Due Date: 8 January 2015, Thursday NAME:.....

Ali Sinan Sertöz STUDENT NO:.....

Math 503 Complex Analysis – Take-Home Final Exam

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Prove that $\dfrac{\zeta'(z)}{\zeta(z)} = -\displaystyle\sum_{n=1}^{\infty} \frac{\Lambda(n)}{n^z}$ for $\operatorname{Re} z > 1$, where $\Lambda(n) = \log p$ if $n = p^m$ for some prime p and $m \geq 1$; and $\Lambda(n) = 0$ otherwise.

Solution:

NAME: STUDENT NO:

Q-2) Show that $\Gamma'(1) = -\gamma$, where $\gamma = \lim_{n\to\infty} \left[\left(1 + \frac{1}{2} + \cdots + \frac{1}{n} \right) - \log n \right]$ is the Euler constant.

Solution:

NAME: STUDENT NO:

Q-3) Show that $\pi = 2\prod_{n=1}^{\infty} \frac{(2n)^2}{(2n-1)(2n+1)}$.

Solution:

NAME: STUDENT NO:

Q-4) Let f be an entire function and let $a, b \in \mathbb{C}$ such that $|a| < R$ and $|b| < R$. If $\gamma_R(t) = Re^{it}$ with $0 \leq t \leq 2\pi$, evaluate $\displaystyle\int_{\gamma_R} \frac{f(z)}{(z-a)(z-b)}\, dz$. Use this result to give another proof of Liouville's Theorem.

Solution: