Bilkent University

Exam # 02
Math 430 Introduction to Complex Geometry
Due: 20 April 2020
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Q-1) In this question we are using the notation of the Hard Lefschetz Theorem, [Griffiths-Harris, PAG, p122].
Here is a reminder about the notation.
M is a compact, complex, Hermitian manifold of complex dimension n.
$L : A^p(M) \to A^{p+2}(M)$, where $L(\alpha) = \alpha \wedge \omega$, with $\alpha \in A^p(M)$ and ω is the associated $(1,1)$-form of the metric of M.
$\Lambda : A^p(M) \to A^{p-2}(M)$ is the adjoint of L.
$h : A^*(M) \to A^*(M)$, where if $\alpha = \sum_{p=0}^{2n} \alpha_p$, with $\alpha_p \in A^p(M)$, then $h(\alpha) = \sum_{p=0}^{2n}(n-p)\alpha_p$.
Recal that we have the relations,

$$[\Lambda, L] = h, \quad [h, L] = -2L, \quad [h, \Lambda] = 2\Lambda.$$

(a) Show that for any positive integer m, we have

$$[\Lambda, L^m] = mhL^{m-1} + m(m-1)L^{m-1},$$

where L^0 is defined as the identity map. In fact you can simplify this expression as

$$[\Lambda, L^m](\alpha) = m(n-k-m+1)L^{m-1}(\alpha),$$

where $\alpha \in A^k(M)$.

(b) Show that for any $\alpha \in H^{n-k}(M)$, we have $L^{k+1}(\alpha) = 0$ if and only if $\Lambda(\alpha) = 0$.