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Math 504 Complex Analysis II - Take-Home Exam 02

| 1 | 2 | 3 | 4 | 5 | TOTAL |
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|  |  |  |  |  |  |
| 25 | 25 | 25 | 25 | 0 | 100 |

Please do not write anything inside the above boxes!
Check that there are $\mathbf{4}$ questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail.

For each question I will post the best student solution on the web. If there are more than one interesting solutions, I will post them all. Having your solution posted on the web will get you extra 10 points for each solution posted. These will be added to your total exam grades before an average is taken at the end of the semester.

Q-1) Find the type and multiplier of each of the following transformations:
(i) $\frac{z+1}{z+3}$,
(ii) $\frac{i z+1}{z+3 i}$,
(iii) $i z+1$,
(iv) $\frac{-z}{z+4}$.
[page 53, Exercise 2B]

## Solution:

Q-2) Let $C$ be a circle in $\Sigma$ passing through $z_{1}, z_{2}, z_{3}$. Prove that $I_{C}(z)=w$ if and only if

$$
\left(w, z_{1} ; z_{2}, z_{3}\right)=\overline{\left(z, z_{1} ; z_{2}, z_{3}\right)} .
$$

[page 55, Exercise 2M]

## Solution:

Q-3) Show that $P S L(2, \mathbb{C})$ is generated by parabolic elements. [page 55, Exercise 2N]

## Solution:

Q-4) Let $F_{\theta}$ be a rotation of $S^{2}$ by an angle $\theta$ around $x_{1}$-axis. This induces a Mobius transformation on $\Sigma$. Write this transformation explicitly. Can you generalize this by replacing $x_{1}$-axis by some other diagonal of $S^{2}$ ?
[Oğuz Gezmiş]

## Solution:

