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## Math 504 Complex Analysis II - Take-Home Exam 06

| 1 | 2 | 3 | 4 | 5 | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| 25 | 25 | 25 | 25 | 0 | 100 |

Please do not write anything inside the above boxes!
Check that there are $\mathbf{4}$ questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail.

For each question I will post the best student solution on the web. If there are more than one interesting solutions, I will post them all. Having your solution posted on the web will get you extra 10 points for each solution posted. These will be added to your total exam grades before an average is taken at the end of the semester.

Q-1) If $z=\frac{a i+b}{c i+d}$ where $a, b, c, d \in \mathbb{R}$ and $a d-b c=1$, prove that

$$
\cosh \rho(i, z)=\frac{1}{2}\left(a^{2}+b^{2}+c^{2}+d^{2}\right) .
$$

[page 268, Exercise 5F]

## Solution:

Q-2) Prove that there is no Fuchsian group isomorphic to $\mathbb{Z} \times \mathbb{Z}$. Can you generalize this?
[page 240, Note on top of the page.]

## Solution:

Q-3) Show that $P S L(2, \mathbb{R})$ is a simple group.
[page 268, Exercise 5K (iii)]

## Solution:

Q-4) Show that as a topological space $\operatorname{PSL}(2, \mathbb{R})$ is homeomorphic to $\mathbb{R}^{2} \times S^{1}$, where $S^{2}$ is a circle. [page 269, Exercise 5N]

## Solution:

