$\qquad$
$\qquad$

Math 431 Algebraic Geometry - Homework

| previous | 7 | 8 | 9 | 10 | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| 60 | 10 | 10 | 10 | 10 | 100 |

Please do not write anything inside the above boxes!

Q-7) Let $\phi: C \rightarrow \mathbb{P}^{2}$ be defined by $\phi[x: y: z]=[x: z]$ where $C$ is a nonsingular projective curve in the projective plane not containing the point $[0: 1: 0]$. Show that if $C$ has degree $d>1$, then $\phi$ has at least one ramification point. Show that if $d=1$, then $\phi$ has no ramification points and is a homeomorphism.

## Answer:

Q-8) Show that the projective curve $D$ defined by $y^{2} z=x^{3}$ has a unique singular point. Show that the $\operatorname{map} f: \mathbb{P}^{1} \rightarrow D$ defined by

$$
f[s: t]=\left[s^{2} t: s^{3}: t^{3}\right]
$$

is a homeomorphism. Deduce that the degree-genus formula cannot be applied to singular curves in $\mathbb{P}^{2}$.

## Answer:

Q-9) Let $C$ be a singular irreducible projective cubic curve in $\mathbb{P}^{2}$. Show that the tangent line to $C$ at a nonsingular point or a line through two distinct nonsingular points of $C$ cannot meet $C$ at a singular point.

## Answer:

Q-10) Show that if $p$ is a point of inflection on a nonsingular cubic curve $C$ in $\mathbb{P}^{2}$, then there are exactly four tangent lines to $C$ which pass through $p$.

## Answer:

