Due Date: March 10, 2014 Monday NAME:..

Instructor: Ali Sinan Sertöz STUDENT NO:..

Math 431 Algebraic Geometry – Midterm Exam 1

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Let $X \subset \mathbb{A}^n$ and $Y \subset \mathbb{A}^m$ be affine varieties.

(a) Show that $X \times Y \subset \mathbb{A}^{n+m}$ with its induced topology is irreducible.

(b) Show that the coordinate ring $k[X \times Y]$ of $X \times Y$ is isomorphic to $k[X] \otimes_k k[Y]$.

(c) Show that $X \times Y$ is a product in the category of varieties.

(d) Show that $\dim X \times Y = \dim X + \dim Y$.

Answer:

NAME: STUDENT NO:

Q-2) Let $X \subset \mathbb{A}^n$ be an affine variety of dimension r. Let $H \subset \mathbb{A}^n$ be a hypersurface such that $X \not\subset H$. Show that every irreducible component of $X \cap H$ has dimension $r - 1$. Give an example where $X \cap H$ has more than one irreducible component.

Solution:

NAME: STUDENT NO:

Q-3) Let $J \subset k[x_1, \ldots, x_n]$ be an ideal generated by r elements. Show that every irreducible component of $Z(J)$ has dimension greater than or equal to $n - r$. Give an example where $Z(J)$ has more than one irreducible component.

Solution:

NAME: STUDENT NO:

Q-4) Let $\phi : \mathbb{A}^1 \to \mathbb{A}^2$ be the map defined by $\phi(t) = (t^2, t^3)$. Show that ϕ is a homeomorphism of $\mathbb{A}^1$ onto the curve $X = Z(y^2 - x^3)$, but is not an isomorphism in the category of algebraic sets.

Solution: