Bilkent University

Take-Home Exam # 04
Math 633 Algebraic Geometry
Due on: 5 December 2019 Thursday - Class Time
Instructor: Ali Sinan Sertöz

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Q-1) Hartshorne Exercise II.1.13 page 67.
Espace Étale of a presheaf. (This exercise is included only to establish the connection between our definition of a sheaf and another definition often found in the literature. See for example Godement [*Topologie Algébrique et Théorie des Faisceaux* Hermann, Paris (1958), Ch. II, §1.2].) Given a presheaf $\mathcal{F}$ on X, we define a topological space $\operatorname{Spé}(\mathcal{F})$, called the *espace étalé* of $\mathcal{F}$, as follows. As a set $\operatorname{Spé}(\mathcal{F}) = \bigcup_{P \in X} \mathcal{F}_P$. We define a projection map $\pi\colon \operatorname{Spé}(\mathcal{F}) \to X$ by sending $s \in \mathcal{F}_P$ to P. For each open set $U \subseteq X$ and each section $s \in \mathcal{F}(U)$, we obtain a map $\bar{s}\colon U \to \operatorname{Spé}(\mathcal{F})$ by sending $P \mapsto s_P$, its germ at P. This map has the property that $\pi \circ \bar{s} = \operatorname{Id}_U$, in other wors, it is a "section" of π over U. We now make $\operatorname{Spé}(\mathcal{F})$ into a topological space by giving it the strongest topology such that all the maps $\bar{s}\colon U \to \operatorname{Spé}(\mathcal{F})$ for all U, and all $s \in \mathcal{F}(U)$, are continuous. Now show that the sheaf $\mathcal{F}^+$ associated to $\mathcal{F}$ can be described as follows: for any open set $U \subseteq X$, $\mathcal{F}^+(U)$ is the set of *continuous* sections of $\operatorname{Spé}(\mathcal{F})$ over U. In particular, the original sheaf $\mathcal{F}$ was a sheaf if and only if for each U, $\mathcal{F}(U)$ is equal to the set of all continuous sections of $\operatorname{Spé}(\mathcal{F})$ over U.

Solution: