Dynamic Heckscher-Ohlin Results from a 2x2x2x2 Overlapping Generations Model with Unequal Population Growth Rates[*]

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ABSTRACT

This paper considers a two-country world where the population in one country grows faster than the other, and investigates the implications of the addition of non-stationary population dynamics to a simple 2-commodity, 2-factor model of international trade within an overlapping-generations framework. The two countries in the world considered are assumed to be identical in every respect except, for their population growth rates initially. The effects of differential speed of population growth on relative factor endowments and patterns of international trade are explored by comparing simulation results obtained from the overlapping-generations general equilibrium model under autarky and trade scenarios. Unequal population growth rates are shown to give rise to differentials in wage rates and rentals for capital under autarky conditions. This, in turn, causes costs of production and relative prices to differ, creating the grounds for trade in the sense of Heckscher-Ohlin (HO). Yet, the results from simulation exercises indicate that static welfare results from the standard 2x2x2 HO model can not be generalized to hold in a dynamic setting with overlapping generations of individuals.

Key Words : Unequal population growth rates, Heckscher-Ohlin model, international trade, overlapping-generations

JEL Classification : F11, F43, D58, D91, J10

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I. Introduction

The standard two-country, two-commodity and two-factor (2x2x2) Heckscher-Ohlin (HO) model of international trade views the differences in relative factor endowments across countries and differing factor intensities in the production of commodities as the primary determinants of comparative advantages of nations. This simple model has provided the grounds for a vast literature of theoretical and empirical studies exploring its implications or testing the validity of its predictions.

By comparison, few studies have attempted to extend the standard Heckscher-Ohlin model into a dynamic framework to explain long-run determinants of comparative advantages. Chen (1992) notes that the lack of such studies was probably due to the fact that trade between two countries that are identical in every respect but the initial capital/labor ratios (relative factor endowments) would come to an end in the long-run, once these ratios have been equalized. So, trade could continue to occur in the long-run only in the presence of additional differences which would cause factor proportions to evolve (and comparative advantages to change) over time. Population growth rate differentials across countries could lead to such an evolution in relative factor endowments –thereby serving as long-run determinants of comparative advantages. Even though some studies in the dynamic trade literature recognized this role of unequal rates of growth in population as early as 1965, this issue has not received much attention in the literature (Deardorff, 1987; Chen, 1992).

Recent demographic projections by the UN and others indicate that the effects of unequal population growth rates on trade deserve and are likely to receive more attention from now on. By these projections, the dissimilarities already existing between relative factor endowments of capital-abundant nations (in North America, Europe and East Asia) and the rest of the world are expected to grow in the decades ahead. If marked differences in the pace of reductions in fertility (and hence, population growth) rates in the relatively capital- and labor-abundant parts of the world are to remain as projected by demographers, not only the labor forces will become more divergent over the next few decades, but also, greater variations will be observed in the age profiles of populations due to the differential speed of population aging in these two areas. Since changes in the age composition of a nation imply changes in relative factor endowments, differences in population growth rates potentially emerge as major determinants of commodity and factor flows across national borders.

In fact, population age pyramids already began to differ significantly (see, for example, McFalls, 1998). Most members of the OECD, for instance, have witnessed a rapid decline in fertility over the past few decades and this has significantly lowered population growth rates. Coupled with the gradual increase in life expectancies, the reductions in population growth rates have already caused the population share of elderly to increase visibly in these nations. This aging of populations is projected to continue well into the 21st century and is expected to alter relative abundance of labor and capital, both by decreasing labor supply and by slowing down capital formation (as a result of the expected decline in savings).[1] Consequently, the relative commodity and factor prices in these countries will change, sooner than the rest of the world, affecting worldwide

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[1] While the resulting change in relative endowments may be in either direction depending upon the magnitude of the effects on each of capital formation and labor supply, Auerbach et.al. (1989) predict an increase in economywide capital-labor ratios in the economies where population aging has already set in.

patterns of trade and factor movements.[2] Exploring the direction and magnitude of likely changes in trade and factor flows requires developing an analytical framework that adds the demographics and hence, the time dimension to the HO model.

The purpose of this paper is to investigate the implications of the addition of non-stationary population dynamics to a simple 2x2x2 model of international trade through simulation experiments carried out within a computable general equilibrium framework, by allowing each country to be populated by two overlapping generations of individuals. The paths that capital per worker and other endogenous variables follow between the initial state and the long-run equilibrium are then simulated under autarky and trade scenarios by assuming that population growth rates differ across countries.

The results obtained from simulations under trade scenario are generally but not invariably consistent with the static HO framework. Consistently with the predictions of the static HO model, the country that is relatively better endowed with a factor specializes in the production of the commodity requiring more intensive use of that factor, and exports that commodity. Contrary to those predictions, however, trade would not necessarily lead to welfare gains for both countries and might not even be Pareto-superior to autarky, as previously suggested in various studies based on OLG models with stationary populations. Galor (1988a, 1988b), for example, showed that free trade may cause a Pareto deterioration for a small country in the absence of international transactions in financial assets. The results in Fried (1980) also revealed that free trade may make everyone in at least one country worse-off as compared to autarky under

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[2] In a non-technical, yet very insightful account of these implications, Peterson (1999) notes that the investigation of the indirect effects of aging populations in the developed world on other countries is one of the critical areas where more research is severely needed.

certain conditions. Mountford (1998) shows how the addition of a simple dynamic structure to the standard convex 2x2x2 Heckscher-Ohlin model can reverse the static implications of international trade in the long-run. So, the results in this paper add unequal population growth rates to the list of reasons explaining why trade may not improve welfare for both parties, in a dynamic, OLG set-up. They also provide the grounds for a rich discussion on possible consequences of the differential speed of demographic transition, fertility decline and population aging in various parts of the world.

The rest of the discussion is organized as follows. The next section briefly surveys the relevant literature. Section III describes the modeling framework, while Section IV presents results and discusses their robustness. Section V concludes the paper by assessing the implications of results.

II. Previous Literature

While the effects of population growth on trade have been investigated in various studies in the North-South trade literature, these studies employed Ricardian models that take international differences in technology (rather than differences in relative factor endowments) as the basis for trade (see Matsuyama, 2000, for an example and a review). Furthermore, these studies did not consider the paths that endogenous variables would follow over time, and examined, instead, the implications of population growth for North-South trade equilibria resulting under various assumptions.

A few studies considered cross-country differences in the speeds of population growth as the source of the evolution in relative factor endowments and studied dynamic trade issues by using growth models. An early and notable attempt in this direction came from Oniki and Uzawa (1965) who extended the two-sector growth model to a two-country setting to study the effects of capital accumulation and labor force growth on international equilibrium over time. Findlay (1970) added a non-traded capital good sector to the Oniki-Uzawa model and studied the behavior of relative factor endowments, by considering a small country facing fixed world terms of trade. He concluded that the long-run pattern of comparative advantage depends ultimately on the propensity to save and the growth rate of labor force. In addition to multi-sector growth models in Oniki and Uzawa (1965) and Findlay (1970), Deardorff (1987) used a one-sector, two-country growth model to investigate what may happen to relative factor endowments over time, if population in one of the countries grows at a higher rate than in the other. Since this one-sector growth model is not suitable for analyzing the long-run commodity composition of trade, however, Deardorff is forced to infer what would happen to trade by linking changes in factor proportions to static HO results from a previously developed model with many goods (Deardorff, 1979). This, of course, amounts essentially to projecting static HO results into a dynamic framework, and hence, could be viewed only as an intuition development exercise to use in the absence of a multi-commodity trade model that is truly dynamic in nature. This treatment lacks rigor and may be potentially misleading, since static HO results do not directly lend themselves to long-run analysis.

Observing changes in relative endowments and the resulting changes in patterns of trade (and factor movements) over time requires developing a dynamic model with at least two commodities and two factors. Even though such a model will be capable of indicating directions and magnitude of changes in trade flows in response to changes in capital/labor ratios, it will not be able to show the effects of changes in the age composition of population on relative endowments. To address these effects, an overlapping generations general equilibrium (OLG-GE) model will be needed.

An early attempt in this direction came from Fried (1980) who used a simple OLG-GE model with two commodities and two generations and compared steady state solutions under free trade and autarky, by assuming zero population growth and a fixed supply of land (and no other factor of production). His results revealed that free trade may make at least one country worse-off relative to autarky under certain conditions. Soon after that, Buiter (1981) showed, using a simple OLG model with one commodity and a variable capital stock, that the differences in time preference rates between two populations (growing at the same rate) are sufficient to induce mobility of capital across countries. Galor (1986) used the OLG approach to model a dynamic migration scheme. Eaton (1987) developed a dynamic model of international trade by considering land and capital as fixed factors of production which, as in Fried (1980), also serve as assets used to transfer income from working periods to retirement. Galor and Lin (1994) analyzed the effects of terms-of-trade deterioration on the current account of a small open economy. The two-sector OLG model developed by Galor (1992) was extended to a two-country set-up by Mountford (1998) and Guilló (1999 and 2001). Guilló (1999) formulated production and capital formation as in Galor (1992) and investigated the effect of relative factor intensities on the relationship between the terms of trade and the capital stock. Mountford (1998) studied different implications of a dynamic structure for the HO model, compared the results obtained in such a dynamic set-up to static HO results in the literature, and discussed their implications for the convergence literature. Neither of these studies, however, allowed for increases in populations of respective nations.

In addition to these theoretical works, numerous applied OLG-GE models have been constructed following Auerbach and Kotlikoff's seminal work (1987), aiming to develop a new framework to examine dynamic effects of fiscal policy on capital formation, growth and intergenerational equity. Recently increased interest in the effects of population aging has made the framework developed by Auerbach and Kotlikoff (1987) very popular for investigations of the implications of changes in old-age dependency ratios for labor supply, savings and fiscal balances. These models have proved particularly useful for analyzing the effects of changes in such age-dependent categories of public expenditures as social security, medicare or education (see Miles, 1999; Joines, et.al, 1999, and Fougère and Mérette, 2000, for examples and surveys of the literature). Most of these applied OLG-GE works assumed closed economies or considered the behavior of a small open economy in a single country setting (see, for example, Raffelhüschen and Risa, 1995; Storesletten, 2000).

Due possibly to computational difficulties involved in solving large scale OLG-GE models in multi-country settings, relatively few studies have put the framework developed by Auerbach and Kotlikoff (1987) into use for studying international commodity and factor flows. A particularly notable example to the use of multi-country/region settings is provided by Juillard and INGENUE Team (2001) where international capital flows induced by differential demographic dynamics in six regions of the world are studied by letting the world interest rate be determined endogenously. Despite the extensive regional coverage of the model, Juillard and his colleagues considered a single commodity, and avoided the discussion of trade patterns, by solely focusing on capital flows instead. In a two-region model, Kenc and Sayan (2001) considered the EU and Turkey together and investigated the transmission of the effects of population aging in Europe onto a small open economy with a young population through international trade and capital flows, by allowing for two tradable and one non-tradable commodity. While both studies recognize the role of differences in the timing of demographic transition in inducing commodity and factor flows across borders, neither model explicitly accounts for international differences in population growth rates.

III. Simulation Framework

The models used here to simulate the long-run autarky equilibrium in each of the two countries considered are similar in spirit to the simple OLG-GE model in Auerbach and Kotlikoff (1987) but they extend it in two directions. First, the autarky models used here have two commodities, instead of one, to allow for the observation of trade patterns. Secondly, unlike the model in Auerbach and Kotlikoff (1987), population in each country is allowed to grow at a positive rate.

The long-run equilibrium to be reached after the opening of trade between these two countries is captured through a separate model that is built on the standard HO assumptions except for differences in initial factor endowments and growing populations so as to highlight the implications of the inequality of population growth rates. Thus, both countries are assumed to produce the same two commodities by employing identical production technologies for each commodity, as in the basic HO model. Likewise, preferences of the corresponding generations are assumed to be the same in both countries. While initial endowments are also assumed to be the same across countries, relative endowments begin to deviate immediately after the first period, due to the differing rates of growth in population and hence, labor supply.

The rest of this section describes first the prototype autarky model and then, the two-country model used to solve for the long-run equilibrium under free trade. Earlier versions of this model were used by Sayan and Uyar (2001) for investigation of trade and labor flows between two nations with differing age compositions, and by Sayan for investigation of trade patterns (2001) and to address trade and migration issues (2002) in a two-country world with unequal population growth rates.

III.1. Prototype Autarky Model

The model used to describe the long-run autarky equilibrium in each country is an infinite horizon OLG-GE model with perfect foresight. Since the countries considered (country A and country B) are assumed to be exactly the same in every respect but the population growth rate, n, it would be sufficient to list the assumptions that are common to both countries and write the equations without the superscripts to distinguish countries.

Each country is populated by individuals who live for two periods. At an arbitrary period t in time, two types of individuals are alive in each country: ‘Youngs’ who were just born and are living the first period of their lives, and ‘olds’ who were born in the previous period and are living the last period of their lives. Each young is assumed to have $(1+n)$ children during the first period of life. Furthermore, the individuals work only when young, each inelastically supplying a fixed amount of labor, and retire in the second period of their lives. Production, exchange and payments are assumed to be made at the end of each period. So, a young born in period t bears children and works in the same period, earning a wage of w_t at the end of that period and decides how much to consume and how much to save. In period $t+1$, he makes his savings, A_{t+1}, available to firms and earns $r_{t+1} \cdot A_{t+1}$, where r_{t+1} is the rental rate on capital in period $t+1$. He consumes all of $(1+r_{t+1}) \cdot A_{t+1}$ in period $t+1$, since no intergenerational transfers (such as bequests, gifts, etc.) are allowed.

Letting η_t denote the number of individuals born in period t, one can write

$$\eta_{t+1} = (1+n) \cdot \eta_t \tag{1}$$

where n denotes the population growth rate.

Within this framework, production and consumption decisions in each economy are characterized as follows.

Production Side

Two commodities indexed by i $(i \in \{1,2\})$ are produced by using capital (K) and labor (L) according to constant returns to scale Cobb-Douglas technologies as in Deardorff (1987), Auerbach and Kotlikoff (1987) or Juillard and INGENUE Team (2001). The parameters of production functions are different across sectors but the same across countries for each sector. Thus, total sectoral outputs at time t, X_{it}, can be expressed in per worker terms as

$$x_{1t} = (k_{1t})^{\alpha} (l_{1t})^{1-\alpha} \tag{2}$$

$$x_{2t} = (k_{2t})^{\beta} (l_{2t})^{1-\beta} \tag{3}$$

where $x_{1t} = \frac{X_{1t}}{\eta_t}$ and $x_{2t} = \frac{X_{2t}}{\eta_t}$; $k_{it} = \frac{K_{it}}{\eta_t}$ is the amount of capital per worker in sector *i*, and $l_{it} = \frac{L_{it}}{\eta_t}$ is the amount of labor supplied by each worker in sector *i*. Since total labor supply at time *t*, L_t, is given by $L_t = \eta_t \cdot \bar{l}$, with $\bar{l}$ representing the exogenous level of labor supplied by an individual,

$$l_{1t} + l_{2t} = \bar{l}\,. \qquad \textbf{(4)}$$

Similarly, total capital stock in period *t*, K_t, is divided between the two sectors yielding

$$k_{1t} + k_{2t} = k_t \qquad \textbf{(5)}$$

in per worker terms.

It is assumed that $\alpha > \beta$ so that commodity 1 is the relatively capital-intensive commodity, whereas commodity 2 is labor-intensive. Commodity 1 is assumed to serve both as a consumption good and as capital, whereas commodity 2 is used only for consumption purposes. Furthermore, commodity 1 is taken as numeraire, and the price of commodity 2 at time *t* is denoted by p_t.

The production sectors are assumed to be competitive so that the representative firm producing commodity 1 maximizes profits, π_{1t}, by solving the problem

$$\max_{k_{1t}, l_{1t}} \pi_{1t} = (k_{1t})^{\alpha}(l_{1t})^{1-\alpha} - r_t \cdot k_{1t} - w_t \cdot l_{1t} \qquad \text{s.t.} \qquad k_{1t} \geq 0,\ l_{1t} \geq 0 \qquad \textbf{(6)}$$

The corresponding problem for the representative firm producing commodity 2, together with the first order conditions, yields

$$r_t = \alpha \cdot (k_{1t})^{\alpha-1}(l_{1t})^{1-\alpha} = p_t \cdot \beta \cdot (k_{2t})^{\beta-1}(l_{2t})^{1-\beta} \qquad \textbf{(7)}$$

$$w_t = (1-\alpha)\cdot(k_{1t})^{\alpha}(l_{1t})^{-\alpha} = p_t \cdot (1-\beta)\cdot(k_{2t})^{\beta}(l_{2t})^{-\beta} \qquad \textbf{(8)}$$

Consumption Side

An individual born at time *t* has a utility function of the form:

$$U_t = \left(C_t\right)^{\mu} \cdot \left(C_{t+1}\right)^{1-\mu} \tag{9}$$

where $C_t = \left(C_{1yt}\right)^{\theta} \cdot \left(C_{2yt}\right)^{1-\theta}$ denotes the utility gained from consumption when young and $C_{t+1} = \left(C_{1ot+1}\right)^{\theta} \cdot \left(C_{2ot+1}\right)^{1-\theta}$ denotes the utility gained from consumption when old. Thus, C_{iyt} denotes the amount of commodity *i* consumed when young, and C_{jot+1} denotes the amount of commodity *i* consumed when old, all in per capita terms. The representative household makes consumption decisions by solving the following problem:

$$\max_{C_{1yt}, C_{2yt}, C_{1ot+1}, C_{2ot+1}} U_t \qquad \text{subject to}$$

$$C_{1yt} + p_t \cdot C_{2yt} + \frac{C_{1ot+1} + p_{t+1} \cdot C_{2ot+1}}{1 + r_{t+1}} = w_t \cdot \bar{l} \tag{10}$$

where r_{t+1} is the rationally anticipated return to capital in period $t+1$ and p_{t+1} is the rationally anticipated price of commodity 2 in period $t+1$.

The solution to this problem results in the following decisions:

$$C_{1yt} = \mu \cdot \theta \cdot w_t \cdot \bar{l} \tag{11}$$

$$C_{2yt} = \frac{\mu \cdot (1-\theta) \cdot w_t \cdot \bar{l}}{p_t} \tag{12}$$

$$C_{1ot+1} = (1-\mu) \cdot \theta \cdot w_t \cdot \bar{l} \cdot \left(1 + r_{t+1}\right) \tag{13}$$

$$C_{2ot+1} = \frac{(1-\mu) \cdot (1-\theta) \cdot w_t \cdot \bar{l} \cdot \left(1 + r_{t+1}\right)}{p_{t+1}} \tag{14}$$

Equilibrium Conditions

Since olds consume all their wealth in the current period, only capital transferred to the next period is the savings of the current young, implying that

$$K_{t+1} = A_{t+1} = \left(w_t \cdot \bar{l} - C_{1yt} - p_t \cdot C_{2yt}\right) \cdot \eta_t . \qquad \textbf{(15)}$$

