Due Date: 22 December 2016, Thursday
Class Time

NAME:..

STUDENT NO:..

Math 503 Complex Analysis - Final Exam

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

Rules for Take-Home Assignments

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you **write your answers alone**.

(2) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)

(3) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you **exhibit your total understanding of the ideas involved**.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

Please sign here:

NAME: STUDENT NO: DEPARTMENT:

Q-1) Show that there is an analytic function f defined on the punctured unit disc

$$D^* = \{z \mid 0 < |z| < 1\},$$

such that f' never vanishes and $f(D^*) = D$, where

$$D = \{z \mid |z| < 1\}.$$

Solution:

NAME: STUDENT NO: DEPARTMENT:

Q-2 Let G be a simply connected and bounded region in $\mathbb{C}$. Fix a point $a \in G$. Assume that for every real valued harmonic function $u(z)$ on ∂G, there exists a real valued harmonic function $U(z)$ on G such that $U(z) = u(z)$ for all $z \in \partial G$. Construct an analytic function $f\colon G \to D$ which vanishes only at a. Here D is the unit disc $|z| < 1$.
This is how Riemann started to prove his famous mapping theorem. After this step he uses some intricate analysis to show that the above constructed f is a conformal equivalence.

Solution:

NAME: STUDENT NO: DEPARTMENT:

Q-3 Show that there is an analytic function f on $D = \{z \mid |z| < 1\}$ which is not analytic on any open set G which properly contains D.

Solution:

NAME: STUDENT NO: DEPARTMENT:

Q-4) Let $\zeta(z)$ be the Riemann zeta function. Prove that for $\operatorname{Re} z > 2$,

$$\frac{\zeta(z-1)}{\zeta(z)} = \sum_{n=1}^{\infty} \frac{\phi(n)}{n^z},$$

where $\phi(n)$ is the Euler totient function which counts the number of positive integers less than n that are relatively prime to n.

Solution: