Due Date: 13 May 2015, Wednesday
Time: Until 17:30
Instructor: Ali Sinan Sertöz

NAME:...

STUDENT NO:.......................................

Math 430 / Math 505 Introduction to Complex Geometry – Homework 4 – Solutions

1	2	3	4	5	TOTAL
50	50	0	0	0	100

Please do not write anything inside the above boxes!

Check that there are **2** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

Rules for Homework and Take-Home Exams

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you write your answers alone. Any similarity with your written words with any other solution or any other source that I happen to know is a direct violation of honesty.

(2) In particular do not lend your written solutions to your friends, nor borrow your friends's written solutions. Oral exchange of ideas is acceptable and is in fact encouraged.

(3) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source can be easily retrieved by the reader. This includes any ideas you borrowed from your friends.

(4) Finally, in your written solution make sure that you exhibit your total understanding of the ideas involved, even mentioning where you quote a result but don't really follow the reasoning. This is an essential ingredient of learning.

Affidavit of compliance with the above rules: *I affirm that I have complied with the above rules in preparing this submitted work. Every solution I wrote reflects my true understanding of the problem. Any sources used, ideas from friends or others are explicitly cited without exception.*

Please sign here:

NAME: STUDENT NO:

Q-1) The Grassmannian $G(k, n)$ is defined to be the set of all k-dimensional complex vector subspaces of $\mathbb{C}^n$. Show that $G(k, n)$ can be given the structure of a complex manifold. What is its dimension? Show that $G(k, n)$ is compact and in fact can be embedded into a projective space. Show also that $G(n-1, n) \simeq G(1, n) \simeq \mathbb{P}^{n-1}$.

Solution:

All of the required information is on pages 193-194 of Griffiths and Harris.

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Q-2) Calculate the volume of the complex projective line with respect to the Fubini-Study metric.

Solution:

This is calculated on page 119 of Huybrechts.

The pull back of the Fubini-Study metric for $\mathbb{P}^1$ back to $\mathbb{C} - \{0\}$ is given by

$$\pi^*\omega = \frac{i}{2\pi}\frac{dz \wedge d\bar{z}}{(1+z\bar{z})^2}.$$

This is also the volume form on $\mathbb{P}^1$. Integrating ω on $\mathbb{P}^1$ we get the volume as follows.

$$\begin{aligned}
\int_{\mathbb{P}^1} \omega &= \int_{\mathbb{C}} \pi^*\omega, \quad \text{integration on } \mathbb{C} \text{ and } \mathbb{C} - \{0\} \text{ are equal} \\
&= \frac{1}{\pi}\int_{\mathbb{R}^2} \frac{dx \wedge dy}{(1+x^2+y^2)^2}, \quad \text{where } z = x + iy \\
&= \frac{1}{\pi}\int_0^{2\pi}\int_0^{\infty} \frac{rdr}{(1+r^2)^2}, \quad \text{after passing to polar coordinates} \\
&= 1.
\end{aligned}$$