Dynamic equilibrium requires clearance of goods' markets in each period *t*, i.e.,

$$X_{1t} = \eta_t \cdot C_{1yt} + \eta_{t-1} \cdot C_{1ot} + (K_{t+1} - K_t) \qquad \textbf{(16)}$$

$$X_{2t} = \eta_t \cdot C_{2yt} + \eta_{t-1} \cdot C_{2ot} \qquad \textbf{(17)}$$

Since equations (16) and (17) imply each other by Walras' Law, one of them can be dropped and commodity market equilibrium can be captured by reexpressing equation (17) in per capita terms as follows:

$$x_{2t} = C_{2yt} + \frac{C_{2ot}}{1+n} = \frac{\mu \cdot (1-\theta) \cdot w_t \cdot \bar{l}}{p_t} + \frac{(1-\mu) \cdot (1-\theta) \cdot w_{t-1} \cdot \bar{l}}{(1+n) \cdot p_t} \cdot (1+r_t) \qquad \textbf{(18)}$$

The equations above make it possible to express the relative price of commodity 2, p_t, as a function $p_t = \phi(k_t)$ of capital per worker k_t.[3] This implies that k_t is the variable that drives adjustment towards long-run equilibrium. When there is no population growth (i.e., when n=0), the long-run equilibrium is reached in the economy's steady state where capital per worker and all other variables are constant. For capital per worker to be constant ($\mathrm{d}k_t/\mathrm{d}t$=0), total capital stock must also stop growing ($\mathrm{d}K_t/\mathrm{d}t$=0). When population growth rate is positive, on the other hand, total capital stock will continue to grow at the constant rate of population growth, n>0, even after k_t has stopped varying

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[3] See Sayan and Uyar (2001) for a more detailed discussion on the model solution and simulations.

over time. So, the stable equilibrium is reached when $\frac{dk_t}{dt} \equiv \frac{d(K_t / \eta_t)}{dt} = 0$ or when $\frac{1}{K_t}\frac{dK_t}{dt} = \frac{1}{\eta_t}\frac{d\eta_t}{dt} = \frac{\eta_{t+1} - \eta_t}{\eta_t}$ which is equal to n by equation (1).

III.2. Trade

Letting country *A* and *B* become partners in trade requires developing a two-country trade model. In this section, the low-(high-)population growth country is assumed to be *A* (*B*), and country specific variables are tagged *A* and *B* in superscripts.

A natural result of free trade will be the equalization of (relative) prices in both countries in each period so that

$$p_t^A = p_t^B, \quad \forall t \tag{19}$$

In this set-up, even when both countries start with the same initial populations and endowments of capital and labor, differences in population growth rates, n^A and n^B ($n^A < n^B$), will let country *A* (*B*) become the relatively capital-(labor-)abundant country immediately after the first period. This is because of two reasons. First, the labor supply in country *B* increases faster than in country *A*. Secondly, since only the youngs are allowed to bear children, the share of youngs in the entire population of country *B* exceeds that in country *A* after the first period. This, in turn, implies that a greater (smaller) portion of the population in *B* (*A*) contributes to the capital accumulation process. As a result of this change in relative endowments, country *A* must be expected to export the relatively more capital-intensive commodity (commodity 1), while the labor-abundant country (country *B*) exporting the relatively more labor-intensive commodity (commodity 2). Then, the market clearing conditions in each country under autarky must now be replaced with the condition that world-wide demand for each commodity is equal to the world-wide supply. This requires that the amount of commodity 1 (commodity 2) that country *A* exports (imports) be equal to the amount of commodity 1 (commodity 2) that country *B* imports (exports), or that the following equations hold:

$$X_{1t}^{A}+K_{t}^{A}-K_{t+1}^{A}-\eta_{t}^{A}\cdot C_{1yt}^{A}-\eta_{t-1}^{A}\cdot C_{1ot}^{A}=\eta_{t}^{B}\cdot C_{1yt}^{B}+\eta_{t-1}^{B}\cdot C_{1ot}^{B}+K_{t+1}^{B}-X_{1t}^{B}-K_{t}^{B} \qquad \textbf{(20)}$$

$$X_{2t}^{A}-\eta_{t}^{A}\cdot C_{2yt}^{A}-\eta_{t-1}^{A}\cdot C_{2ot}^{A}=\eta_{t}^{B}\cdot C_{2yt}^{B}+\eta_{t-1}^{B}\cdot C_{2ot}^{B}-X_{2t}^{B} \qquad \textbf{(21)}$$

where $\eta_{t}^{A}=\left(1+n^{A}\right)\cdot\eta_{t-1}^{A}$ and $\eta_{t}^{B}=\left(1+n^{B}\right)\cdot\eta_{t-1}^{B}$.

These equations make it possible to determine the (common) world price ratio, once k_t^A and k_t^B are known.

Formulated as such, the model does not directly allow for foreign direct investment and factor movements between countries. However, given that commodity 1 is used for both consumption and investment purposes, capital stock at time *t* comes from commodity 1 produced at time *t-1*. Therefore, while installed capital itself is immobile between countries, some of commodity 1 exported by country *A* in the previous period may be used as capital by the importing country *B* in the current period.

IV. Results

In order to observe the paths that endogenous variables would follow until their growth rates have stopped varying over time under each of autarky and trade scenarios, the models described above have been simulated with the configuration of parameters and initial conditions given in Table 1.

TABLE 1.

Initial Conditions and Parameters used in Simulations

Production Parameters		Utility Parameters		Initial Population of Youngs		Initial Price Ratio	Labor Supply per Young	Population Growth Rates	
α	β	θ	μ	η_A	η_B	p(t=1)	$\bar{l}$	n_A	n_B
0.40	0.10	0.15	0.10	1	1	1	1	0.025	0.075

While picking production function parameters common to both countries, α was taken to be greater than β so as to make production of commodity 1 (which serves as a consumption as well as an investment good) relatively capital-intensive, and that of commodity 2 (serving as a consumption good alone) labor-intensive. θ and μ parameters are again common to both countries and describe consumers' choices between commodities in each period, and between current and future consumption. Both countries start with an initial young population of 1 and the same (calibrated) level of capital stock. Thus, the relative price of commodity 2 in terms of commodity 1 (i.e., the numeraire) is the same and equal to 1 in both countries initially. Since population in country *B* grows faster than in country *A*, populations begin to diverge after the first period (Figure 1), causing the labor supply in *B* to exceed that in *A*, and total population share of unproductive elderly in country *A* to be higher than that in country *B*.

Figure 1.

Working and Total Population in Country *A* and *B* over Time

The result is a gradual change in relative factor endowments and hence, relative commodity and factor prices lasting until growth rates of all variables have stabilized at the end of the 120th period. The change in relative factor endowments can be observed through the developments in capital stock per worker –which is, in fact, the variable that drives the dynamic adjustment process towards long-run equilibrium within the current OLG-GE framework (Sayan and Uyar, 2001). Figure 2 shows the behavior of capital per worker over time in both countries under autarky and trade. As shown in the figure, autarky value of capital per worker in *A* exceeds that in *B* immediately after the initial period, and remains higher until stable growth rates are achieved for all variables, due to the faster population growth in *B*.

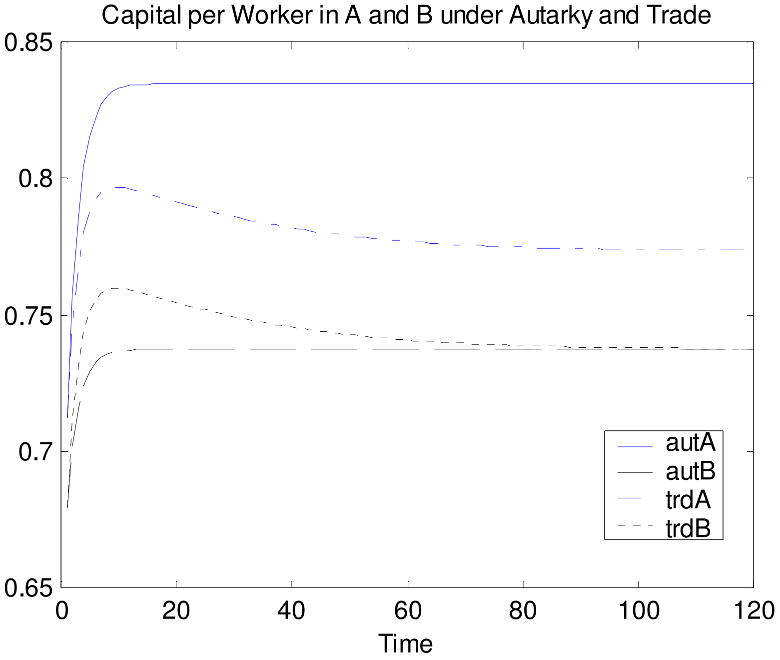

Figure 2.

Capital per Worker under Autarky and Trade in Country *A* and *B*

Underlying the paths plotted in Figure 2 are the variations in the growth rate of capital stock across countries, and autarky and trade scenarios over time. These variations can be observed from Figure 3 which compares the growth rate of total capital stock, K_t, under autarky and trade scenarios for each country for the first 60 periods. As shown in the upper panel of Figure 3, K_t initially grows faster under autarky as compared to trade in country *A*, but its growth rate under both scenarios converges to the population growth rate (0.025) in the long-run. A similar pattern emerges in country *B*, except for the first 8 periods when the growth in K_t under trade exceeds that under autarky. As before, this growth rate converges to the population growth rate (0.075) in the long-run.

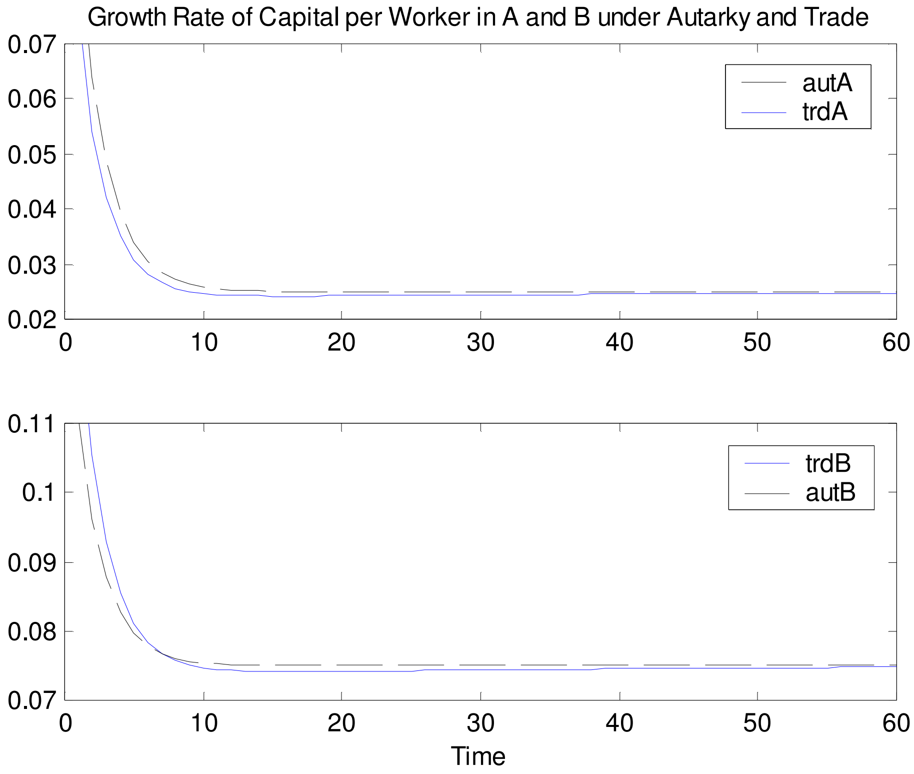

Figure 3.

Growth Rates of Capital Stock under Autarky and Trade
in Country *A* and *B*

As country *B* (*A*) becomes relatively labor-(capital-)abundant, resulting differences in relative prices make trade possible. When trade between two countries is opened up, capital-abundant country *A* begins to export to labor-abundant country *B* the capital-intensive commodity (1) which it produces relatively more efficiently (with a higher productivity) as shown in the upper panel of Figure 4. Similarly, labor-abundant country *B* produces labor-intensive commodity (2) at a relatively higher level of productivity (lower panel of Figure 4) and exports it to the capital-abundant country *A* (Figure 5).

Figure 4.

Output per Worker by Sector under Trade

Since trade allows country *B* to import capital through the imports of capital-intensive investment good exported by the capital-abundant *A*, capital per worker in *A* (*B*) falls below (jumps over) the autarky level. Once the growth rates of capital per worker stops changing over time, each worker in country *A* (*B*) ends up with a long-run capital stock that is lower (higher) than he would have under autarky. Yet, the rapid population growth in country *B* eventually pulls the capital stock per worker under trade towards its autarky value (Figure 2).

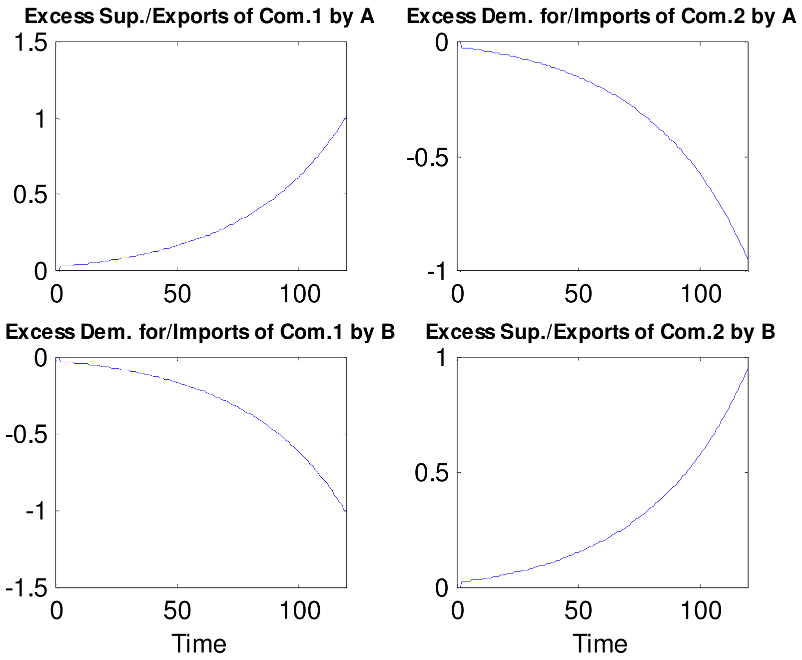

Figure 5.

Sectoral Trade between *A* and *B* over Time

The opening of trade leads to an equalization of relative commodity and factor prices, as predicted by the Heckscher-Ohlin theory (Figures 6-8). Figure 6 shows that autarky relative price of the labor-intensive commodity 2 in the labor-abundant country *B* always lies below that in country *A*, but a common price ratio is established after the countries start trading with each other. This common price ratio stays strictly between autarky relative prices at first, but then begins to converge towards the autarky relative price of commodity 2 in country *B*. In other words, this country begins to act as though it is a large country in the international trade theory sense of the term. This follows from the faster growth in the population of *B* which enables it to produce and demand an increasingly higher portion of total world output over time. In aggregate (as opposed to per capita) terms, Country *A*'s share of production and demand get smaller and smaller compared to the aggregate production and demand in country *B* as time passes. Thus, solving the model for the world becomes nearly the same as solving it only for country *B*.

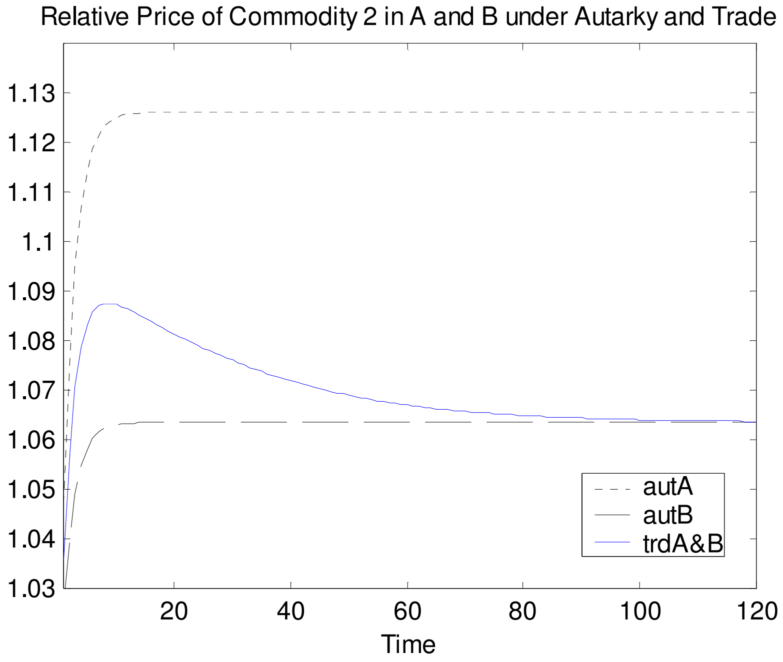

Figure 6.

Relative Price of Commodity 2 under Autarky and Trade in Country *A* and *B* over Time

A similar pattern is observed concerning the behavior of factor prices over time. Figure 7 shows that the wage rate in the labor-abundant *B* remains below *A* under autarky. Consistently with the Heckscher-Ohlin framework, opening of trade between *A* and *B* results in a common wage rate lying between autarky wage rates. This common rate begins to fall towards 20th period, as total labor supply in country *B* continues to increase. The wage rate eventually converges to the autarky rate in *B* after this country's share in worldwide supply of labor has significantly outgrown that of country *A* in the long-run.

Figure 7.

Wage Rate under Autarky and Trade in Country *A* and *B*

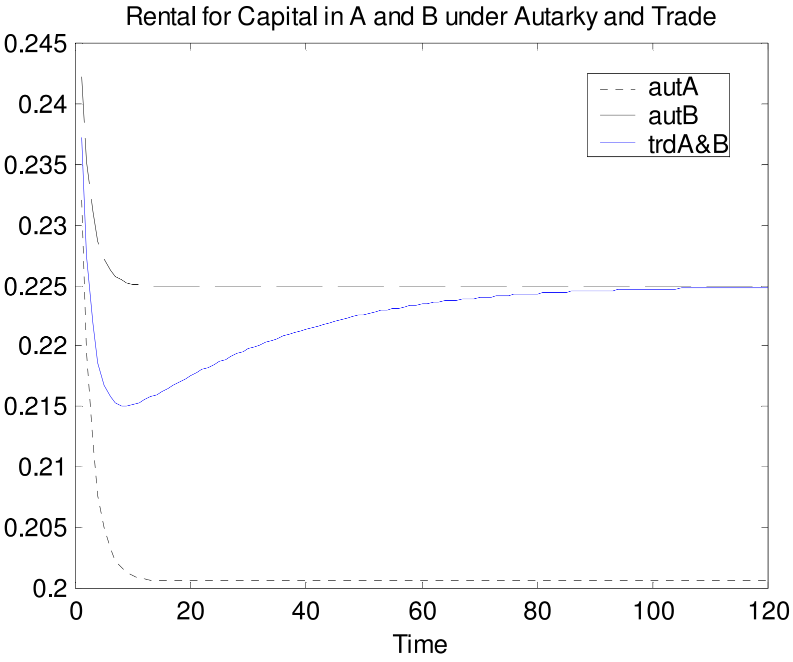

Figure 8.

Rental for Capital under Autarky and Trade in Country A and B

Similar observations can be made also for the rental rate (Figure 8). Trade between *A* and *B* again results in a common rental rate lying between autarky rates. This common rate begins to increase towards 20th period, eventually converging to the autarky rate in *B* after this country's share in total world stock of capital has significantly exceeded hat of country *A* in the long-run.

As a result of the changes in relative commodity and factor prices under trade, per capita GNP in country *A* falls below its autarky level, converging to the per capita GNP of country *B* in the long-run. While country *B* gains from trade in the short- to medium-run, its per capita GNP gains do not endure in the long-run, as rapid growth of population in this country *B* causes individuals' income to converge to the autarky level (Figure 9).

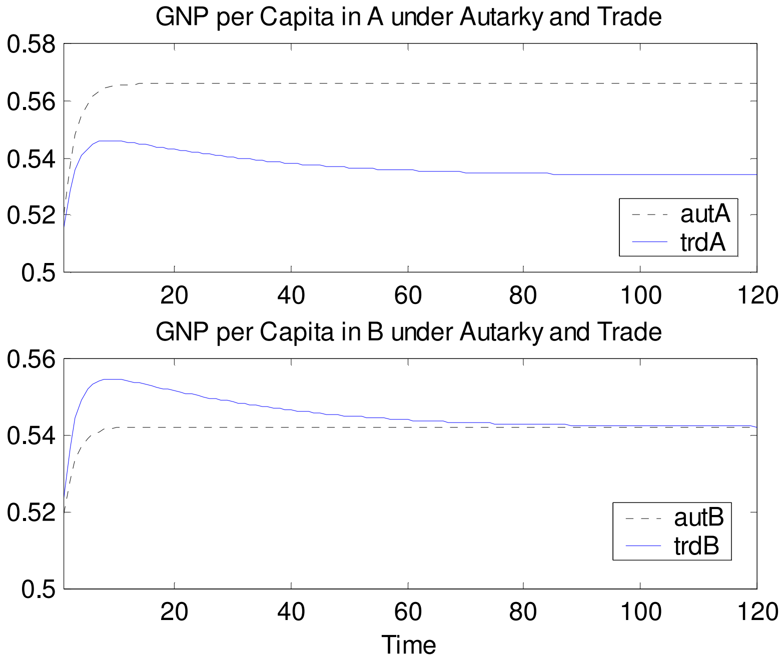

Figure 9.

Per Capita GNP in Countries *A* and *B* under Autarky and Trade

The loss of per capita income in country *A* and the temporary gains in country *B* under trade affect consumption patterns. As seen in Figure 10, per capita consumption of both commodities is higher in *A* than in *B* for both age groups under autarky. Trade lowers consumption in *A*, whereas it temporarily raises consumption in country *B* above its autarky level. Per capita consumption in both countries eventually converges to the autarky level in country *B*, as in the case of other variables.

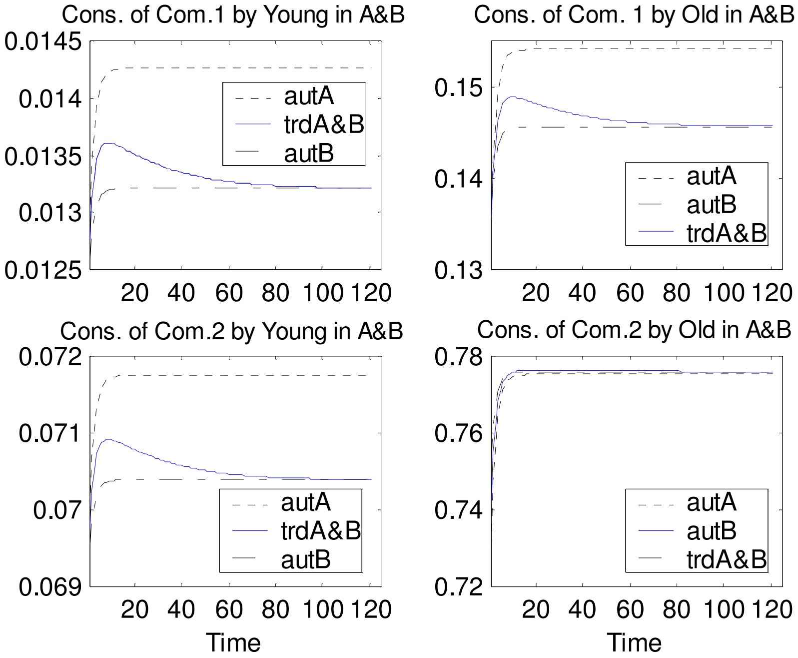

Figure 10.

Per Capita Consumption of Commodities by Age under Autarky and Trade

As would be expected from the previous discussion, trade would not produce any welfare gains for country *A*. In fact, this country would be made worse off by trade, whereas country *B* obtains temporary gains that disappear in the long-run (Figure 11).

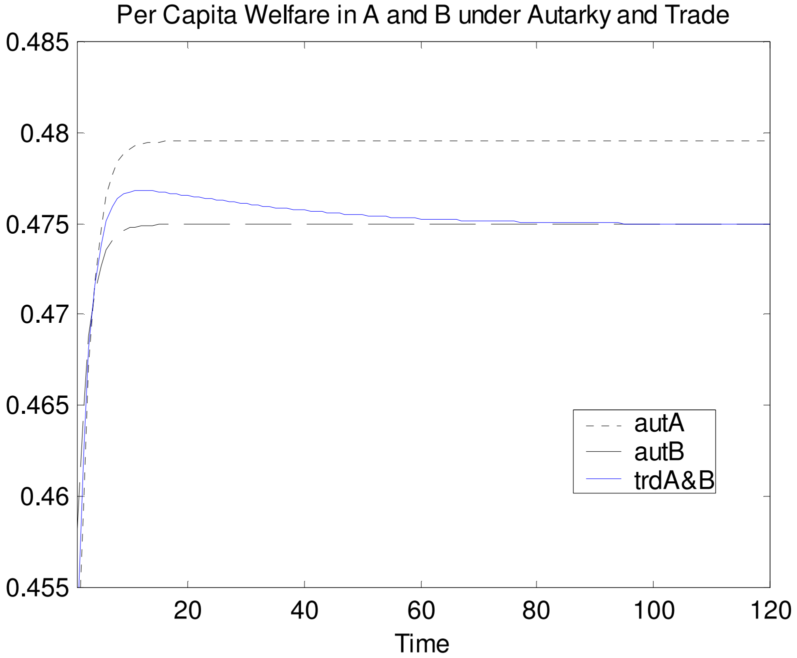

Figure 11.

