Due Date: February 24, 2014 Monday

NAME:....

Ali Sinan Sertöz

STUDENT NO:

#### Math 431 Algebraic Geometry – Homework 1

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

### **Q-1**) (General Mathematics)

What are Fields, Abel and Gauss prizes? Who were the most recent recipients? What are IMU, MSRI and IHES?

Answer:

#### NAME:

# Q-2) (Topology)

Show that in a Noetherian topological space, every non-empty closed set can be expressed as a finite union of irreducible closed sets, unique up to permutation and up to redundancy.

## Solution:

### **Q-3**) (Commutative Algebra)

Show, using only a sketch of ideas, that there exists a Noetherian ring with infinite (Krull) dimension. You can find such an example on page 203 of Nagata's book *Local Rings* (1962). For understanding this example you will need to learn what it means to localize a ring at a multiplicatively closed set.

Solution:

### **Q-4**) (Algebraic Geometry)

Let k be an algebraically closed field of characteristic  $p \ge 0$  but  $p \ne 2$ . Let  $f \in k[x, y]$  be an irreducible quadratic polynomial. How many different (i.e. non-isomorphic)  $Z(f) \subset \mathbb{A}^2$  does there exist? What about p = 2 case?

#### Solution: