Econ 222-01
2013-2014 Spring
Homework III
Due Date: March 14th.

1) Let N be an integer. Consider a population consisting of the integers $\{1, 2, \ldots, N\}$.[1]

   a. Assume that $N = 10$. Using a spreadsheet simulate the experiment of choosing a sample of size 4 from this population and finding the maximum of the sample, 1000 times. Using the results you obtained construct a relative frequency distribution table for the values you obtained and graph it (use the spreadsheet to construct the table and the graph, only include the printout of the table and the graph in your homework). This will be an approximation to the distribution of the maximum of the sample, i.e., the sampling distribution of the sample maximum.

   b. We would like test $H_1 : N < 10$ against $H_0 : N \geq 10$ at a significance level 0.05. To conduct this test, we take a random sample (with replacement) of size 4 for the population and use the sample maximum as a test statistic. Assume that the distribution of the test statistic is as you found in part a. What is the decision rule for the given test statistic?

   c. What would your decision be if the sample consisted of: 2, 6, 3, 4?

2) Consider a population that is uniformly distributed with a range of 1. We would like to test if the mean of this population is different than 0 at a significance level of 0.2. For this purpose we take a random sample of size 5 from this population. We will conduct the test with two test statistics (the probability density functions and cumulative distribution functions of these test statistics are given at the end.):

   $\bar{X} - \mu$**:** The "adjusted" sample mean.

      a. What would the decision rule be if you used the "adjusted" sample mean, $\bar{X} - \mu$, as the test statistic?

      b. What would your decision be if the random sample yields the following values: 0.1, 0.2, -0.4, -0.6, -0.1 (based on the test statistics $\bar{X} - \mu$?

      c. What is the corresponding p-value?

   $\bar{X}_{n,x} - \mu$**:** The "adjusted" mean of the sample minimum and sample maximum.

      d. What would the decision rule be if you used the "adjusted" sample mean, $\bar{X} - \mu$, as the test statistic?

      e. What would your decision be if the random sample yields the following values: 0.1, 0.2, -0.4, -0.6, -0.1 (based on the test statistics $\bar{X} - \mu$?

[1] The values in this population may represent the number of products produced by a firm in a given day. In this case i will represent the ith product produced today. Alternatively the population may be the fish in a lake which contains N fish.

f. What is the corresponding p-value?

If you had to choose one of the above test statistics to conduct this test, which one would you use? Explain your decision.

The PDF and CDF of $\bar{X} - \mu$ and $\bar{X}_{n,x} - \mu$
(The blue graphs correspond to $\bar{X}_{n,x} - \mu$ and
the black graphs corresponds to $\bar{X} - \mu$.)