Econ 222-01
2013-2014 Spring
Homework V
Due Date: April 4th.

1) Let $H_0 : \sigma_1^2 = \sigma_2^2$ and $H_1 : \sigma_1^2 \neq \sigma_2^2$. We take a sample of size 5 from population 1 and a sample of size 7 from population 2 and use s_1^2/s_2^2 as the test statistic.

   a. If the observed value of the test statistic is 2.50 what can you say about the p-value?

   b. Assume that we conduct the test at a significance level of 0.1. What would the probability of a Type II error be, if $\sigma_1^2 = \sigma_2^2/2$?

2) Consider the hypotheses test, where $H_1 : \sigma^2 < 4$ is tested against $H_0 : \sigma^2 \geq 4$, using the test statistic $(n-1)s^2/4$ (we assume that the population is normally distributed). Using a `spreadsheet` graph the probability of Type I error and Type II errors as a function of the "true" population variance σ^2 for the following values of n and α:

   a. $n = 10$, $\alpha = 0.01$.

   b. $n = 10$, $\alpha = 0.05$.

   c. $n = 20$, $\alpha = 0.01$.

   d. $n = 20$, $\alpha = 0.05$.

3) Consider a population that is uniformly distributed with a range of 1. We would like to test if the mean of this population is greater than 0 ($H_1 : \mu > 0$) at a significance level of 0.1. For this purpose we take a random sample of size 5 from this population. We will conduct the test with two test statistics: $\bar{X} - \mu$ and $\bar{X}_{n,x} - \mu$ (see Homework III).

   a. Answer the following questions for the case where you use $\bar{X} - \mu$ as the test statistic:

      (i) What is the probability of a Type I error when $\mu = -0.05$?

      (ii) What is the probability of a Type II error when $\mu = 0.05$?

   b. Answer the following questions for the case where you use $\bar{X}_{n,x} - \mu$ as the test statistic:

      (i) What is the probability of a Type I error when $\mu = -0.05$?

      (ii) What is the probability of a Type II error when $\mu = 0.05$?

   c. Are the answers to part a and b of this question in accord with your expectations (are the difference in the probabilities of Type I and Type II errors, if there is any, in accord with your intuition about the given test statistics)? Explain.

Table 1: CDF of $\bar{X} - \mu$
(The number is the cell corresponding to row a and column b gives
$\mathrm{P}\left(\bar{X} - \mu \leq a + b\right)$)

a \ b	0.0000	0.0005	0.0010	0.0015	0.0020	0.0025	0.0030	0.0035	0.0040	0.0045
0.000	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
0.005	0.5150	0.5165	0.5180	0.5195	0.5210	0.5224	0.5239	0.5254	0.5269	0.5284
0.010	0.5299	0.5314	0.5329	0.5344	0.5359	0.5374	0.5389	0.5404	0.5419	0.5433
0.015	0.5448	0.5463	0.5478	0.5493	0.5508	0.5523	0.5538	0.5552	0.5567	0.5582
0.020	0.5597	0.5612	0.5627	0.5641	0.5656	0.5671	0.5686	0.5700	0.5715	0.5730
0.025	0.5745	0.5759	0.5774	0.5789	0.5803	0.5818	0.5833	0.5848	0.5862	0.5877
0.030	0.5891	0.5906	0.5921	0.5935	0.5950	0.5964	0.5979	0.5994	0.6008	0.6023
0.035	0.6037	0.6052	0.6066	0.6081	0.6095	0.6109	0.6124	0.6138	0.6153	0.6167
0.040	0.6181	0.6196	0.6210	0.6224	0.6239	0.6253	0.6267	0.6282	0.6296	0.6310
0.045	0.6324	0.6338	0.6353	0.6367	0.6381	0.6395	0.6409	0.6423	0.6437	0.6451
0.050	0.6465	0.6479	0.6493	0.6507	0.6521	0.6535	0.6549	0.6563	0.6577	0.6591
0.055	0.6605	0.6618	0.6632	0.6646	0.6660	0.6673	0.6687	0.6701	0.6715	0.6728
0.060	0.6742	0.6755	0.6769	0.6783	0.6796	0.6810	0.6823	0.6837	0.6850	0.6864
0.065	0.6877	0.6890	0.6904	0.6917	0.6930	0.6944	0.6957	0.6970	0.6983	0.6996
0.070	0.7010	0.7023	0.7036	0.7049	0.7062	0.7075	0.7088	0.7101	0.7114	0.7127
0.075	0.7140	0.7153	0.7166	0.7179	0.7191	0.7204	0.7217	0.7230	0.7242	0.7255
0.080	0.7268	0.7280	0.7293	0.7305	0.7318	0.7330	0.7343	0.7355	0.7368	0.7380
0.085	0.7393	0.7405	0.7417	0.7430	0.7442	0.7454	0.7466	0.7478	0.7491	0.7503
0.090	0.7515	0.7527	0.7539	0.7551	0.7563	0.7575	0.7587	0.7598	0.7610	0.7622
0.095	0.7634	0.7646	0.7657	0.7669	0.7681	0.7692	0.7704	0.7715	0.7727	0.7739
0.100	0.7750	0.7761	0.7773	0.7784	0.7796	0.7807	0.7818	0.7829	0.7841	0.7852
0.105	0.7863	0.7874	0.7885	0.7896	0.7907	0.7918	0.7929	0.7940	0.7951	0.7962
0.110	0.7973	0.7984	0.7994	0.8005	0.8016	0.8027	0.8037	0.8048	0.8058	0.8069
0.115	0.8079	0.8090	0.8100	0.8111	0.8121	0.8131	0.8142	0.8152	0.8162	0.8172
0.120	0.8183	0.8193	0.8203	0.8213	0.8223	0.8233	0.8243	0.8253	0.8263	0.8273
0.125	0.8283	0.8292	0.8302	0.8312	0.8322	0.8331	0.8341	0.8350	0.8360	0.8370
0.130	0.8379	0.8389	0.8398	0.8407	0.8417	0.8426	0.8435	0.8445	0.8454	0.8463
0.135	0.8472	0.8481	0.8490	0.8499	0.8508	0.8517	0.8526	0.8535	0.8544	0.8553
0.140	0.8562	0.8571	0.8579	0.8588	0.8597	0.8605	0.8614	0.8623	0.8631	0.8640
0.145	0.8648	0.8657	0.8665	0.8673	0.8682	0.8690	0.8698	0.8707	0.8715	0.8723
0.150	0.8731	0.8739	0.8747	0.8755	0.8763	0.8771	0.8779	0.8787	0.8795	0.8803
0.155	0.8811	0.8818	0.8826	0.8834	0.8842	0.8849	0.8857	0.8864	0.8872	0.8879
0.160	0.8887	0.8894	0.8902	0.8909	0.8916	0.8924	0.8931	0.8938	0.8945	0.8953
0.165	0.8960	0.8967	0.8974	0.8981	0.8988	0.8995	0.9002	0.9009	0.9016	0.9022
0.170	0.9029	0.9036	0.9043	0.9049	0.9056	0.9063	0.9069	0.9076	0.9082	0.9089
0.175	0.9095	0.9102	0.9108	0.9115	0.9121	0.9127	0.9134	0.9140	0.9146	0.9152
0.180	0.9159	0.9165	0.9171	0.9177	0.9183	0.9189	0.9195	0.9201	0.9207	0.9213
0.185	0.9219	0.9224	0.9230	0.9236	0.9242	0.9247	0.9253	0.9259	0.9264	0.9270
0.190	0.9275	0.9281	0.9286	0.9292	0.9297	0.9303	0.9308	0.9313	0.9319	0.9324
0.195	0.9329	0.9335	0.9340	0.9345	0.9350	0.9355	0.9360	0.9365	0.9370	0.9375
0.200	0.9380	0.9385	0.9390	0.9395	0.9400	0.9405	0.9409	0.9414	0.9419	0.9424
0.205	0.9428	0.9433	0.9438	0.9442	0.9447	0.9451	0.9456	0.9460	0.9465	0.9469
0.210	0.9474	0.9478	0.9482	0.9487	0.9491	0.9495	0.9499	0.9504	0.9508	0.9512
0.215	0.9516	0.9520	0.9524	0.9528	0.9532	0.9536	0.9540	0.9544	0.9548	0.9552
0.220	0.9556	0.9560	0.9564	0.9568	0.9571	0.9575	0.9579	0.9583	0.9586	0.9590
0.225	0.9594	0.9597	0.9601	0.9604	0.9608	0.9611	0.9615	0.9618	0.9622	0.9625

Table 2: CDF of $\bar{X}_{n,x} - \mu$
(The number is the cell corresponding to row a and column b gives
$\mathrm{P}\left(\bar{X}_{n,x} - \mu \leq a + b\right)$)

a \ b	0.0000	0.0005	0.0010	0.0015	0.0020	0.0025	0.0030	0.0035	0.0040	0.0045
0.000	0.5000	0.5025	0.5050	0.5075	0.5099	0.5124	0.5148	0.5173	0.5197	0.5221
0.005	0.5245	0.5269	0.5293	0.5317	0.5340	0.5364	0.5387	0.5411	0.5434	0.5457
0.010	0.5480	0.5503	0.5526	0.5549	0.5572	0.5595	0.5617	0.5640	0.5662	0.5684
0.015	0.5706	0.5728	0.5750	0.5772	0.5794	0.5816	0.5837	0.5859	0.5880	0.5902
0.020	0.5923	0.5944	0.5965	0.5986	0.6007	0.6028	0.6049	0.6070	0.6090	0.6111
0.025	0.6131	0.6151	0.6172	0.6192	0.6212	0.6232	0.6252	0.6272	0.6291	0.6311
0.030	0.6330	0.6350	0.6369	0.6389	0.6408	0.6427	0.6446	0.6465	0.6484	0.6503
0.035	0.6522	0.6540	0.6559	0.6577	0.6596	0.6614	0.6632	0.6651	0.6669	0.6687
0.040	0.6705	0.6722	0.6740	0.6758	0.6776	0.6793	0.6811	0.6828	0.6845	0.6863
0.045	0.6880	0.6897	0.6914	0.6931	0.6948	0.6965	0.6981	0.6998	0.7015	0.7031
0.050	0.7048	0.7064	0.7080	0.7096	0.7113	0.7129	0.7145	0.7161	0.7176	0.7192
0.055	0.7208	0.7224	0.7239	0.7255	0.7270	0.7286	0.7301	0.7316	0.7331	0.7346
0.060	0.7361	0.7376	0.7391	0.7406	0.7421	0.7435	0.7450	0.7465	0.7479	0.7494
0.065	0.7508	0.7522	0.7536	0.7551	0.7565	0.7579	0.7593	0.7607	0.7620	0.7634
0.070	0.7648	0.7662	0.7675	0.7689	0.7702	0.7715	0.7729	0.7742	0.7755	0.7768
0.075	0.7781	0.7794	0.7807	0.7820	0.7833	0.7846	0.7859	0.7871	0.7884	0.7896
0.080	0.7909	0.7921	0.7934	0.7946	0.7958	0.7970	0.7983	0.7995	0.8007	0.8019
0.085	0.8030	0.8042	0.8054	0.8066	0.8077	0.8089	0.8101	0.8112	0.8124	0.8135
0.090	0.8146	0.8158	0.8169	0.8180	0.8191	0.8202	0.8213	0.8224	0.8235	0.8246
0.095	0.8257	0.8267	0.8278	0.8289	0.8299	0.8310	0.8320	0.8331	0.8341	0.8351
0.100	0.8362	0.8372	0.8382	0.8392	0.8402	0.8412	0.8422	0.8432	0.8442	0.8452
0.105	0.8461	0.8471	0.8481	0.8490	0.8500	0.8510	0.8519	0.8528	0.8538	0.8547
0.110	0.8556	0.8566	0.8575	0.8584	0.8593	0.8602	0.8611	0.8620	0.8629	0.8638
0.115	0.8647	0.8655	0.8664	0.8673	0.8681	0.8690	0.8699	0.8707	0.8715	0.8724
0.120	0.8732	0.8741	0.8749	0.8757	0.8765	0.8773	0.8781	0.8790	0.8798	0.8806
0.125	0.8813	0.8821	0.8829	0.8837	0.8845	0.8853	0.8860	0.8868	0.8875	0.8883
0.130	0.8890	0.8898	0.8905	0.8913	0.8920	0.8927	0.8935	0.8942	0.8949	0.8956
0.135	0.8963	0.8971	0.8978	0.8985	0.8992	0.8998	0.9005	0.9012	0.9019	0.9026
0.140	0.9033	0.9039	0.9046	0.9053	0.9059	0.9066	0.9072	0.9079	0.9085	0.9092
0.145	0.9098	0.9104	0.9111	0.9117	0.9123	0.9129	0.9135	0.9141	0.9148	0.9154
0.150	0.9160	0.9166	0.9172	0.9178	0.9183	0.9189	0.9195	0.9201	0.9207	0.9212
0.155	0.9218	0.9224	0.9229	0.9235	0.9240	0.9246	0.9251	0.9257	0.9262	0.9268
0.160	0.9273	0.9278	0.9284	0.9289	0.9294	0.9299	0.9305	0.9310	0.9315	0.9320
0.165	0.9325	0.9330	0.9335	0.9340	0.9345	0.9350	0.9355	0.9359	0.9364	0.9369
0.170	0.9374	0.9379	0.9383	0.9388	0.9393	0.9397	0.9402	0.9406	0.9411	0.9415
0.175	0.9420	0.9424	0.9429	0.9433	0.9437	0.9442	0.9446	0.9450	0.9455	0.9459
0.180	0.9463	0.9467	0.9471	0.9476	0.9480	0.9484	0.9488	0.9492	0.9496	0.9500
0.185	0.9504	0.9508	0.9512	0.9515	0.9519	0.9523	0.9527	0.9531	0.9534	0.9538
0.190	0.9542	0.9546	0.9549	0.9553	0.9557	0.9560	0.9564	0.9567	0.9571	0.9574
0.195	0.9578	0.9581	0.9585	0.9588	0.9591	0.9595	0.9598	0.9601	0.9605	0.9608
0.200	0.9611	0.9614	0.9618	0.9621	0.9624	0.9627	0.9630	0.9633	0.9636	0.9639
0.205	0.9643	0.9646	0.9649	0.9652	0.9654	0.9657	0.9660	0.9663	0.9666	0.9669
0.210	0.9672	0.9675	0.9677	0.9680	0.9683	0.9686	0.9688	0.9691	0.9694	0.9697
0.215	0.9699	0.9702	0.9704	0.9707	0.9710	0.9712	0.9715	0.9717	0.9720	0.9722
0.220	0.9725	0.9727	0.9730	0.9732	0.9734	0.9737	0.9739	0.9741	0.9744	0.9746
0.225	0.9748	0.9751	0.9753	0.9755	0.9757	0.9760	0.9762	0.9764	0.9766	0.9768