or any other source that I happen to know is a direct violation of honesty.

(2) In particular do not lend your written solutions to your friends, nor borrow your friends's written solutions. Oral exchange of ideas is acceptable and is in fact encouraged.

(3) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source can be easily retrieved by the reader. This includes any ideas you borrowed from your friends.

(4) Finally, in your written solution make sure that you exhibit your total understanding of the ideas involved, even mentioning where you quote a result but don't really follow the reasoning. This is an essential ingredient of learning.

Affidavit of compliance with the above rules: *I affirm that I have complied with the above rules in preparing this submitted work. Every solution I wrote reflects my true understanding of the problem. Any sources used, ideas from friends or others are explicitly cited without exception.*

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Q-1) Let $T_{n,d}$ be the space of all hypersurfaces of degree d in $\mathbb{P}^n$. Let $N_{n,d} = \dim T_{n,d}$. ($n > 1,\ d > 0$.)

1) Calculate $N_{n,d}$.

Let G_n be the space of all projective lines in $\mathbb{P}^n$.

2) Calculate $\dim G_n$.

Let $I_{n,d}$ denote the incidence space of pairs $(L, X) \in G_n \times T_{n,d}$ such that $L \subset X$. Consider the projection maps $pr_1 : I_{n,d} \to G_n$ and $pr_2 : I_{n,d} \to T_{n,d}$.

3) Find the dimension of a generic fibre of pr_1.

4) What is the dimension of $I_{n,d}$.

5) Show that a generic hypersurface $X \in T_{n,d}$ contains no line when $d > 2n - 3$.

Solution:

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Q-2) We know that the Hodge conjecture holds for threefolds, but with rational coefficients. Here we will show that the Hodge conjecture fails with integer coefficients.
Let $X \in \mathbb{P}^4$ be a generic hypersurface of degree 6.

1) Show that $H^2(X, \mathbb{Z}) \cong H^2(\mathbb{P}^4, \mathbb{Z}) \cong \mathbb{Z}$.
2) Show that $H^4(X, \mathbb{Z}) \cong \mathbb{Z}$ and is generated by a class α which is of type $(2, 2)$.
3) Show that α is not the Poincare dual of an algebraic cycle.

Solution: