Due Date: 28 April 2017, Friday
Time: Class time
Instructor: Ali Sinan Sertöz

NAME:..

STUDENT NO:.......................................

Math 430 / Math 505 Introduction to Complex Geometry – Midterm 2 – Solutions

1	2	3	4	5	TOTAL
30	30	40	0	0	100

Please do not write anything inside the above boxes!

Check that there are **3** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

Rules for Homework and Take-Home Exams

(1) You may discuss the problems with your classmates or with me, or even with people who took this course before. It is not considered good behavior to ask these questions at online forums without mentioning that these are homework questions of an introductory course.

(2) You may use any written source be it printed or online. Google search is perfectly acceptable.

(3) **It is absolutely mandatory that you write your answers alone.**

(4) You must obey the usual rues of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. *(It is always nice to flatter your friends by using their ideas and thanking them.)*

(5) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you exhibit your total understanding of the ideas involved.

(6) Do not lend your written work to your friends and do not ask to borrow their written work. You may explain your solutions to your friends to any degree of detail you like, or you may ask them as many questions as they are willing to answer. **But the final writing process should be done alone.**

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work. Moreover I declare that every solution I wrote reflects my true understanding of the problem, and any sources used, including ideas from friends, are explicitly cited without exception.

Please sign here: *Date:*

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Q-1) Show that $H^k(M, \mathcal{O}) = 0$ for $k > n$, where M is a complex manifold of dimension n.

Solution:

All subsequent page references are to Griffiths and Harris's book *Principles of Algebraic Geometry*.

We have the Dolbeault Theorem (page 45) which says that for a complex manifold M

$$H^q(M, \Omega^p) \cong H^{p,q}_{\bar{\partial}}(M).$$

Here Ω^p denotes the sheaf of holomorphic p-forms. In particular the holomorphic 0-forms are the holomorphic functions; $\Omega^0 = \mathcal{O}$. Thus we have

$$H^q(M, \mathcal{O}) \cong H^{0,q}_{\bar{\partial}}(M).$$

Now it is clear that if $\dim_{\mathbb{C}} M = n$, then there can be no q-form with $q > n$ since repeatition of coordinate forms will kill the q-form.

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Q-2) Show that $H^k(\mathbb{C}^n, \mathcal{O}^*) = 0$ for $k > 0$.

Solution:

Since $\mathbb{C}^n$ is contractable, its cohomology is the same as the cohomology of a polydisc $\Delta \subset \mathbb{C}^n$. On page 25 we have the $\bar{\partial}$-Poincare Lemma which says that

$$H^{p,q}_{\bar{\partial}}(\Delta) = 0, \quad \text{for } q > 0 \quad .$$

In particular we have, putting $p = 0$,

$$H^{0,q}_{\bar{\partial}}(M) \cong H^q(\Delta, \mathcal{O}).$$

Thus we have

$$H^q(\mathbb{C}^n, \mathcal{O}) \cong H^q(\Delta, \mathcal{O}) = 0, \quad \text{for } q > 0 \quad .$$

This is *Computations 2* on page 46. (You can use it directly without giving the above explanations.)

Next we consider the exponential short exact sequence

$$0 \to \mathbb{Z} \to \mathcal{O} \to \mathcal{O}^* \to 0.$$

We then consider only the following part of the associated long exact sequence in cohomology:

$$\cdots \to H^q(\mathbb{C}^n, \mathcal{O}) \to H^q(\mathbb{C}^n, \mathcal{O}^*) \to H^{q+1}(\mathbb{C}^n, \mathbb{Z}) \to \cdots$$

We just showed that $H^q(\mathbb{C}^n, \mathcal{O}) = 0$ when $q > 0$. On the other hand $H^{q+1}(\mathbb{C}^n, \mathbb{Z}) = 0$ when $q > 0$ since $\mathbb{C}^n$ is contractible. From the exactness of the sequence we then conclude that

$$H^q(\mathbb{C}^n, \mathcal{O}^*) = 0 \quad \text{for } q > 0 \quad .$$

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Q-3) Show that any analytic hypersurface (a subvariety locally given as the zero set of a holomorphic function) in $\mathbb{C}^n$ is actually the zero set of a global holomorphic function.

Solution:

On page 47 we have

Any analytic hypersurface in $\mathbb{C}^n$ is the zero locus of an entire function.

Its proof is given there in detail and therefore I will not repeat it here. Just make sure that you explained the details in your own words.