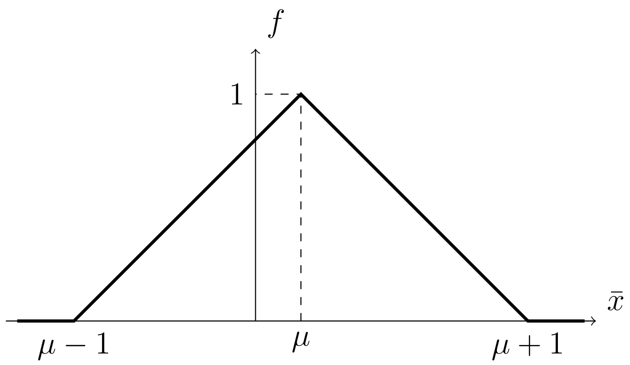
Econ 222-01
2013-2014 Spring
Homework 1
Due Date: March 7th.

1) For a normally distributed population with variance 4 we test $H_1 : \mu < 10$ against $H_0 : \mu \geq 10$. In order to conduct the test we took a random sample of size 16 and used $(\bar{x} - \mu)/(\sigma/\sqrt{n})$ as our test statistic. Based on the sample we failed to reject H_0 at a significance level of 0.01 but rejected H_0 at a significance level 0.04.

   What can you say about the value of the sample mean that led to these conclusions?

2) In order to test $H_1 : \sigma^2 < 16$ against $H_0 : \sigma^2 \geq 16$, for a normally distributed population, we take a sample of size 9. We use $(n-1)s^2/\sigma^2$ as the test statistic. What can you say about the p-value corresponding to $s^2 = 3$?

3) Consider a population which is uniformly distributed with a range of 2. The population mean is not known but is believed to be equal to 0 ($H_0 : \mu = 0$). We would like to test if the population mean is different than 0 ($H_1 : \mu \neq 0$). For this purpose we take a random sample of size 2 and use the sample mean as the test statistics. If the population mean is μ then it is known that the distribution of the test statistic is as shown below:



The graph of the density function, f, of $\bar{x}$.

   a. Write the decision rule for a significance level of $\alpha = 0.04$.

   b. If the observed value of the test statistic (sample mean) is 0.49 would you reject the null hypothesis?

   c. What is the p-value corresponding to the observed value of 0.9 for the test statistic?

4) We would like to estimate the mean of a uniformly distributed population that has a range of 1. In order to estimate the mean we will take a random sample of size 5 from this population. We will compare two estimators: One is to use the sample mean, $\bar{X} = (x_1 + \cdots + x_5)/5$. The other is the mean of the sample maximum and sample minimum, $\bar{X}_{n,x} = (\max\{x_1, \ldots, x_5\} + \min\{x_1, \ldots, x_5\})/2$.

a. If the sample is: 0.8, 0.4, 0.9, 0.7, 0.7 what would your estimate based on the sample mean, i.e., $\bar{X}$, be and what would your estimate based on the mean of the sample minimum and sample maximum, i.e., $\bar{X}_{n,x}$ be?

b. In order to conduct a hypothesis test about the mean of such a population based on these estimators we need to find their (probability) density functions (pdf) and their (cumulative) distribution functions (cdf). Since our background at this stage is not sufficient to find the exact pdf's and cdf's of the estimators, we will construct them by simulation. Apply the following steps in order to find approximations of the pdf's and cdf's:

- Using a spreadsheet generate 5 random number between -0.5 and 0.5. This will simulate the experiment of choosing 5 numbers, at random, from the interval $(-0.5\,,\,0.5)$.
- Repeat this 10,000 times and for each of the 5 values, calculate the mean and the average of the minimum and maximum of the 5 numbers. These will give you the values of $\bar{X} - \mu$ and $\bar{X}_{nx} - \mu$ for each sample.
- Draw a relative frequency polygons for $\bar{X} - \mu$ and $\bar{X}_{nx} - \mu$, using the class intervals of size 0.1 (i.e., $(-0.5\,,\,-0.4]$, $(-0.4\,,\,-0.3]$, ... $(0.3\,,\,0.4]$, $(0.4\,,\,-0.5)$). These are approximations for the pdf's of $\bar{X} - \mu$ and $\bar{X}_{nx} - \mu$.
- Construct tables like the table cumulative probability table of the standard normal distribution that I distributed in class. These tables are approximations for the cdf's of $\bar{X} - \mu$ and $\bar{X}_{nx} - \mu$.

We will be making use of these tables in our next homework.