Due Date: 20 March 2013, Wednesday NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 504 Complex Analysis II – Take-Home Exam 03

1	2	3	4	5	TOTAL
25	25	25	25	0	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail.

For each question I will post the best student solution on the web. If there are more than one interesting solutions, I will post them all. Having your solution posted on the web will get you extra 10 points for each solution posted. These will be added to your total exam grades before an average is taken at the end of the semester.

NAME: STUDENT NO:

Q-1)

(i) Show that $\mathbb{Z}[i]$ and $\mathbb{Z}[\rho]$ are discrete subgroups of $\mathbb{C}$, where $\rho = \frac{1}{2}(1+\sqrt{-3})$.

(ii) Show that $\mathbb{Z}[\sqrt{2}]$ is not a discrete subgroup of $\mathbb{C}$.

[page 120, Exercise 1A]

Solution:

NAME: STUDENT NO:

Q-2) Find conditions on the integers a and b such that $a\omega_1 + b\omega_2$ is a basis element of the lattice generated by ω_1 and ω_2.
[page 120, Exercise 3E]

Solution:

NAME: STUDENT NO:

Q-3) Let Ω be a lattice. For which λ, the map $\omega \mapsto \lambda\omega$ is an automorphism of Ω?
[page 120, Exercise 3G]

Solution:

NAME: STUDENT NO:

Q-4)

(i) Show that $\displaystyle\sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$ converges normally on compact subsets of $\mathbb{C} \setminus \mathbb{Z}$.

(ii) Show that $\displaystyle\pi^2 \operatorname{cosec}^2 \pi z = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$.

[page 121, Exercise 3I and 3J]

Solution: