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Ottoman Mathematical Culture in the Nineteenth Century

Spurred on by increasing military, political and economic needs, the Ottomans had, by the nineteenth century, already begun to adopt Western science. To be specific, when the Treaty of Karlowitz was signed in 1699, following the first major defeat of the Ottomans, they realised the gulf between themselves and the Europeans, who had made great advances in science and technology. Coming to terms with their losses, and attempting to deal with the apparent superiority of the West, marks the beginning of Ottoman westernisation, the first such transformation of its kind anywhere. In this transition period of westernisation, the adoption of Western mathematics and related military techniques, like firearms and cartography, played an important role.

As a first step, military engineering schools were established so as to better prepare the Ottoman Empire against future military attacks. These were the first schools of higher education in the Ottoman Empire and were based on western models One of these schools, the Naval Engineering School, was established in 1773, under the guidance of Baron de Tott, a Hungarian in the service of the Ottoman Empire, and was reformed under Abdulhamid I (1774-1789). The aim of the school was to educate naval officers in geometry and geography. The subjects taught were arithmetic, geometry, geography, trigonometry, algebra, topography, mechanics and astronomy, as well as the Turkish language, Arabic and French. Some of the teachers were from France.

The second engineering school to be established was the Army Engineering School, founded in 1795. The aim was to train artillerymen and military engineers. Among the subjects taught were, again, French, Arabic, geometry, arithmetic, geography, algebra, trigonometry and astronomy, together with fortification.

These engineering schools had an important influence on mathematical activities at the time. Text books for the schools were prepared with translations and adaptations from European sources. The chief instructors and other teachers of these schools wrote and translated mathematical books. Hence mathematical teachers from Europe, and students who had been sent to the West, introduced modern mathematics to the Ottomans. Meanwhile, the printing press, which was established in the eighteenth century in Istanbul, served the Ottoman westernisation in the field of mathematics by printing the mathematical books.

The following were some of the key contributors to Ottoman westernisation in the field of mathematics.

Gelenbevi Ismail Efendi (1730-1790)

Among the chief instructors of the Naval Engineering School, Gelenbevi Ismail Efendi, from Gelenbe, near Aydın in western Anatolia, was of outstanding mathematical ability. Active at the end of the eighteenth century, when Ottoman science was already in decline, he was the last great Ottoman mathematician of the classical tradition, still working in the Islamic practice of algebra. He also used the traditional sexagesimal fractions in trigonometry. He wrote four books on mathematics, three in Turkish and one in Arabic: *Hisab al Kusur* (Calculation of the fractions), *Sharh-i Jadavil al Ensab* (The Explanation of the Sine Tables), *Usul Jadavil-i Ensab-ı Sittini* (Roots of the Sexagesimal Sine Tables), *Adla-i Musallasat* (The Sides of Triangles).

The *Hisab al Kusur*, on arithmetic and algebra, is a detailed work with five chapters. The fifth chapter, the most important one, explains positional notation for numbers and operations with them, as well as the rules of finding the unknown using algebra. The chapter deals with the solutions of different types of equations with integer coefficients, known in Islamic algebra as *masail-i sitte* (six problems, equations). Gelenbevi did not give geometrical justifications of the solutions as earlier mathematicians such as Khwarizmi had done.

Gelenbevi's *Risala fi Sharh-i Jadavil al Ensab* concerns tables of logarithms and their use, a topic becoming very well known in Istanbul at the time. This treatise is the first independent work on logarithms in the Ottoman era.
In his *Adla-i Musallasat*, written in Turkish and one of the rare original Ottoman trigonometry books, Gelenbevi studied the relations between sides and angles of triangles. Here, three kinds of theorems related to the triangles were presented, the Pythagorean Theorem, and the tangent and sine theorems.

Huseyin Rıfkı Tamani (? – 1817)

Huseyin Rıfkı Tamani, from the city of Taman in the Crimea, was among the first teachers appointed by Sultan Selim III to teach at the Army Engineering School. He was employed as the chief instructor at the school for a long time. He tried to organise the mathematical courses and wrote text books for them. His mathematical books were reprinted several times and were used as textbooks at the Engineering School for many years.

Huseyin Rıfkı knew English, French, Italian and Latin, in addition to oriental languages. He was a pioneer for bringing modern mathematics to the Ottoman Empire through translating many books into Turkish. The most important being his translation of a modernised and revised version of Euclid's *Elements,* published by Bonnycastle. He made the translation with the help and collaboration of an English engineer called Selim, who had entered the Ottoman service, and had converted to Islam. Both the British writer and traveller MacFarlane and the engineer Sang, who had been invited to Istanbul to work in manufacturing, highly praised Huseyin Rıfkı's *Elements* and stated that his Geometry was the best book to read and understand, existing in any language. It is thought that the final two sections on plane trigonometry were Huseyin Rıfkı's own work.

Huseyin Rıfkı was the author of *Article on Logarithms*, the third book on logarithms to be published in the Ottoman era. He gives the rules of logarithms and demonstrated the conversion of sexagesimal and decimal fractions to each other in the introductory part. This book was partly original and partly a translation, and was written to support the widespread use of logarithms by the Ottomans.

Work on the translation of western scientific books at the School, which was very active during the time of Huseyin Rıfkı (1806-1817), decreased in the time of Sayyid Ali Pasha who was the chief instructor following Huseyin Rıfkı. However, translations from the West were taken up again under Ishak Efendi who succeeded Sayyid Ali Pasha.

Ishak Efendi (1748? – 1834)

Ishak Efendi, the famous chief instructor of the Army Engineering School, was a student of Huseyin Rıfkı. He was born in the town of Narta in Yanya (Janina) and was a Jewish convert to Islam. He played an important role in the transmission of modern science to the Ottomans through his numerous books and lectures. Among his treatises, of special importance is the four volume *Mecmua-i Ulum-u Riyaziya* (Compendium of Mathematical Sciences) (1831), written for students of the School. The first and the second volumes are devoted to mathematics. The first volume treats arithmetic, algebra and geometry, while the second volume considers plane trigonometry, geometrical operations, applications of algebra to geometry, conic sections and differential and integral calculus. Higher mathematics was introduced to the Ottomans for the first time with these subjects. Again, for the first time, the terminology of modern sciences was translated into Turkish by Ishak Efendi. Being well versed in the subject matter as well as western languages, he coined many Ottoman Turkish equivalents for scientific terms (including mathematical ones) used in Europe.

Mecmua-i Ulum-u

The second volume of the treatise has two chapters – plane trigonometry and conics, and the calculus. Higher mathematics appears in the third section of the first chapter. Here we find geometric curves, including the loci of linear and higher degree equations, as well as irrational curves, and the helix.

In the second chapter, the first section treats the differential calculus, including methods for finding the differentials of the second and third degree, the differentials of the sine and cosine and the differentials of logarithmic functions.

The second section, on the integral calculus, looks at the possibilities or impossibilities of geometrical integrals of differentials with two terms, the quadrature of curves, calculations of volumes, the integral calculus of sine and cosine functions, approximate integrals, some transformations used in the integral calculus, integrals of surd expressions, integrals of differential equations, and differential equations of second, third and higher degrees..

It is clear that the understanding of the modern treatment of numbers, as well as the differential and integral calculus were introduced to Ottoman mathematics by *Mecmua-i Ulum-u Riyaziya*, the work drawing on several western sources. Ishak Efendi dedicated the first volume of his work to Sultan Mahmud II. The Sultan approved the book and its publication and distribution were provided by the government. The book was also published in Cairo in 1845 for use by the students of a new military school set up by Mehmed Ali Pasha, the reformist governor of Egypt. Ishak Efendi's influence on the transmission of modern mathematical sciences therefore spread to the wider Islamic world beyond Istanbul. The *Mecmua-i Ulum-u Riyaziya* was at the same level as the mathematical books of the technical schools in Europe at that time.

Emin Pasha (d.1851)

Emin Pasha was the son of Huseyin Rıfkı. He was sent to Cambridge University in 1835 after finishing the Army Engineering School.

He returned to Istanbul in 1841 and became the director of the Military School. He tried to develop it as a modern institution of education by bringing in experienced teachers and new methods.

Although he was a brilliant mathematician, Enin Pasha could not write many mathematical treatises, because so much of his time was taken up with administration. At Cambridge, his doctoral thesis was a paper of 27 pages called *Calcul de Variation*. Here, he explains how one could solve problems that could not be treated with calculus. He referred to the works by French Poisson (1781-1840) and Yakoli. His book on mathematics, physics and the art of war, *Memoire sur un nouveau Système de Confection des Fusées de Guerre*, was published in 1840.

Ibrahim Edhem Pasha (1785? -1865)

Ibrahim Edhem Pasha was another of the mathematicians who was in contact with European mathematics. Not much is known about his life since he lived in Egypt and does not appear in Turkish sources. Furthermore, since he wrote in Turkish, he is not found in Arabic catalogues. Ibrahim Edhem Pasha translated some mathematical books into Turkish, the most important being a translation of Legendre's *Éléments de Géométrie*. He copied the theorems of Lacroix, as did some mathematicians in France, rejecting Legendre's theorems, because of their complexity. Also in this translation, he uses the measurements of the meridian circle determined by Delambre (1749-1822) and Mechain (1744-1804), as a unit length, which shows that he also knew something of practical subjects such as geodesy, and the measurement units in France.

Huseyin Tevfik Pasha (1833-1901)

Huseyin Tevfik Pasha, from the city of Vidin in Rumelia, was one of the most important representatives of modern mathematics among the Ottomans. He graduated from the military school of Istanbul in 1859 and was appointed as a teacher of algebra to the same school. He taught higher algebra, analytic geometry, calculus, mechanics and astronomy.

Huseyin Tevfik could follow the mathematical developments in Europe and America closely, because of his knowledge of English and French languages. He was often sent abroad for official duties. Meanwhile, he found time to devote to mathematics, collected a rich library, and encouraged scientific discussions with talented students close to him (for example Salih Zeki). He also tried to create an intellectual environment by publishing some periodicals.

Huseyin Tevfik was sent to Germany in 1886 to inspect the Mauser rifles invented by Peter Paul Mauser (1838-1914). He gave lectures on the Ottoman State and Islam in French, in the course of appointment to the membership of the Historical Society of New Island.

Although many of Huseyin Tevfik's books were not published, his most important mathematical work, *Linear Algebra* (1882), written in English, was successful enough to be republished in 1892. In this book, which had been written in 1872 while he was in America on official duties, Huseyin Tevfik he extended Argand's multiplication of complex numbers to three dimensions.

The preface to *Linear Algebra* which is the one of the first books on the subject, declares:

> This linear algebra is the product of applying of Argand's complex and imaginary quantities to three dimensional space. The "Quaternions", invented by Sir William Hamilton, was similar to that as well. However, almost there are no features in common between these two systems. The Argand's system, can be applied only to plane geometry, is not a specific condition of Hamilton's system. Hence, after the discovery of Hamilton's "calculus of directed lines", Argand's algebra is far from the being complete.
>
> Cauchy, one of the eminent mathematicians, used the Argand's system in his some

> important works. “Methode des Equipollences” by M. Bellavitis was a generalised system of the plane analytic geometry and in reality was a developed form of the Argand’s algebra.
>
> The Argand’s system provided the best and the most perfect proof for the most important theorem of the theory of algebraic equations. Argand’s method, on the imaginary quantities, gave the geometrical commentary of some expressions in ordinary algebra like $a + b\sqrt{-1}$ and $a - b\sqrt{-1}$. So, it can’t be thought that the ordinary algebra is not fully completed without Argand’s system.
>
> For this reason, I established a new algebra instead of Hamilton’s calculus.
>
> The applying of linear algebra to three dimensional geometry and to the plane geometry is possible. When it was applied to the plane geometry, it becomes the Argand’s algebra except its notation.
>
> What we can say in behalf of the Quaternions, is also valid for the linear algebra.
>
> The most important point in the view of education is that almost every problem is independent each other.

This preface clearly shows the importance and originality of this book, which was not a translation of an existing European work.

Mehmed Nadir Beg (1856-1927)

Mehmed Nadir Beg was a teacher of Salih Zeki (see below) in Darüşşafaka (high school for orphan children). When a department of Number Theory was established in Istanbul University under the direction of Salih Zeki in 1919, Mehmed Nadir was appointed the head of this department. He published the solutions of many problems in the theory of numbers in the journal *L’Intermédiaire des Mathématiciens*, which first appeared in 1894.

Most of Nadir Beg’s works were on Diophantine equations. He also prepared a text book on the theory of numbers and published its first volume.

Salih Zeki (1864-1921)

Salih Zeki, one of the mathematicians who

lived in the final period of the Ottoman empire, contributed to mathematical studies and to higher education in Turkey. His mathematical talent was evident when he was a student in high school. He began work as an officer in the Office of Mails and Telegrammes after graduation from Darüşşafaka. He was then sent to Paris by the office to further his study. There he completed his education in electrical engineering and returned to his country. Although he was working in administrative positions, such as director of the observatory, Salih Zeki could find time for his scientific activities, as Huseyin Tevfik Pasha had done, and finally, on appointment as director of Darülfünun (the house of sciences – today’s Istanbul University), he had the opportunity to study mathematics. He established the Departments of Mathematics, Astronomy and Physics. He prepared text books for the departments and contributed as a teacher. He wrote numerous textbooks on mathematics, physics and astronomy, and on algebra, theory of numbers, plane geometry, theory of probability, arithmetic, plane trigonometry, and on Lobatchevsky and Riemann geometries.

Salih Zeki is recognised as the founder of studies related to Turkish history of science and wrote two important works on the history of mathematics and astronomy. The first of these, entitled *Asar-ı Bakiya* (Immortal Works), consists of four volumes, of which only the first two were published. The first volume discusses trigonometrical studies in the Islamic World, while the second volume treats the Muslim contributions to arithmetic and algebra. He wrote the treatise using western sources such as Montucla, Tannery, Delambre and Cantor, and also drew on oriental source material from manuscripts in the libraries of Istanbul.

Salih Zeki's other book on the history of mathematics is the *Kamus-ı Riyaziyyat* (Dictionary of Mathematical Sciences). Here, he explains mathematical and astronomical terms and describes the lives and studies of all western and oriental mathematicians and astronomers. Only the first volume of the ten volumes was published, in 1892.

Salih Zeki was also interested in the philosophy of science. He translated into Turkish Poincaré's *La Valeur de la* Science (1905) in 1912, *La Science et l'Hypothèse* (1903) in 1927, and *Science et Méthode* (1908) in 1928. He also translated *Philosophy of Science* by Alexandre Bertrand in 1914/15. He also published a book on logic, *Mizan-i Tefekkur* (The Balance of Thought), in 1916. Thus he was instrumental in laying the foundations of the philosophy of mathematics as well as philosophy of science in Turkey.

Salih Zeki gave lectures for mathematics teachers and amateurs in Istanbul University in the academic years of 1912-13 and 1913-14. In these lectures, he mentioned modern topics in mathematics, such as non-Euclidean geometries and imaginary quantities.

In conclusion, although Ottoman mathematicians kept faith with the classical Islamic tradition in the period of the empire's growth, when the scientific revolutions occurred in Europe a gulf emerged between Ottoman and Western mathematics.

Ottoman mathematicians followed mathematical developments in Europe and used European knowledge to catch up with the modern world. Modern mathematical concepts and methods slowly began to be absorbed. However, the gulf between them could not be easily bridged. The Ottomans seemed to be informed about Western sciences mainly by means of translations and adaptations. Their most significant source was France, the centre of the Enlightenment, where talented students were sent for their higher education. It is significant to see that Ottoman mathematicians thought that development in mathematics could only be accomplished through adopting Western mathematics. There is still a great deal of mathematical papers to be studied to be able to make clear the success of the last Ottoman mathematicians.

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Announcements of events

8th International Conference of The Mathematics Education into the 21st Century Project "Reform, Revolution and Paradigm Shifts in Mathematics Education"

November 25-December 1, 2005
See Newsletter 59 and further information from
arogerson@vsg.edu.au.

Third International Conference on Ethnomathematics – Cultural Connections and Mathematical Manipulations

February 12-16, 2006
Auckland, New Zealand.
http://www.math.auckland.ac.nz/Events/2006/ICEM-3/

Espace Mathématique Francophone : Colloque EMF 2006

May 26-31, 2006
Québec, Canada
http://emf2006.educ.usherbrooke.ca/

3rd International Conference on the Teaching of Mathematics (ICTM-3)

June 30-July 6, 2006
Istanbul, Turkey
Following on the success of earlier conferences held in Samos, Greece (1998) and Crete, Greece (2002), the 2006 conference intends to focus on "new ways of teaching undergraduate mathematics". The conference will be co-sponsored by the MAA. For more information, including information on how to submit a paper, see http://www.tmd.org.tr/ictm3.

International Leibniz Congress - Unity in Plurality

July 24-29, 2006
Hannover, Germany
http://www.gwlb.de/Leibniz/Gesellschaft/Veranstaltungen/Kongress/Circular/

5th European Summer University on the History and Epistemology in mathematics education (ESU-5)

July 19-24, 2007
Prague, Czech Republic
The fifth conference in this series will focus on

- history and epistemology as tools for an interdisciplinary approach in the teaching and learning of mathematics and the sciences
- introducing an historical dimension in the teaching and learning of mathematics
- history and epistemology in mathematics teachers' education
- cultures and mathematics
- the history of mathematics education in Europe
- mathematics in Central Europe

The official languages of ESU-5 are three: English, Czech and French.
For more information, see Newsletter 58 or http://www.pedf.cuni.cz/kmdm/esu5.

ICME-11

July 6-13, 2008
Monterrey, Mexico

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Please note that the webpage http://www.mathedu-jp.org/hpm/index.htm is currently not updated due to technical reasons. Please use http://www.clab.edc.uoc.gr/hpm/ for the time being.