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# A Hub Covering Model for Cargo Delivery Systems

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**The hub location problem appears in a variety of applications including airline systems, cargo delivery systems, and telecommunication network design. When we analyze the application areas separately, we observe that each area has its own characteristics. In this research we focus on cargo delivery systems. Our interviews with various cargo delivery firms operating in Turkey enabled us to determine the constraints, requirements, and criteria of the hub location problem specific to the cargo delivery sector. We present integer programming formulations and large-scale implementations of the models within Turkey. The results are compared with the current structure of a cargo delivery firm operating in Turkey. © 2006 Wiley Periodicals, Inc. NETWORKS, Vol. 49(1), 28–39 2007**

**Keywords:** hub location problem; cargo delivery systems; integer programming; network design

## 1. INTRODUCTION

The hub location problem can be thought of as a special "network design" problem. In the generic version, there are $n$ demand nodes, each of which generates and/or absorbs demands. It is customarily assumed that there is a positive flow of traffic between each pair. The simplest method of handling the flow between these nodes would be to connect each pair of nodes directly; however, this would be highly inefficient. In the hub location problem, flows from the same origin with different destinations are consolidated on their route at a hub node, and they are then combined with flows from different origins going to the same destinations. This "flow consolidation and dissemination" is called *hubbing*. The advantage of hubbing is that by consolidating the flow, economies of scale can be achieved due to bulk transportation. Hubbing is encountered in airline systems, cargo delivery systems, and telecommunication network design. In this research we focus on cargo delivery systems.

The locations of the hubs and the allocations of demand nodes to the hubs are the main decisions in the hub location problem. We remark here that this definition of the hub location problem relies on a basic assumption that all hub pairs will have direct connection. In general, the network connecting the hubs is also a decision. This may be a questionable assumption for some application areas, especially if hub–hub connections are costly. As will be explained in Section 2, this assumption is valid for our problem.

The first description of the hub location problem is given by O'Kelly [19]; the author presents real-world examples and simple models for the location of one or two hubs. O'Kelly [20] describes the quadratic structure in hubbing and defines the "single-assignment hub location problem" where each node is allocated to exactly one hub. That is, all the inflow and outflow of each demand center is to be routed via one hub. Even though this structure is the most commonly used one in the literature, there is also a multi-assignment version of the problem in which a demand node can send/receive flow to/from multiple hubs. Because single assignment is the more common structure, in this article we will focus on single-assignment hub location.

O'Kelly [21] presents a quadratic integer program that minimizes the total transportation cost; in the literature, this quadratic integer program has become the basic model for the hub location problems. Different linearizations of this basic model are proposed in the literature [1, 4, 6, 22, 24].

In the literature, there are some policy-oriented studies, which mainly examine the necessity of hubbing in real life. Kanafani and Ghobrial [10] and Toh and Higgins [25] examine the impacts of hubbing on airline systems. Both articles are mainly discussions on advantages and disadvantages of hubbing. Hall [8] focuses on overnight deliveries. The author analyzes the impacts of express package delivery time restrictions on network design. The analysis is based on fixed hub locations. Ghobrial and Kanafani [7] propose some policies for airline systems. They emphasize that using a fewer number of hubs increases aircraft utilization and customer satisfaction, but at the same time, it causes congestion at the hubs.

When we consider the application-oriented articles related to the distribution network of cargo delivery companies, we observe that most of the articles assume fixed hub locations and focus on the network design aspect of the problem.

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The first application study is due to Marsten and Muller [18], who study the design of the service network and utilization of the aircraft fleet of Flying Tiger Line. The authors assume known hub locations and propose mixed integer linear programming models for different fixed network structures. Powell and Sheffi [23] analyze the truck-load planning problem and provide an optimization formulation that minimizes total cost. In this study, the location of hubs are assumed to be known, and the main decisions are the segments to be used in transportation. Kuby and Gray [16] propose a mixed integer linear program that minimizes total network cost. The authors assume that the hub locations are known in advance. A different approach for airline systems is considered by Jaillet et al. [9]. In that study, hubbing is not forced by constraints but the resulting model will have hubs if cost-efficient. The authors propose integer linear programming formulations that are dependent on different service policies and heuristic solution procedures. Another application-oriented study comes from Lin [17]. The author focuses on a "freight routing problem," which can be considered as a variant of the hub location problem. Assuming the hubs are fixed, the author proposes mixed integer programs and Lagrangean-based solution algorithms for the decisions of segments to be used. The algorithms are tested with a network from Taiwan that includes 65 demand centers and three hubs.

Klincewicz [15] and Bryan and O'Kelly [2] present reviews for communication network systems and airline systems, respectively. Campbell et al. [5] provides a state-of-the-art review including recent trends in hub location research. Different variants of the basic hub location model are classified according to their objectives, network components, and constraints.

We note here that most of the literature on hub location problems focuses on the objective of minimizing total transportation cost. Campbell [3] is the first study where different objectives in addition to minisum are defined together with real-life examples. The author defines the *p-hub center* and *hub covering problems* and presents mathematical models. Later, Kara and Tansel [11, 13] analyze the *p-hub center* and *hub covering problems* in more detail.

It has been observed in Kara and Tansel [12] that the standard hub location model is not appropriate for cargo delivery systems because it does not compute the total travel times correctly. The transient times spent waiting at hubs are not incorporated. The authors observe this deficiency and propose mathematical models for the hub location problem in which the transient times are also considered. The authors mainly focus on the minimax version and call the problem *The Latest Arrival Hub Location Problem*.

The *Latest Arrival Hub Location Problem* under the minimax objective can also be modeled as the *p-hub center problem*. Transient times spent at hubs can actually be ignored while modeling, and the resulting solution will still be optimum [26]. However, the CPU times of CPLEX for the *Latest Arrival Hub Center* model are much better than those of the *p-hub center model*. The maximum CPU time requirement of the *p-hub center model* is 11.3 hours [11], whereas that of the *Latest Arrival Hub Center* model—over the same data set and with the same computing power— is 4.4 hours [12]. Also, *The Latest Arrival Hub Location Problem* provides more insights and it is more realistic. Based on these, we believe the latest arrival version is more appropriate for our model, and we continue with the latest arrival formulations.

In this article, we analyze the covering version, namely *The Latest Arrival Hub Covering Problem*. We first conduct a survey on real-life companies to clarify the exact structure of cargo delivery systems. Most of the cargo delivery companies utilize a ground transportation service network in Turkey. For these companies, overnight delivery is considered VIP service, provided only between certain city pairs. Thus, in this research we focus on the transportation of general cargo. We provide a detailed analysis of the structure of the cargo delivery companies operating in Turkey in Section 2. In Sections 3 and 4, respectively, we present an integer programming (IP) formulation and a large-scale implementation of the model within Turkey. In Section 5 we observe certain additional characteristics and derive two variations of the basic model developed in Section 3. In Section 6 we compare our results with a cargo delivery firm's existing structure, from different perspectives, and in Section 7 we summarize our results.

## 2. CARGO DELIVERY SYSTEMS IN TURKEY

To comprehend the structure of the cargo delivery firms operating in Turkey, we interviewed the National Postal Service (PTT) and four different private cargo delivery companies. Our investigations indicate that speed and reliability are more significant factors than cost in cargo delivery. Delivering the cargo in a timely manner is the key element in the market.

The transportation of the cargo from origin to consignees is carried out via operation centers (which can also be considered hubs). Each demand center is assigned to an operation center that handles collection and distribution operations for the demand centers that it serves. A parcel that originates from a demand center first travels to the assigned operation center. At the operation center, all the parcels are sorted according to their consignees and loaded into larger and more specialized vehicles based on destinations (if the destinations are assigned to different operation centers) to travel to the operation centers that handle the destination nodes. There, all parcels are again sorted according to the final destinations and transported to the final consignees.

The single-assignment strategy is adopted in all five of the companies that we interviewed mainly for simplicity of management. Transportation between two operation centers (hub-to-hub transfer) is by larger and more specialized trucks. As well as having more capacity, these trucks are also faster than ordinary trucks. Also, there is direct transportation between each operation center pair; even if the shortest path between two operation centers passes through another operation center, the trucks do not stop on the way. Thus, the fully connected hub network assumption is valid.

For cargo delivery companies, the most important factor is the vehicle departure times from operation centers. Vehicles departing from an operation center need to wait for all incoming vehicles. Otherwise, parcels that arrive at the operation center after the departure of the vehicle and directed to the same operation center will require a second vehicle. In our discussions with company representatives we noted that all the companies measure their service quality by their delivery times; they would like to guarantee their customers service within specific time limits (e.g., 24 or 48 hours). Note that providing service within a predetermined time limit immediately signals that covering models are more appropriate.

The rest of our analysis will focus on a specific cargo delivery firm which has a wide service network within Turkey. This firm has over 1000 service centers and approximately 5000 qualified personnel. It offers service via 562 branches/agents, 26 operation centers, and 34 regional directorates.

## 3. LATEST ARRIVAL HUB COVERING PROBLEM

Cargo delivery systems are time-sensitive rather than price-sensitive. Delivering the cargo within a specified time interval is the most important factor in quality of service. This stresses the need to correctly compute delivery times, which leads us to the *Latest Arrival Hub Location Problem* of Kara and Tansel [12]. According to our analysis with the companies, we observed that covering is the appropriate criterion for the cargo delivery systems, and thus, in this article we focus on the *Latest Arrival Hub Covering Problem*. We adopt the terminology developed in [12].

Let $G = (N, E)$ be a connected transportation network with node set $N = \{1, \dots, n\}$ and arc set $E$. We assume that the nodes $1, \dots, n$ generate and absorb a positive flow to and from the rest of the nodes. The arc set $E$ includes transportation network links. For each pair of nodes $i, j \in N$, let $c_{ij}$ be the time spent in travelling on a shortest path connecting $i$ and $j$. Note that under the assumption of a connected network, $c_{ij}$ is always finite even if $(i,j) \notin E$, $c_{ij} = 0$ iff $i = j$, $c_{ij} = c_{ji}$ and $c_{ij} + c_{jk} \geq c_{ik}$ $\forall i, j, k$. Let $\alpha$ be the scaling factor to be used in hub-to-hub transportation (the travel time of carrying flow between two hub nodes $k$ and $r$ is taken as $\alpha c_{rk}$.). Let $\beta > 0$ be the predetermined time bound that restricts the total delivery time.

The *Latest Arrival Hub Covering Problem* is to select a subset $H = \{h_1, \dots, h_p\}$ of $N$ and allocate the rest of the nodes to the selected hub nodes $h_1, \dots, h_p$ so as to minimize the number of hubs while keeping the delivery time within $\beta$.

The most important characteristic of the *Latest Arrival Hub Covering Problem* is the vehicle departure times from a node. These departure times are subject to the arrival time of the vehicles that are coming to that node. To compute the total delivery time, consider an origin destination pair $i$–$j$ that are assigned to two distinct hubs $k$ and $m$, respectively. The vehicle that departs from hub $k$ towards hub $m$ transports not only the units that come from $i$ but also the units that come from other nonhub node(s) that are assigned to hub $k$. Due to the complete hub network, a vehicle that departs from hub $k$ towards other hubs does not transport the units that come from other hubs. Hence, the latest arriving vehicle at hub $k$ from the nodes that it serves determines the departure time toward all other hubs. Similarly, a vehicle from hub $m$ that is destined to go to a final destination should also wait for the vehicles coming from other hubs. Thus, the vehicle departure times at hub $m$ toward the destination nodes will all be the same (determined by the latest arriving cargo) regardless of where the vehicle is going. These observations are true assuming the existence of a positive flow between each origin/destination pair: this is called the full crosstraffic assumption [12].

To sum up, there are two different departure times from any hub $k$. The first one is the departure time for vehicles that are destined to go to other hubs, and the second one is the departure time for vehicles that are destined to go to the final destinations served by that hub. Let $\widehat{DT}_k$ and $DT_k$ be these departure times, respectively. Let $X_{jk}$ be a zero/one variable that takes on the value 1 if node $j$ is assigned to hub $k$ and 0 otherwise. Note that $X_{kk} = 1$ means there is a hub at node $k$ and $X_{kk} = 0$ means there is no hub at node $k$. An IP formulation for the latest arrival hub covering problem is:

$$\text{(Latest Cover-0)} \quad \min \sum_k X_{kk}$$

s.t.

$$\sum_k X_{ik} = 1 \qquad \forall i \qquad (1)$$

$$X_{ik} \leq X_{kk} \qquad \forall i, k \qquad (2)$$

$$\widehat{DT}_k \geq c_{jk} X_{jk} \qquad \forall j, k \qquad (3)$$

$$DT_k \geq \widehat{DT}_r + \alpha c_{rk} X_{rr} \qquad \forall r, k \qquad (4)$$

$$(DT_k + c_{jk}) X_{jk} \leq \beta \qquad \forall k, j \qquad (5)$$

$$X_{ik} \in \{0, 1\} \qquad \forall i, k \qquad (6)$$

$$DT_k, \widehat{DT}_k \geq 0 \qquad \forall k \qquad (7)$$

The objective function minimizes the total number of hubs. Constraints (1) and (6) ensure that each node is assigned to exactly one hub. Constraint (2) allows the allocations to be made to hub nodes only. Constraints (3) and (4) ensure that $\widehat{DT}_k$ and $DT_k$ take on the intended values as mentioned before. Constraint (5) forces the total delivery time to be less than or equal the upper bound $\beta$ for every origin–destination pair. Last, constraint (7) is the nonnegativity constraint.

(Latest Cover-0) is a nonlinear mixed integer program due to constraint (5). Constraint (5) can be replaced by constraint (8), and the resulting linear model is:

$$\text{(Latest Cover)} \quad \min \sum_k X_{kk}$$

$$\text{s.t.} \quad DT_k + c_{jk} X_{jk} \leq \beta \qquad \forall k, j \qquad (8)$$

(1)–(4), (6)–(7)

The correctness of this linearization can be justified by the following observation:

**Observation 1.** Constraint (8) correctly linearizes the constraint (5).

**Proof.** There are two cases to consider depending on the value of $X_{jk}$.
CASE 1, $X_{jk} = 1$: (5) and (8) yield the same left-hand sides.
CASE 2, $X_{jk} = 0$: (8) yields $DT_k \leq \beta$ while (5) yields $0 \leq \beta$. Due to the full crosstraffic assumption, node $k$ is a destination node. The total delivery time between every origin destination pair is required to be within the time limit $\beta$. Hence, putting a limit $\beta$ on the variable $DT_k$ will not affect the optimal solution. ∎

Thus, we provide a linear IP with $n^2$ binary and $2n$ real variables and $4n^2 + n$ constraints.

## 4. COMPUTATIONAL ANALYSIS

We tested the computational performance of (Latest Cover) using data from a Turkish highway map. We have 81 demand centers corresponding to 81 cities in Turkey. There are three types of parameters that we use in our IP model. The first one is $c_{ij}$, which is the travelling time on a shortest path connecting $i$ and $j$. We used specialized software [AndRoute 2.0 which displays the distance, time, and route(s) for a specified origin-destination pair] to calculate the travelling time between each city pair. (The data is available from the authors upon request.) The second parameter is the discount factor $\alpha$. Customarily, this parameter is taken as 0.2, 0.4, 0.6, 0.8, or 1 in the hub location literature. In our case, this parameter comes from the company we focus on. The firm representatives stated that $\alpha$ is approximately 0.9 in cargo delivery systems using highway transportation. To see the effect of the parameter $\alpha$, we also consider the cases where $\alpha = 0.8$ and 1. The last parameter is the predetermined time limit $\beta$ that restricts the total delivery time. Clearly, it is impossible to decrease $\beta$ below a certain value, determined by $\alpha$ and the travelling time between the pair of cities that are farthest apart. This limit is realized by locating hubs at these cities. For Turkey, this city pair is Hakkari and Çanakkale and the travelling time between these cities is 1950 minutes. For each value of $\alpha$, the minimum possible limits are-shown in the Table 1.

For each value of $\alpha$, we generate different instances by changing $\beta$. We start from 36 hours and decrease by decrements of 2 hours until reaching the lowest limit. We solve the model via CPLEX 8.1 on an Intel Pentium TV 1.133-GHz computer with 256 MB RAM, and 512 KB Cache. In Table 2, we present the objective function values, the optimum hub locations, and the CPU hours provided by CPLEX for each $(\alpha, \beta)$ combination. We terminate when the CPU time reaches 25 hours. For the last two instances of $\alpha = 0.8$ and 0.9 (when the limit $\beta$ is near to the lowest possible) we terminate the instances at 25 hours. For those four cases (marked with an $*$ in the table), we report (possibly) suboptimal solutions.

TABLE 1. The minimum possible time limits for each value of $\alpha$.

| $\alpha$ | Minimum possible $\beta$ (min) |
|---|---|
| 1 | $1950 \times 1 = 1950$ (32.5 hours) |
| 0.9 | $1950 \times 0.9 = 1755$ (29.25 hours) |
| 0.8 | $1950 \times 0.8 = 1560$ (26 hours) |

Observe from Table 2 that for a fixed value of $\alpha$, when $\beta$ decreases (when the limit $\beta$ becomes tighter), the CPU times increase significantly. The number of hubs also increases as expected due to the need of less circuitous routings to satisfy the time limit.

We now focus on the locations of the selected hubs with different $(\alpha, \beta)$ combinations. Figure 1 provides visual help.

One interesting observation from Table 2 is that when $\beta$ is tighter (when $p \geq 3$), Hakkari is among the selected hub sets in six of the seven instances. In the instances where Hakkari is not a hub, a nearby city, Şırnak, is in the hub set. Observe also that, for $\alpha = 0.9$ and 0.8 when $\beta$ is tightest, the cities which determine the time bound, Çanakkale and Hakkari, are among the hub sets. Another observation from the table is that in each of the 14 instances, there is always one hub from the "central" region of Turkey (from the set Amasya, Ankara, Kayseri, Sivas, Tokat). For $\beta = 2160$ and 2040 one hub is enough. For $\beta = 2160$ four of the central cities, Amasya, Kayseri, Sivas, and Tokat, could all be alternative locations for the single hub. However, when $\beta$ is decreased to 2040, only Tokat satisfies the criterion. When $\beta$ decreases further (to 1950 for $\alpha = 1.0$ and 1920 for $\alpha = 0.9$) one hub is no longer enough, and the model requires three hubs. This increase in the number of required hubs is due to the fact that the longest intercity travel time (Çanakkale-Hakkari with 1950 minutes) is already greater than $\beta$. We note here that if $\beta = 1920$, the optimum number of hubs decreases to 2 when we decrease $\alpha$ to 0.8. Also observe from Table 2 that, for fixed $\alpha$, when $\beta$ approaches its minimum value, the hubs move towards the boundaries of the country. This is to capitalize on economies of scale over longer distances.

For the cargo delivery companies the "service within 24 hours" concept is very important. We have already seen that in Turkey, it is not possible to provide service between every city pair within 24 hours (the farthest city pair is 29.25 hours apart when they have discounted travel time). In the next section we define a variation of the model to cope with this requirement. However, it appeared during our discussions with the company representatives that another important measure of service quality is the number of cities that can be reached within 24 hours from every other city in Turkey. We analyze the solutions with respect to this criterion in Table 3. Observe that if the travel time between a city and its hub, plus the departure time $DT_k$ of that hub, is less than or equal to 24 hours then any cargo to this city will be delivered within 24 hours. We derived the number of cities that can be served in 24 hours for each $(\alpha, \beta)$ combination with the hub locations given in Table 2. The results are given in Table 3.

TABLE 2. The computational results with different $\alpha$ and $\beta$ values.

| $\alpha$ | $\beta$ (minutes) | No. of hubs | Hub locations | CPU time (hours) |
|---|---|---|---|---|
| 1 | 2160 | 1 | Kayseri | 0.20 |
| | 2040 | 1 | Tokat | 0.05 |
| | 1950 | 3 | Ankara, Hakkari, Tokat | 4.55 |
| 0.9 | 2160 | 1 | Kayseri | 0.21 |
| | 2040 | 1 | Tokat | 1.99 |
| | 1920 | 3 | Amasya, Hakkari, Elaziğ | 5.03 |
| | 1800 | 5 | Afyon, Hakkari, Sivas, Tekirdağ, Tokat | 25* |
| | 1755 | 7 | Ankara, Ardahan, Çanakkale, Denizli, Hakkari, Mardin, Sivas | 25* |
| 0.8 | 2160 | 1 | Kayseri | 0.05 |
| | 2040 | 1 | Tokat | 0.84 |
| | 1920 | 2 | Ankara, Sivas | 3.79 |
| | 1800 | 3 | Ankara, Hakkari, Sivas | 11.33 |
| | 1680 | 6 | Afyon, Erzincan, Muğla, Sivas, Tekirdağ, Şırnak | 25* |
| | 1560 | 9 | Afyon, Bitlis, Çanakkale, Denizli, Erzurum, Hakkari, Hatay, Kayseri, Kocaeli | 25* |

If we use only one hub, the optimum locations are Kayseri and Tokat for $\beta = 2160$ and 2040, respectively. Although Kayseri serves more cities in 24 hours (22), it is not feasible for $\beta = 2040$. Tokat, on the other hand, is the only city which could be a hub for $\beta = 2040$, but the number of cities to receive service within 24 hours decreases to 21. We note that there are 13 cities that receive service within 24 hours from both Kayseri and Tokat. Observe from Table 3 that, for fixed $\alpha$ when $\beta$ is decreased, the number of hubs increases, usually resulting in an increase in the number of cities served within 24 hours.

In the literature, hub location models are usually tested by using standard test data, the CAB Data set [21]. The set contains the travel distances between 25 U.S. cities obtained from the Civil Aeronautics Board Survey of 1970. We also test the performance of our proposed model with this benchmark data set. Following the conventional approach, we take the number of nodes $n$ from the set {10, 15, 20, 25}. For $\alpha$ we again used 0.8, 0.9, and 1. For the parameter $\beta$ we used the values calculated for the CAB data set by Kara and Tansel [13]. The results are given in Table 4.

As can be seen from the table, the performance of the proposed model with the benchmark data set is very good because the results are obtained within seconds.

## 5. MODEL VARIATIONS

We noticed from the results of (Latest Cover) that the most populated cities of Turkey (e.g., Istanbul) are not selected as hubs (Table 2). The location literature reports that covering problems usually have alternative solutions. Thus, we wondered if there were alternative optimum solutions that would use more populated cities as hubs. We analyze this issue as the first model variation in Section 5.1. We propose another model variation in Section 5.2, in which service within 24 hours is considered.


FIG. 1. The political map of Turkey. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

TABLE 3. The number of cities that can be served within 24 hours with different $(\alpha, \beta)$ combinations.

| $\alpha$ | $\beta$ (minutes) | No. of opened hubs | The number of cities that can be served within 24 hours |
|---|---|---|---|
| 1 | 2160 | 1 | 22 |
| | 2040 | 1 | 21 |
| | 1950 | 3 | 21 |
| | 2160 | 1 | 22 |
| | 2040 | 1 | 21 |
| 0.9 | 1920 | 3 | 22 |
| | 1800 | 5 | 31 |
| | 1755 | 7 | 40 |
| | 2160 | 1 | 22 |
| | 2040 | 1 | 21 |
| 0.8 | 1920 | 2 | 31 |
| | 1800 | 3 | 33 |
| | 1680 | 6 | 42 |
| | 1560 | 9 | 54 |

### 5.1. Model Variation-I (Incorporating Weights)

In (Latest Cover) all cities have the same possibility of being a hub. However, certain cities are more "reasonable" according to the firm's managerial requirements. For these reasons, we assign weights to the cities and incorporate these weights in the objective function. The model that we propose is as follows:

$$\text{(Latest Cover-1)} \quad \min \sum_k F_k X_{kk}$$
$$\text{s.t. (1)–(4), (6)–(8)}$$

where $F_k$ is the weight factor for city $k$. This $F_k$ value reflects the "reasonability" criteria of the firm. The representatives of the firm that we focus on selected five main criteria. Three of those criteria are related to each other and taken together they quantify the desirability level of each city. These three criteria are: *industrialization level*, the *in and out cargo intensity*, and the *number of branches of the firm*. For the *industrialization level* criterion, we used data from Statistics Turkey, which provides an ordering of all cities of Turkey into five categories (very high, high, average, low, and very low). For *cargo intensity* and the *number of branches* we used the firm's numbers. The remaining two criteria are: *land price* and *highway intensity*. The *land price* criterion is included to reflect the "cost" of opening a hub. We again utilize data from Statistics Turkey which provides unit land costs in TL/m$^2$ for each city. The *highway intensity* criterion is included to capture the properties of the road network. We utilized Arcview GIS Version 3.1 to determine the quality on a three-point scale: good, average, and bad. Once these criteria and the values for each of the 81 cities were determined, we calculated the $F_k$ value for each city using MAUT (Multi Attribute Utility Theory, [14]). Ultimately, the most desirable city for the firm is the one with the lowest weight. For example, the weight for İstanbul is 0.229, whereas that of Batman is 0.781. The model (Latest Cover-1) is again solved via CPLEX with the same $(\alpha, \beta)$ combinations. In Table 5, we present the results of the two models with the same parameter settings. The truncated solutions are marked with an *.

Even though the number of hubs are similar for both of the models, the cities selected are different. For example for $\alpha = 0.9$ and $\beta = 1920$, (Latest Cover) produces Amasya, Hakkari, and Elaziğ as the three hubs whereas (Latest Cover-1) gives Ankara, Bursa, and Erzincan. Observe here that the optimum number of hubs of both models need not be identical due to different objective functions. Observe from the solutions of (Latest-Cover-1) in Table 5 that, for the $\beta$ values where the optimum number of hubs is 3, Ankara is always selected as a hub. Actually, for $\beta \leq 1950$, either Ankara or Sivas is in the selected hub set. This is mainly due to the fact that they are among the "central" cities with lower weights. We observe from Table 5 that the CPU time requirement of the model (Latest Cover-1) is less than the CPU time requirement of the (Latest Cover) for the same $(\alpha, \beta)$ combinations. This is mainly due to the fact that giving weights to possible hub locations helps branching in CPLEX. Also observe from Table 5 that with (Latest Cover-1) we get better results for the cases in which we terminated the solutions after 25 hours. For example for $\alpha = 0.9$ and $\beta = 1755$, with

TABLE 4. The computational results with the CAB Data set.

| $n$ | $\alpha$ | $\beta$ (minutes) | No. of hubs | Hub locations | CPU time (seconds) |
|---|---|---|---|---|---|
| 10 | 1.0 | 1766 | 4 | Boston, Chicago, Denver, Houston | 0.12 |
| | 0.9 | 1590 | 5 | Boston, Cincinnati, Dallas, Denver, Houston | 0.08 |
| | 0.8 | 1413 | 5 | Boston, Chicago, Dallas, Denver, Houston | 0.12 |
| 15 | 1.0 | 2600 | 3 | Boston, Los Angeles, Memphis | 0.79 |
| | 0.9 | 2340 | 3 | Boston, Los Angeles, Memphis | 0.42 |
| | 0.8 | 2080 | 4 | Boston, Kansas City, Los Angeles, Miami | 0.42 |
| 20 | 1.0 | 2600 | 3 | Boston, Kansas City, Los Angeles | 2.79 |
| | 0.9 | 2340 | 4 | Boston, Kansas City, Los Angeles, Miami | 1.36 |
| | 0.8 | 2118 | 5 | Boston, Chicago, Denver, Los Angeles, New Orleans | 2.09 |
| 25 | 1.0 | 2725 | 6 | Cincinnati, Memphis, Miami, Phoenix, San Francisco, Seattle | 0.61 |
| | 0.9 | 2453 | 6 | Cincinnati, Miami, Phoenix, St. Louis, San Francisco, Seattle | 0.41 |
| | 0.8 | 2307 | 6 | Cincinnati, New Orleans, Phoenix, San Francisco, Seattle, Tampa | 0.34 |

TABLE 5. The comparison of (Latest Cover) and (Latest Cover-1) with different values of $\alpha$ and $\beta$.

| $\alpha$ | $\beta$ (minutes) | (Latest Cover) The locations and the number of hubs | (Latest Cover) CPU time (hours) | (Latest Cover-1) The locations and the number of hubs | (Latest Cover-1) CPU time (hours) |
|---|---|---|---|---|---|
| 1 | 2160 | Kayseri (1) | 0.20 | Kayseri (1) | 0.02 |
| | 2040 | Tokat (1) | 0.05 | Tokat (1) | 0.05 |
| | 1950 | Ankara, Hakkari, Tokat (3) | 4.55 | Ankara, Diyarbakır, Erzurum (3) | 2.81 |
| 0.9 | 2160 | Kayseri (1) | 0.21 | Kayseri (1) | 0.02 |
| | 2040 | Tokat (1) | 1.99 | Tokat (1) | 0.88 |
| | 1920 | Amasya, Hakkari, Elaziğ (3) | 5.03 | Ankara, Bursa, Erzincan (3) | 3.52 |
| | 1800 | Afyon, Hakkari, Sivas, Tekirdağ, Tokat (5) | 25* | Bursa, Denizli, İstanbul, Sivas, Şırnak (5) | 25* |
| | 1755 | Ankara, Denizli, Sivas, Mardin, Çanakkale, Ardahan, Hakkari (7) | 25* | Afyon, Çanakkale, İstanbul, Diyarbaku, Hakkari, Sivas (6) | 25* |
| 0.8 | 2160 | Kayseri (1) | 0.05 | Kayseri (1) | 0.02 |
| | 2040 | Tokat (1) | 0.84 | Tokat (1) | 0.32 |
| | 1920 | Ankara, Sivas (2) | 3.79 | Ankara, Malatya (2) | 1.41 |
| | 1800 | Ankara, Hakkari, Sivas (3) | 11.33 | Ankara, Diyarbakır, Erzurum (3) | 9.25 |
| | 1680 | Afyon, Erzincan, Muğla, Sivas, Tekirdağ Şırnak (6) | 25* | Ankara, Elaziğ, Erzurum, Hakkari (4) | 25* |
| | 1560 | Afyon, Bitlis, Çanakkale, Hatay, Denizli, Erzurum, Hakkari, Kayseri, Kocaeli (9) | 25* | Ankara, Çanakkale, Denizli, İstanbul, Hakkari, Kayseri, Diyarbakır, Erzurum (8) | 25* |

*Denotes truncated solutions.

(Latest Cover-1) we find a solution with six hubs, whereas with the original model, we had stopped at a solution utilizing seven hubs.

### 5.2. *Model Variation-II (Service within 24 hours)*

We noted in Section 4 that "the service within 24 hours" concept is important for cargo delivery companies, and that it is impossible for a company to promise service within 24 hours for each city within Turkey unless airlines are used. Nevertheless, the companies want to promise delivery within 24 hours for certain city pairs. This would mean that these city pairs are served within 24 hours; the rest of them are served within a longer time limit, usually 48 hours, by the next day's vehicle. Thus, there are actually two different $\beta$ limits; service within certain pairs is provided within-limit $\beta_1$, whereas the rest of the city pairs are served within limit $\beta_2$ where $\beta_2 > \beta_1$. In (Latest Cover) the departure times were set so that each vehicle waited for all the incoming vehicles. In the present variation, the departing vehicles will wait for *some* of the arriving vehicles, but cargo from the rest will be carried by another vehicle departing at a later hour that satisfies the looser $\beta$ limit. The question arises: which departing vehicles should wait for which arriving vehicles?

Our discussions with the representatives of the studied firm indicate that they do not want to assign two different vehicles for hub-to-hub transportation because those vehicles are more costly. Thus, the vehicles departing toward other hubs should still wait for all the incoming vehicles from the origins that the hub serves; that is, the definition of $\widehat{DT}_k$ given in Section 3 is still valid. The second vehicle is justified for hub-to-destination deliveries. Remember that the departure time from hub $k$ to all of its destinations, denoted by $DT_k$, is determined by the latest arriving vehicle at that hub (including the ones coming from the other hubs). Now, to improve service quality, two different vehicles will depart from the hub towards each destination. We need to introduce $DT1_{kj}$ and $DT2_{kj}$ for the departure time from hub $k$ towards destination $j$. The first vehicle departing at $DT1_{kj}$ will finish the deliveries within the $\beta_1$ time limit and the second vehicle departing at $DT2_{kj}$ will do the deliveries within the $\beta_2$ limit. To finish the deliveries within a tighter time frame, the first vehicle should not wait for all of the arriving vehicles. Note that because the departing vehicles do not wait for all the incoming vehicles, in this model the departure time will be dependent on the destination index, $j$. For destination $j$, we require $DT1_{kj} + c_{jk} \le \beta_1$ if $j$ is served by the hub at $k$. Remember from constraint (4) that $DT_k \ge \widehat{DT}_r + \alpha c_{rk} X_{rr}$. Now we will have $DT1_{kj} \ge \widehat{DT}_r + \alpha c_{rk} X_{rr} (4')$, and we will impose this constraint only for certain $(r, k, j)$ triplets. The triplets of the constraint $(4')$ will determine for each hub $k$, towards each destination $j$, which of the arriving vehicles $r$ should be waited for. It can be seen that the constraint should be imposed when $\widehat{DT}_r + \alpha c_{rk} + c_{jk} \le \beta_1$, that is for the triplets for which $\alpha c_{rk} + c_{jk} \le \beta_1 - \widehat{DT}_r$. However, $\widehat{DT}_r$ is also a variable and so an output of the model. Thus, we need to define a new parameter $\gamma$ as an upper limit on $\widehat{DT}_r$ and impose the constraint $(4')$ when $\alpha c_{rk} + c_{kj} \le \beta_1 - \gamma$. The mathematical model for the stated problem is as follows:

(Latest Cover-2-0)

$$\min \sum_k X_{kk}$$

s.t.

$$DT1_{kj} \ge \widehat{DT}_r + \alpha c_{rk} X_{rr} \quad \forall r,k,j \quad \text{if } \alpha c_{rk} + c_{kj} \le \beta_1 - \gamma \tag{9}$$

$$(DT1_{kj} + c_{jk})X_{jk} \le \beta_1 \quad \forall k,j \tag{10}$$

$$DT2_{kj} \ge \widehat{DT}_r + \alpha c_{rk} X_{rr} \quad \forall r,k,j \quad \text{if } \alpha c_{rk} + c_{kj} > \beta_1 - \gamma \tag{11}$$

$$(DT2_{kj} + c_{jk})X_{jk} \le \beta_2 \quad \forall k,j \tag{12}$$

$$DT1_{kj}, DT2_{kj}, \widehat{DT}_k \ge 0 \quad \forall k,j \tag{7'}$$

(1)–(3), (6)

Constraints (9) and (11) determine the vehicle departure times from hub $k$ towards destination $j$. Constraints (10) and (12) force the total delivery time to be within the corresponding time limits. Note that the right-hand side of constraints (9) and (11) seem to be independent of index $j$. However, these variables do depend on $j$ because the corresponding "if statements" include index $j$.

(Latest Cover-2-0) is nonlinear due to constraints (10) and (12). To linearize the model, constraints (10) and (12) can be replaced by constraints (13) and (14), respectively. The resulting model is:

(Latest Cover-2)

$$\min \sum_k X_{kk}$$

s.t.

$$\mathrm{DT1}_{kj} + c_{jk}X_{jk} \le \beta_1 X_{jk} + \beta_2(1 - X_{jk}) \quad \forall r,k,j \quad \text{if } \alpha c_{rk} + c_{kj} \le \beta_1 - \gamma \tag{13}$$

$$\mathrm{DT2}_{kj} + c_{jk}X_{jk} \le \beta_2 \quad \forall r,k,j \quad \text{if } \alpha c_{rk} + c_{kj} > \beta_1 - \gamma \tag{14}$$

(1)–(3), (6), (7′), (9), (11)

The correctness of this linearization can be justified by the following observation:

**Observation 2.** (13) and (14) correctly linearize the constraints (10) and (12), respectively.

**Proof.** First observe that $\beta_2 > \beta_1$ and all the delivery times should be within $\beta_2$. Thus, (14) is a correct linearization. For (13), if $X_{jk} = 1$ both (10) and (13) yield the same right-hand sides. If $X_{jk} = 0$ then (10) yields $0 \le \beta_1$, whereas (13) yields $DT1_{jk} \le \beta_2$. As mentioned before putting a $\beta_2$ limit on any of the variables should not result in suboptimal solutions. ∎

(Latest Cover-2) is a linear mixed integer program with $n^2$ binary and $2n^2 + n$ real variables and $4n^3 + 2n^2 + n$ constraints.

In real life, the second departure time is usually 24 hours (=1440 minutes) after the first departure time (the next day's vehicle). We tested the computational performance of (Latest Cover-2) with the parameters $\alpha = 0.9$, $\beta_1 = 1440$ minutes (24 hours), $\beta_2 = 2880$ minutes (48 hours). For the parameter $\gamma$, we used 8 and 6 hours to see the performance of the model. We again terminate the instances after 25 hours.

TABLE 6. The computational results of (Latest Cover-3).

| | $\gamma$ (hours) | CPU (seconds) | No. of hubs |
|---|---|---|---|
| $\alpha = 0.9$ | 4 | 19.68 | 13 |
| $\beta_1$ = 24 h | 5 | 20.56 | 10 |
| $\beta_2$ = 48 h | 6 | 20.14 | 10 |
| | 7 | 23.01 | 6 |
| | 8 | 22.86 | 4 |

For $\gamma = 8$ hours, the model stopped at a solution with 10 hubs and for $\gamma = 6$ hours the model resulted in a solution with 13 hubs. Both of the solutions are possibly suboptimal, since they are truncated.

Observe here that we needed to include an additional parameter $\gamma$ to differentiate the departure times. The parameter $\gamma$ actually puts a limit on travel time from the demand centers to their assigned hubs. When we discussed this parameter with the firm representatives, we learned of a legislative restriction stating that a commercial driver can travel no more than 6 hours continuously. This time limit is considered for only nonhub-to-hub transportation, because hub-to-hub vehicles usually have two assigned drivers. In (Latest Cover-2), even though we define the parameter $\gamma$, we do not impose it as a constraint on the departure times $\widehat{DT}_r$. In the following model, we will restrict the model so that travel time between a nonhub and its hub finishes within the $\gamma$ bound.

(Latest Cover-3) $\min \sum_k X_{kk}$

s.t.

$$c_{jk}X_{jk} \le \gamma \qquad \forall j,k \tag{15}$$

(1)–(3), (6), (7′), (9), (11), (13), (14)

We tested the performance of the model for $\alpha = 0.9$, $\beta_1 = 24$ hours, $\beta_2 = 48$ hours and with different legislation parameters $\gamma$ (the legally determined 6 hours plus 4, 5, 7, and 8 hours). Table 6 summarizes the results.

Observe from Table 6 that the inclusion of constraint (15) improved the CPU times drastically. Within seconds we get results for all of the instances. Remember that we needed to terminate CPLEX after 25 hours for (Latest Cover-2). For $\gamma = 8$ hours, without constraint (15), the model was terminated after 25 hours with a solution using 10 hubs. As can be seen from Table 6, when we incorporate constraint (15), the model results in a solution using four hubs within half a minute.

As constraint (15) arises from a legislative requirement, this constraint is also valid for the original model (Latest Cover) developed in Section 3. To observe the effect of this constraint, we appended constraint (15) to (Latest Cover) and tested the performance of the new model with the instances for which $\alpha = 0.9$, $\gamma = 6$ hours (the legal value) and with the tightest three $\beta$ values (1920, 1800, and 1755). The results are given in Table 7.

TABLE 7. The comparison of (Latest Cover) with and without constraint (15).

| | (Latest Cover) | | (Latest Cover) with (15) where $\gamma = 6$ h | |
|---|---|---|---|---|
| | The location and the number of hubs | CPU time | The location and the number of hubs | CPU time |
| $\alpha = 0.9$ $\beta = 1920$ | Amasya, Hakkari, Elaziğ (3) | 5.03 h | Bitlis, Çorum, Erzurum, Kocaeli, Uşak, Osmaniye (6) | 7.64 s |
| $\alpha = 0.9$ $\beta = 1800$ | Afyon, Hakkari, Sivas, Tekirdağ, Tokat (5) | 25* h | Çorum, Erzurum, Hakkari, Kocaeli, Uşak, Batman, Osmaniye (7) | 35.7 s |
| $\alpha = 0.9$ $\beta = 1755$ | Ankara, Çanakkale, Hakkari, Ardahan, Denizli, Sivas (7) | 25* h | Adana, Çorum, Diyarbakır, Erzurum, Çanakkale, Denizli, Hakkari, Sakarya (8) | 41.1 s |

Observe from Table 7 that there is again a significant decrease in the CPU times. For the cases where $\beta$ equals 1800 and 1755 (Latest Cover) was terminated after 25 hours with 5 and 7 hubs. However, when we restrict the nonhub–hub transportation time, the model produces an optimum solution within seconds. However, the number of hubs increases with this new model. Even though the $\beta$ limit can be satisfied with (say) 3 hubs for $\beta = 920$, constraint (15) cannot be satisfied and the model needs to open three more hubs.

Recall that we have a variant of the model in which the cities have different "desirability coefficients": (Latest Cover-1). We also appended the constraint (15) to (Latest Cover-1). The results are given in Table 8. The observations that can be derived from Table 8 are very similar to those of Table 7. The inclusion of constraint (15) decreases the CPU times significantly (more than 25 hours versus 11 seconds). However, the required number of hubs increases to satisfy the 6-hour limit (constraint 15).

We note here that we continue to present the (Latest Cover) as the basic model because constraint (15) may be country specific, whereas the model (Latest Cover) is more general.

## 6. EXISTING STRUCTURE AND COMPARISONS

In this section, we compare the current structure of the firm with the results of the *Latest Arrival Hub Covering* models proposed in the previous sections. In the current structure, the firm has 26 hubs in 25 cities (one in each of the Anatolian and European sides of İstanbul). The reason for hubs,

TABLE 8. The comparison of (Latest Cover-1) with and without constraint (15).

| | | (Latest Cover-1) | | (Latest Cover-1) with (15) | |
|---|---|---|---|---|---|
| | | No of hubs | CPU time | No of hubs | CPU time |
| $\alpha = 0.9$ | $\beta = 1920$ | 3 | 3.52 h | 7 | 2.54 s |
| $\gamma = 6$ h | $\beta = 1800$ | 5 | 25* h | 7 | 10.36 s |
| | $\beta = 1755$ | 7 | 25* h | 8 | 9.59 s |

which seems very ineffective to us, comes from the managerial structure of the firm. The hub location policy is based on locating hubs in cities where there is a regional directorate. The firm has 34 region directorates in 25 cities: six in İstanbul, three in Ankara, and three in İzmir.

We provided eight alternative solutions based on our models, and compared these solutions with the current structure of the firm in terms of service quality (number of cities receiving service within 24 hours) and cost. We fixed the parameter $\alpha = 0.9$ (as this is the realized value). In the first two proposed solutions, we used the results of (Latest Cover) with $\beta = 1920$ and $\beta = 1800$ because they are the minimum possible time limits for the corresponding value of $\alpha$. For the third and fourth proposed solutions, we used the results of (Latest Cover-1), for the fifth and sixth proposed solutions, we used the results of the model (Latest Cover) with constraint (15) appended, and finally for the seventh and eighth proposed solutions we used the results of (Latest Cover-1) with constraint (15). For each model, we set $\beta = 1920$ and $\beta = 1800$.

We first wanted to conduct a cost-based analysis. For each solution, there is a fixed setup cost and an operational cost. The fixed setup cost is a function of the number of hubs and it is usually very high compared to the operational cost. Thus, we decided to use operational cost in the comparison; for the operational cost we focus on the routing costs only. The routing cost is a function of the distances travelled and the number of vehicles travelling. To calculate the number of vehicles and the diesel fuel cost, we needed to define additional parameters: we needed to find the correct cargo values (between each origin–destination pair) to calculate the number of vehicles required. However, we were only able to get estimates for the total outgoing cargo of each city. Let $w_i$ be the amount of flow that originates from an origin $i$. Define $cap1$ as the amount of flow that can be transported on a vehicle used for nonhub–hub transportation. Then the number of vehicles required from origin $i$ to its hub is

$$v_i = w_i/cap1.$$

Let $cap2$ denote the amount of flow that can be transported in the large sized vehicles used in hub-to-hub transportation.

