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## Math 503 Complex Analysis - Homework 4

| 1 | 2 | 3 | 4 | 5 | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: |
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| 100 | 0 | 0 | 0 | 0 | 100 |
| Please do not write anything inside the above boxes! |  |  |  |  |  |

Check that there is $\mathbf{1}$ question on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

Q-1) [Conway, page 133, Exercise 5] Let $f$ be analytic in $D=\{z| | z \mid<1\}$ and suppose that $|f(z)| \leq M$ for all $z$ in $D$.
(a) If $f\left(z_{k}\right)=0$ for $1 \leq k \leq n$ show that

$$
|f(z)| \leq M \prod_{k=1}^{n} \frac{\left|z-z_{k}\right|}{\left|1-\bar{z}_{k} z\right|}
$$

for $|z|<1$.
(b) If $f\left(z_{k}\right)=0$ for $1 \leq k \leq n$, each $z_{k} \neq 0$, and $f(0)=M\left(z_{1} z_{2} \cdots z_{n}\right)$, find a formula for $f$.

## Solution:

