

Due Date: 6 May 2013, Monday

NAME:.....

Ali Sinan Sertöz

STUDENT NO:.....

**Math 504 Complex Analysis II – Take-Home Exam 07**

| 1  | 2  | 3  | 4  | 5 | TOTAL |
|----|----|----|----|---|-------|
|    |    |    |    |   |       |
| 25 | 25 | 25 | 25 | 0 | 100   |

*Please do not write anything inside the above boxes!*

Check that there are **4** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail.

For each question I will post the best student solution on the web. If there are more than one interesting solutions, I will post them all. Having your solution posted on the web will get you extra 10 points for each solution posted. These will be added to your total exam grades before an average is taken at the end of the semester.

---

NAME:

STUDENT NO:

**Q-1)** Let  $F$  be a Dirichlet region for a Fuchsian group  $\Lambda$  and let  $s$  be a side of  $F$ . If  $T \in \Lambda$  and  $T(s)$  is a side of  $F$  prove that

$$F \cap T(F) = T(s).$$

Moreover show that if there is  $T' \in \Lambda \setminus \{I\}$  such that  $T'(s)$  is also a side of  $F$ , then  $T = T'$ , hence no three sides of  $F$  can be congruent.

[page 269, Exercise 5R]

**Solution:**

NAME:

STUDENT NO:

**Q-2)** On page 265, Corollary 5.11.4 says “A *Hurwitz group of smallest order is simple.*”  
Explain what is meant by *smallest order*, and prove the corollary.

**Solution:**

NAME:

STUDENT NO:

**Q-3)** Let  $p = g - 1 > 84$  be a prime number. Show that there is no compact Riemann surface of genus  $g$  admitting  $84(g - 1)$  automorphisms.

[page 269, Exercise 5U]

**Solution:**

NAME:

STUDENT NO:

**Q-4)** Summarize the ideas involved in showing that there are infinitely many compact Riemann surfaces whose automorphism groups are Hurwitz groups.

**Solution:**