TABLE 9. The comparison of the alternative solutions and the firm's current structure.

| | | The location and the number of hubs | No. of cities that can be served within 24 hours | The diesel fuel cost (billion TL.) |
|---|---|---|---|---|
| Article I. | Current structure of the firm | 25 | 34 | 243.1 |
| Proposed solution-1 | (Latest Cover) $\beta = 1920$ | Amasya, Hakkari, Elaziğ (3) | 22 | 280.0 |
| Proposed solution-2 | (Latest Cover) $\beta = 1800$ | Afyon, Hakkari, Sivas, Tekirdağ, Tokat (5) | 31 | 166.5 |
| Proposed solution-3 | (Latest Cover-1) $\beta = 1920$ | Ankara, Bursa, Erzincan (3) | 9 | 155.0 |
| Proposed solution-4 | (Latest Cover-1) $\beta = 1800$ | Bursa, Denizli, İstanbul, Sivas, Şırnak (5) | 23 | 142.6 |
| Proposed solution-5 | (Latest Cover) + const. (15) $\beta = 1920$ | Bitlis, Çorum, Erzurum, Kocaeli, Osmaniye, Uşak (6) | 29 | 119.4 |
| Proposed solution-6 | (Latest Cover) + const. (15) $\beta = 1800$ | Batman, Çorum, Erzurum, Hakkari, Kocaeli, Osmaniye, Uşak (7) | 30 | 126.9 |
| Proposed solution-7 | (Latest Cover-1) + const. (15) $\beta = 1920$ | Ankara, İstanbul, Bitlis, Çorum, Denizli, Erzurum, Gaziantep (7) | 25 | 97.23 |
| Proposed solution-8 | (Latest Cover-1) + const. (15) $\beta = 1800$ | Çorum, Denizli, Diyarbakır, Erzurum, Hakkari, İçel, Kocaeli (7) | 35 | 100.12 |

To determine the number of vehicles required in hub–hub transportation, we assumed that the total flow arriving at any hub will be distributed towards other hubs based on the populations of the destination hubs. That is, the number of vehicles required between hubs $k$ and $r$ is

$$v_{kr} = \frac{\frac{pop_r}{\sum_{j \neq k} pop_j} \sum_i w_i X_{ik}}{cap2}.$$

In the formula of $v_{kr}$, the total flow into hub $k$ is first determined, and then apportioned according to the population of the destination hub $r$.

Let $d_{ik}$ be the distance between nodes $i$ and $k$. The company representatives provided the diesel fuel cost as 292.500 TL/km (≅20 cents/km). The total diesel fuel cost DFC is calculated via the following formula.

$$\text{DFC} = 292.500 * \left[ 2 * \left[ \sum_i \sum_k v_i d_{ik} X_{ik} + \sum_k \sum_r v_{kr} d_{kr} X_{rr} X_{kk} \right] \right]$$

We now provide a summary table (Table 9) for the current structure of the firm with 25 hubs, and for the eight proposed solutions. We report the "number of cities served within 24 hours" and the total DFC for each alternative.

It is evident from Table 9 that proposed solution 8 dominates proposed solutions 1, 2, 3, 4, 5, and 6 in terms of both total diesel fuel cost (DFC) and the number of cities served within 24 hours. Proposed solution 7 has the least routing cost. However, the number of cities served within 24 hours is only 25. On the other hand, the proposed solution 8 serves 35 cities in 24 hours (even more than the firm's current structure) by increasing the diesel costs by approximately 3%. The proposed solution 8 is the only alternative that dominates the firm's current structure both in terms of DFC and the number of cities served within 24 hours. The total routing cost will decrease approximately by 59%.

Table 9 also signals that the current hub locations of the firm can be improved in terms of total routing costs. All the proposed solutions except proposed solution 1 give smaller costs than the current structure. However, the number of cities receiving service in 24 hours (which is a measure of service quality) is good at the firm's current structure. Actually, this is expected because the current structure uses 25 hubs. The proposed solution 8, on the other hand, captures 35 cities with only seven hubs and also with less routing cost. We remark here that the firm representatives appreciated the suggested solutions and decided to thoroughly investigate their current service network.

## 7. CONCLUSIONS AND REMARKS

In this article we study the *Latest Arrival Hub Covering Problem*, which is the hub location problem encountered by cargo delivery systems. Our interviews with different cargo delivery companies operating in Turkey justify the correctness of the *Latest Arrival Hub Location Problem* proposed initially by Kara and Tansel [12] for cargo delivery companies. We propose a linear integer program for the covering version of the problem because delivering the cargo within a limited time interval seems to be an important aspect for the cargo delivery companies.

To increase the applicability of the model, we proposed two different variations. The first variation of the model incorporates weights for each alternative hub location; this constitutes the "weighted latest arrival hub covering problems." To calculate the appropriate weights, we had several discussions with the real decision makers and derived weights according to certain criteria they had developed. We utilized the Multiattribute Utility Theory to combine all the criteria into a single weight. The computational performance of this model was slightly better than that of the original model. As expected, the outputs of the model were also more reasonable for our decision makers.

The second variant of the model reflects the real-life requirement that some of the cargo may have to wait for the "next day's vehicle." We formulated an integer program that would consider two different service schedules. A special case of the proposed model arises when the second deadline is 24 hours after the first. We also observed a legislative requirement putting a limit on the driving time of a commercial driver. Inclusion of this constraint (15) into the models improved the CPU times significantly.

We have tested all the models proposed on an 81-node network, namely the Turkish postal network. A comparison of the firm's current structure with the results of the proposed models shows that large reductions in cost can be achieved (59%).

In summary, in this research we have clarified the structure of cargo delivery systems and we have proposed mathematical models specific to the cargo delivery sector. We then implemented the models on a large-scale (81-node) network to get solutions. We remark here that the models proposed are somewhat realistic but the LP bounds of all the models are weak. Thus, all the models proposed are open to improvement in strengthening the bounds.

One deficiency of the proposed models is that cost does not appear in the integer programs. Even though the cargo delivery companies are time-sensitive rather than money-sensitive, ultimately they must remain economically viable. We do provide a cost based analysis but it is actually a byproduct of our models. Somehow incorporating the cost in the mathematical models would be a better approach, one that is in the immediate research agenda of the authors. A second future research direction is the inclusion of different modes of transportation. As observed in Section 4, 24-hour delivery is not possible for every city pair within Turkey without using airlines; therefore, the authors aim to include the airline transfer possibility in the integer programs.

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