Midterm-1 for Math 431 **Due Date: 1 April 2022**

Your Take-Home Exam assignment consists of three exercises from Hartshorne's book: Exercises I.5.1, I.5.2 and I.5.7.

These are reproduced below for your convenience. You may find several solutions on the Internet but first attack these problems on your own. In particular for I.5.7 first try $f(x,y,z) = x^2+y^2+z^2$ and draw the real picture. This will tell you what is happening in general. Check what happens when $\deg f = 1$. After playing with equations on your own you can then search the Internet and talk with your friends.

Write your solutions on A4 papers and staple them together. Do not use other fancy covers!

Thank you.

I.5.1. **Locate the singular points and sketch the following curves in $\mathbb{A}^3$ (assume char $k \neq 2$). Which is which in Figure 4?**

(a) $x^2 = x^4 + y^4$;

(b) $xy = x^6 + y^6$;

(c) $x^3 = y^2 + x^4 + y^4$;

(d) $x^2y + xy^2 = x^4 + y^4$.

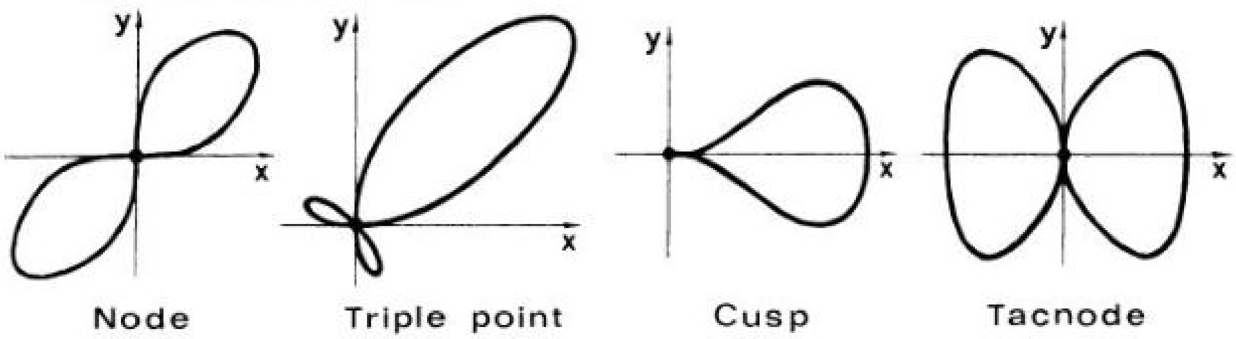

Figure 4. Singularities of plane curves.

I.5.2. **Locate the singular points and describe the singularities of the following surfaces in $\mathbb{A}^3$ (assume char $k \neq 0$). Which is which in Figure 5?**

(a) $xy^2 = z^2$;

(b) $x^2 + y^2 = z^2$;

(c) $xy + x^3 + y^3 = 0$.

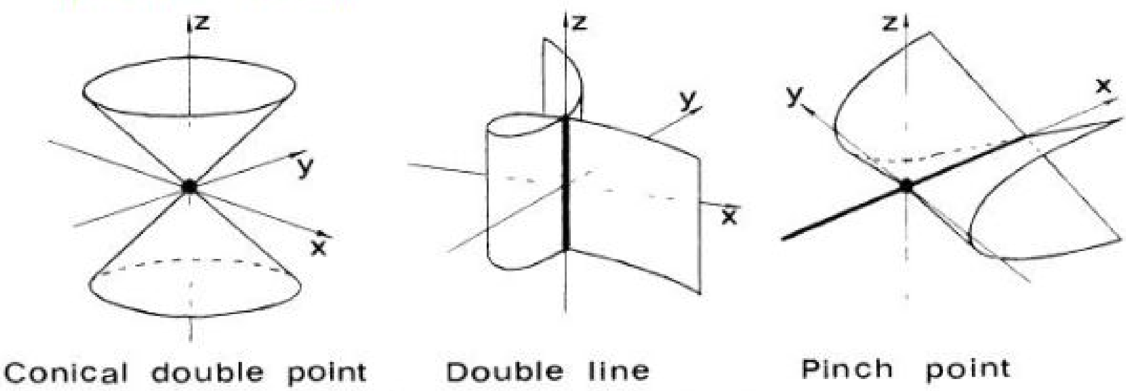

Figure 5. Surface singularities.

I.5.7. **Let $Y \subseteq \mathbb{P}^n$ be a nonsingular plane curve of degree > 1, defined by the equation $f(x, y, z) = 0$. Let $X \subseteq \mathbb{A}^{n+1}$ be the affine variety defined by f (this is the cone over Y). Let p be the point $(0, 0, 0)$, which is the vertex of the cone. Let $\phi\colon \tilde{X} \to X$ be the blow-up of X at p.**

(a) **Show that X has just one singular point, namely p.**

(b) **Show that $\tilde{X}$ is nonsingular.**

(c) **Show that $\phi^{-1}(p)$ is isomorphic to Y.**