Utility in Countries *A* and *B* under Autarky and Trade

These long-run results are summarized in Table 2 which ranks equilibrium values of key variables under autarky and trade in a descending order. A comparison of the ranking in Table 2 with the previously reported results in Sayan and Uyar (2001) and Sayan (2002) indicates that the results here are robust to changes in the values of parameters and initial conditions given in Table 1. One of the key parameters that affect the results is μ. Since the saving rate depends on the value of this parameter, μ determines whether k_t follows an upward or downward trend over time, thereby affecting growth and welfare results. It is interesting to note that even when the values of μ assumed are as high as to turn the upward trend of k_t in Figure 2 downward (see Sayan and Uyar, 2001; Sayan, 2002), the ranking in Table 2 is preserved. In general, even though parameter values and initial conditions in Table 1 affect the magnitude of long-run equilibrium values of variables and the paths leading to long-run equilibrium, the rankings of country-specific values of variables across autarky and trade scenarios remain the same within a wide range of parameter values.

TABLE 2.

Ranking of Long-Run Equilibrium Values of Key Variables
under Autarky and Trade in Country *A* and *B*

k	*r*	*W*	*p*	*c1y*	*c2y*	*c1o*	*c2o*	*u*	*gnp*
Aut*A*	Aut*B* = Trd*B* = Trd***A***	**Aut***A*	**Aut***A*	**Aut***A*	**Aut***A*	**Aut***A*	Aut*B* = Trd*B* = Trd***A***	**Aut***A*	**Aut***A*
Trd***A***		Aut*B* = Trd*B* = Trd***A***	Aut*B* = Trd*B* = Trd***A***	Aut*B* = Trd*B* = Trd***A***	Aut*B* = Trd*B* = Trd***A***	Aut*B* = Trd*B* = Trd***A***		Aut*B* = Trd*B* = Trd***A***	Aut*B* = Trd*B*
Aut*B*									
Trd*B*	**Aut***A*						**Aut***A*		Trd***A***

V. Conclusions

This paper addressed a largely overlooked issue in the dynamic trade literature by considering the role that differences in the speed of population growth across countries may play as long-run determinants of comparative advantages within a Heckscher-Ohlin framework. The implications of unequal population growth rates for trade patterns and the behavior of different variables over time were explored by developing a simple 2-factor, 2-commodity and 2-country model of international trade which allows each country to be populated by two overlapping generations of individuals. The model was then used to compare the paths that endogenous variables would follow from an initial state until the economies reach long-run equilibrium through simulations run under autarky and trade scenarios.

The countries were assumed to have exactly the same preferences, and to produce the same commodities by employing identical production technologies for each commodity. They were further assumed to be endowed with the same amounts of capital and labor initially. The required differences in autarky relative prices for opening of trade between the countries were shown to arise immediately after the initial state, as inequality of population growth rates begins to create *à la*-Heckscher-Ohlin differences in relative factor endowments. In particular, the country with the lower population growth rate becomes relatively capital abundant, whereas the country with the higher population growth rate becomes relatively labor abundant over time.

Within the framework of OLG general equilibrium model here, the long-run equilibrium is reached when the variations in the growth rate of capital per worker over time have stopped and this growth rate has become equal to the population growth rate. Naturally, the paths of endogenous variables and the nature of long-run equilibrium reached are different under autarky and trade, allowing for a comparison across scenarios. When such a comparison was made based on the simulation results obtained here, the model was observed to produce results that are generally but not invariably consistent with the static HO framework. The country that is relatively abundant in one of the factors specializes in the production of the commodity which uses that factor more intensively and exports that commodity, as predicted by the static HO model. Moreover, trade leads to an equalization of factor prices, as in the static HO framework. Contrary to those predictions, however, trade would not necessarily represent a Pareto improvement over the state of autarky. While the static HO theory rules out welfare losses for either of the countries, the results in this paper revealed that trade may be immiserizing for the country whose population grows slowly relative to the other. This is due to the continuously increasing difference in the size of populations which enables the country with the faster growing population to have a commanding share in total world supply of factors as well as output. In other words, the labor-abundant country starts acting as a large country (in the international trade theory sense of the term) after a while.

A nation with faster population growth than its trade partners is indeed likely to become capable of setting prices in at least certain sectors. To use Leamer's (1992) example, Mexico may not seem economically large enough to have a significant effect on the prices of goods and the earnings of workers in the U.S. today, but population growth and productivity gains will make the Mexico of the future much larger than it is now. This will assign Mexico the role of a large country in a number of tradable good sectors, particularly the sectors that intensively use low-skilled that is abundant in this country. As such, Leamer (1992) argues, the size of Mexico of the future will be sufficiently large to undo the effects of any protection designed to maintain wages of low-skilled workers in the U.S.

Going back to the simulation results, the values of such variables as capital stock per worker, wage rate and per capita income in the capital-abundant country are observed to be pulled down towards corresponding autarky values in the labor-abundant country, and in the absence of free movement of labor and capital across countries, trade causes welfare losses for this country.

This is a significant finding which provides additional simulation evidence to previous studies pointing out that welfare implications of trade predicted from the standard HO model might not be preserved under different dynamic set-ups. Furthermore, the results here offer the inequality of population growth rates across trading nations as a possible and previously unreported reason explaining why and how trade may not be Pareto superior to autarky.

In general, the analysis here lays the initial grounds for a rich discussion on possible consequences of the differential speed of demographic transition, fertility decline and population aging currently observed in various parts of the world. The introduction of the overlapping generations structure to the 2x2x2 general equilibrium framework of the HO model made the model developed here particularly suitable for analyzing the consequences of projected changes in relative factor endowments in the capital-abundant countries with low and fast declining population growth rates and aging populations vis-a-vis labor-abundant countries with high and slowly declining population growth rates.

Given the ease of transferring technologies, differential speeds of population increases are indeed likely to become major determinants of trade and factor flows in the decades ahead, as the changing sizes of labor forces and age profiles of populations affect relative abundance of labor and capital. The OLG framework described in this paper facilitates modeling the effects of demographic differences on the evolution of relative factor endowments, and the resulting patterns of trade and factor flows. Suggestions for future research include the introduction of capital mobility and migration to the trade model, and repeating the analysis here by endogenizing population growth rates so as to allow for a slow down in the speed of population increases in response to increased income as observed in real life. Such extensions will certainly provide additional insights into the possible nature of economic relations between countries across which the timing of demographic transition differs considerably, and increase the policy relevance of results from simulation exercises.

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