Due Date: 13 October 2014, Monday – Class time NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 503 Complex Analysis – Homework 1

1	2	3	4	5	TOTAL
40	60	0	0	0	100

Please do not write anything inside the above boxes!

Check that there are **2** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Starting from the following basic facts that we proved in class

$$
\begin{aligned}
e^{i\theta} &= \cos\theta + i\sin\theta \quad \text{for} \quad \theta \in \mathbb{R}, \\
\sin z &= \sin x \cosh y + i \cos x \sinh y, \quad \text{and} \\
\cos z &= \cos x \cosh y - i \sin x \sinh y, \quad \text{for} \quad x, y \in \mathbb{R},
\end{aligned}
$$

show that

$$e^{iz} = \cos z + i \sin z, \quad \text{for} \quad z \in \mathbb{C}.$$

Also show that for any $z_1, z_2 \in \mathbb{C}$, we have the addition rules

$$
\begin{aligned}
\sin(z_1 + z_2) &= \sin z_1 \cos z_2 + \sin z_2 \cos z_1, \\
\cos(z_1 + z_2) &= \cos z_1 \cos z_2 - \sin z_1 \sin z_2.
\end{aligned}
$$

Solution:

NAME: STUDENT NO:

Q-2) Consider the function

$$z \mapsto w = z + \frac{1}{z}, \quad \text{for} \quad z \in \mathbb{C},\ z \neq 0.$$

Describe the mapping properties of this map. In other words define a Riemann surface S such that the map is one-to-one and onto S.

In particular find a contour C in the z-plane such that (a) it goes around the point $z = 1$ once and totally lies in the right hand plane $\mathrm{Re}\, z > 0$, and (b) its image can be easily described under the above map. Then describe its image. How many times does it go around the branch point $w = 2$?

Solution: