

# **Effect of Moments on Aggregation and Long Memory in Inflation**

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## **Abstract**

There are two crucial conditions for cross-sectional aggregation of AR(1) parameters to produce long memory: 1) heterogeneity and 2) proximity to the unit root. We analyze role of moments, namely the mean and variance, of the distribution of the AR(1) coefficients in generating long memory. The positive relation between these moments and the order of integration suggests that the degree of fractional integration should decrease with a lower mean or variance. We investigate this result by first modeling long memory in inflation as a result of the aggregation of individual inflation expectations and then showing how the adoption of inflation targeting decreases the memory length in seven countries due to its moderating effect on individual inflation expectations.

**JEL Classifications: C500, E310, E520**

**Keywords: Long Memory, Aggregation, Inflation Expectations, Inflation Targeting**

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We would like to thank Karim Abadir, Jesus Gonzalo, Mark Wohar, Jack Strauss, Hailong Qian, Muhammad Islam, and Kivilcim Metin-Ozcan for their useful comments and suggestions in earlier versions of this paper. The usual disclaimer applies.

## 1 Introduction

The 1990s have witnessed a large body of literature investigating fractional integration (or more generally long memory<sup>1</sup>) in economic models. Despite substantial evidence of its relevance in many macroeconomic series<sup>2</sup>, there have not been many papers establishing its economic origins. Until recently, the most common<sup>3</sup> explanation for fractionally integrated processes in economics has been Granger's (1980) cross-sectional aggregation<sup>4</sup> of a large number of heterogeneous dynamic processes. Aggregation over individuals or firms has been advanced as the source of long memory in many empirical studies on aggregate economic series. We look deeper into this aggregation issue to achieve a better understanding of the link between econometric theory and occurrence of long memory in observed data.

There are two necessary conditions for cross-sectional aggregation of AR(1) parameters to produce long memory in the sum: 1) heterogeneity, and 2) proximity to the unit root. It is trivial to show that the sum of  $N$  AR(1) series with identical parameters will be an ARMA process, implying that heterogeneity of AR(1) coefficients during aggregation is essential for obtaining long memory. Granger (1980) and Lippi & Zaffaroni (2000) also show that unless these individual AR(1) coefficients are allowed to approach to 1, the aggregate series will not have a fractional degree of integration. These necessary conditions form the motivation behind our paper, namely, analyzing the role of the moments of distribution of AR(1) coefficients in aggregation towards the emergence of long memory.

We illustrate the impact of the moments on two different distributions used in cross-sectional aggregation of AR(1) coefficients, the beta distribution of Granger (1980) and a more general semiparametric distribution by Lippi & Zaffaroni (2000). Deriving the analytical forms for the degree of fractional differencing,  $d$ , in terms of the mean and variance of each distribution, we observe that  $d$  increases with both of these moments. In other words, greater heterogeneity in the AR(1) coefficients and a closer proximity of their mean to 1 will lead to an increase in the degree of persistence, possibly to nonstationary levels. Such a finding necessitates satisfying some initial conditions on the distribution of AR(1) parameters before assigning existence of long memory to cross-sectional aggregation.<sup>5</sup>

We find support for our findings by reexamining previous evidence of long memory in international inflation series (Hassler & Wolters, 1995; Baillie, Chung & Tieslau, 1996; and Baum, Barkoulas & Caglayan, 1999). We conjecture that this observed persistence in inflation is due to the aggregation of heterogeneous inflation expectations and that it will disappear once there is convergence in expectations. We exploit the switch to inflation targeting as an initiator of such a decline in the variation of expectations. Inflation targeting, if credible, will cause the public to form their expectations closer to the announced target. Bernanke, Laubach, Mishkin, & Posen (1999) use a combination of surveys and interest rate differentials to show that the public announcement of inflation targets and strict adherence to them help moderate inflation expectations. From their evidence and our contentions, we would expect to see a decrease in long memory in inflation after the switch to this new type of monetary policy.

Section 2 elaborates on the relation between the degree of fractional differencing and the moments of AR(1) coefficients having the two distributions mentioned above. Section 3 applies these ideas to a model of inflation. Section 4 includes the estimation process, and is followed by an interpretation of the results. Section 6 provides some concluding remarks.

## 2 Moments and Memory Length

Granger (1980) considers the cross-sectional aggregation of a large number of heterogeneous AR(1) processes ( $i = 1, \dots, N$ )

$$x_{it} = \alpha_i x_{it-1} + \varepsilon_{it} \quad (1)$$

where  $\varepsilon_{it}$  is white noise,  $E(\varepsilon_{it}, \varepsilon_{jt}) = 0$ , and  $E(\alpha_i, \varepsilon_{jt}) = 0$  for all  $i, j, t$ . When  $\alpha_i$  has the beta distribution<sup>6</sup>

$$f(\alpha) = \frac{2}{B(p, q)} \alpha^{2p-1} (1-\alpha^2)^{q-1} \quad \text{for } 0 \leq \alpha \leq 1 \quad (2)$$

(where  $B(p, q)$  is the beta function) and  $N$  gets large, the aggregate series  $x_t = \sum_{i=1}^N x_{it}$  will exhibit long memory (a slowly decaying autocovariance function) and have a fractional order of integration<sup>7</sup>,  $d = 1 - q/2$ . Granger shows that decreasing the range of  $\alpha$  from above (i.e., when  $\alpha$  is not allowed to be close to 1) results in the disappearance of long memory and that the conclusions do not change when  $b < \alpha \leq 1$  (where  $b > 0$ ). This condition demonstrates that for fractional integration,  $x_t \sim I(d)$ , heterogeneity alone is not sufficient, but the coefficients  $\alpha_i$  should also be allowed to approach to one (i.e., mean should be high).

Our analysis extends Granger's by illustrating the analytical relation of the degree of fractional differencing to the moments<sup>8</sup>, namely, the mean ( $\mu_\alpha$ ) and variance ( $\sigma_\alpha^2$ ), of the coefficient  $\alpha$ . Mean and variance of the beta distribution are

$$\mu_\alpha = \frac{p}{p+q} \quad (3)$$

$$\sigma_\alpha^2 = \frac{pq}{(1+p+q)(p+q)^2} \quad (4)$$

Combining them with the previously mentioned fractional order of integration,  $d = 1 - q/2$ , helps us to illustrate the relation between the order of integration and these moments. Substituting out  $p$  and  $q$  gives us

$$d = \frac{3\sigma_\alpha^2 - \mu_\alpha \sigma_\alpha^2 - (1 - \mu_\alpha)^2 \mu_\alpha}{2\sigma_\alpha^2} \quad (5)$$

The relations  $\partial d / \partial \sigma_\alpha^2 > 0$  and  $\partial d / \partial \mu_\alpha > 0$  indicate that the degree of persistence crucially depends on the tail probability of the distribution of  $\alpha$  close to one. A decrease in the variation or mean of  $\alpha$  unambiguously lowers the degree of fractional differencing, and in extreme cases may eliminate it completely.

Lippi & Zaffaroni (2000) use a more general semiparametric distribution to illustrate how cross sectional aggregation can lead to long memory in the aggregate series. In a model similar to Granger's

$$x_{it} = \alpha_i x_{it-1} + u_t + \varepsilon_{it} \quad (6)$$

they divide the disturbance term into common ( $u_t$ ) and idiosyncratic ( $\varepsilon_{it}$ ) shocks. Using a family of continuous distributions  $\beta$

$$\beta(\alpha, b) \sim C_b (1 - \alpha)^b \quad (7)$$

where  $\alpha \in [0,1)$ ,  $b \in (-1,\infty)$ , and  $C_b$  is an appropriate positive constant, they display that aggregation will lead to long memory models depending on the density of the distribution of  $\alpha_i$  around 1. As  $b$  approaches  $-1$ , this density will become greater, resulting in stronger persistence. At negative values of  $b$ , the aggregation of the idiosyncratic or the common components will produce the degrees of differencing,  $d = (1-b)/2$  or  $d = -b$ , respectively.

Deriving the mean of  $\alpha$  for the distribution suggested by Lippi & Zaffaroni (2000), we find that

$$\mu_\alpha = \frac{C}{(b+1)(b+2)} \quad (8)$$

for  $b \neq -1$ . Since  $b$  is inversely related to  $d$ , persistence increases with higher means. As the non-central moments of their distribution are recurrent<sup>9</sup>,  $d$  is also positively related to variance of  $\alpha_i$ . Like Granger, not allowing  $\alpha_i$  to vary *or* approach 1 (by pushing  $b$  away from  $-1$  toward positive values) will lead to an exponentially decaying autocovariance function, which is a property of short memory models.

These findings strengthen Granger's conclusions and illustrate our claim that cross-sectional aggregation will lead to long memory *only if* the AR coefficients show sufficient heterogeneity *and* proximity to 1. In the next two sections, we show empirical support for our analytical findings by first formulating the relation between inflation and its expectation, and then examining how the degree of fractional differencing responds to changes in moments.

### 3 Inflation Expectations

To illustrate our result, we need two things: first, aggregation of a large number of AR(1) series with sufficient variation and large mean; and second, a clear shift in the distribution of these parameters. Previous evidence of long memory in inflation fits these requirements because of the link between inflation and aggregated inflation expectations. A regime switch to inflation targeting is a good candidate due to its impact on the formation of inflation expectations.

Earlier theoretical (Crettez & Michel, 1992; Naish, 1993) and empirical (Figlewski & Wachtel, 1981; Zarnowitz, 1985; Evans & Wachtel, 1993) studies have shown that when information acquisition is costly, inflation expectations are not consistent with the assumptions of rational expectations theory<sup>10</sup>. These papers show that the use of adaptive expectations can be optimal in environments of costly information, and also confirm that adaptive expectations models fit inflation forecasts better than rational expectations models.

Figlewski & Wachtel (1981) find that the rates of adjustment of inflation expectations differ from one agent to the next, and that this rate is a positive function of past inflation levels and a negative function of the diversity of opinion about future price increases. Utilizing their adaptive expectations representation, the inflation expectation,  $\pi^{ie}$ , for agent  $i$  is

$$\pi_{t+1}^{ie} = \theta_i \pi_t + (1 - \theta_i) \pi_t^{ie} + \eta_{t+1}^i \quad (9)$$

where  $i = 1, \dots, N$  and  $\eta^i$  is a white noise disturbance term. We adopt an inflation process as a linear function of inflation expectations<sup>11</sup>

$$\pi_t = K_t + \gamma \pi_{t+1}^e + z_t \quad (10)$$

where  $\pi_{t+1}^e$  is the aggregate expectation of inflation level  $\pi_t$ ,  $K_t$  represents variables like money growth rate or output gap, and  $z_t$  is a white noise supply shock. Assuming the aggregate inflation expectation to be the mean of the individual forecasts (i.e.,  $\pi_{t+1}^e = (1/N) \sum \omega_i \pi_{t+1}^{ie}$ , where  $\omega_i$  is an appropriate weight factor that discounts extreme inflation expectations), it can be shown that the reduced form for the individual inflation expectation follows an AR(1) process

$$\pi_{t+1}^{ie} = \theta_i K_{it}^* + \alpha_i \pi_t^{ie} + \eta_{t+1}^{i*} \quad (11)$$

where  $\alpha_i$  is<sup>12</sup>

$$0 \leq \alpha_i = \frac{N(1-\theta_i)}{N-\gamma\theta_i\omega_i} \leq 1 \quad (12)$$

and is approximately equal to  $1 - \theta_i$  for large  $N$ .

Granger (1980) and Lippi & Zaffaroni (2000) have shown that the aggregation of AR(1) models, as in equation (11), results in a fractionally integrated process. Thus, aggregation of the individual expectations,  $\pi_{t+1}^{ie}$ , above (to obtain the mean  $\pi_{t+1}^e$ ) could induce a long memory process in the aggregate inflation expectation, which would in turn translate into long memory in inflation<sup>13</sup> via equation (10).

$$\pi_{t+1}^e \sim I(d) \rightarrow \pi_t \sim I(d) \quad (13)$$

Such a derivation offers one possible reason for the evidence of long memory in the inflation process. Other potential reasons suggested to date are persistence in money

supply (Scacciavillani, 1994) and the aggregation of individual prices into a price index (Hassler & Wolters, 1995). To differentiate our model from the others, we look at the impact of the adoption of inflation targeting.

It is widely accepted that an activist central bank can create an inflationary bias because of its opportunism in surprising the public to stimulate production. As a result, persistent inflation will become ingrained in the system via the public's expectations without any compensating increase in output (Equation 10). The adoption of inflation targeting is aimed at moderating inflation expectations by not only providing discipline in the setting of monetary policy, but also by improving the communication between the policy makers and the public. In their comprehensive work on inflation targeting, Bernanke *et al* (1999) analyze the effects of inflation targeting on inflation expectations. Using a combination of surveys and interest rate differentials, they conclude that the targeting framework increases the public's understanding of monetary policy, and lowers inflation expectations, i.e. decreasing heterogeneity and mean. Therefore, if inflation targeting is successful in decreasing the variability of inflation expectations, evidence of long memory processes present before the regime switch should disappear or be significantly reduced afterwards. Such empirical evidence would support our theory since the adoption of inflation targeting should not have any affect on money supply persistence or price aggregation.

#### **4 Results**

We use monthly price data from International Financial Statistics of the IMF for the sample period of 1960 to 1999 for seven inflation-targeting countries, namely Canada, Finland, Israel, Spain, Sweden, the United Kingdom and Australia<sup>14</sup>. We also use CPI

data for the first six countries, and manufacturing input prices for Australia, due to the unavailability of a monthly CPI series in that country. Inflation series are derived by log differencing the twelfth lag to remove seasonality from the data.

Prior to the estimation of the long memory parameters, we first examine whether our assumption of changing moments in the inflation processes is valid. A quick glance at the descriptive statistics in Table 1 shows that all of the moments for the sample inflation series decrease with the adoption of the new monetary regime. However, as there is considerable evidence on the positive relation between inflation and its volatility, we have to make sure that such a decrease was not caused by the moderation of inflation, but rather by the inflation expectations being concentrated around the target. For that reason, we next utilize the methodology in Lewbel (1994) to analyze how the public's inflation expectations have changed with the adoption of inflation targeting.

(Insert Table 1 here)

In his 1994 paper, Lewbel shows that the distribution of individual AR(1) coefficients,  $\alpha_i$  of Equation (1), can be identified from the dynamic behavior of aggregate (macro)economic data. One very useful result in the present circumstances is that the first two autoregressive coefficients in the aggregate data will be exactly equal to the mean and variance, respectively, of the distribution of AR(1) coefficients across the population. Accordingly, ARIMA analysis of aggregate inflation expectations in a country gives us an idea about the individual inflation expectations in that nation. Table 2 displays these results for a proxy of inflation expectations, the interest rate differential between the nominal and index-linked bonds in the UK and Australia<sup>15</sup> (starting from 1985:01 for the UK and 1986:07 for Australia). The results show different reactions by these two

countries to the regime switch: the UK shows an increase in the mean (from .77 to .91) while Australia exhibits an increase in the dispersion (from .014 to .043). One needs to use caution in interpreting these results, due to the problems (e.g., low levels of inflation risk and liquidity) associated with using the above spread as a proxy for inflation expectations; however, the UK seems to have had more success in pulling inflation expectations closer to the announced target.

(Insert Table 2 Here)

Next, we estimate the fractional differencing parameter in the inflation processes for our sample countries to observe if the decrease in moments induces a decline in the estimated value as suggested by our theory. Since there is a lack of consensus on the most appropriate ARFIMA estimation technique (due to poor performance at high orders or levels of ARMA dynamics, and at low number of observations), we are compelled to utilize four estimation techniques to check the robustness of our results. These include two semiparametric methods, namely the log periodogram regression of Geweke & Porter-Hudak (henceforth GPH, 1983) and the Gaussian semiparametric estimation described in Robinson & Henry (GSP, 1999), along with two maximum likelihood estimations: frequency domain approximate MLE by Fox & Taqqu (FDML, 1986), and time domain Modified Profile Likelihood suggested by Cox & Reid (MPL, 1987) and implemented by An & Bloomfield (1993). Estimations are carried out using ARFIMA packages in GAUSS and OX (Doornik, 1998). Data is differenced when necessary since all methods require stationarity. Since specifics of these methods are beyond the scope of this paper we refer the reader to an excellent survey by Baillie (1996) and Ooms & Doornik (1999) for further details.

The estimation results for  $\hat{d}$  are displayed in Table 3, and neither the values nor the orders of the ARMA parameters (in MLE) are reported to conserve space. The values in parentheses below the estimates represent their corresponding t-statistics. Examination of the results shows that *i*) the semiparametric estimation techniques, GPH and GSP, consistently underestimate the  $d$  parameter, and *ii*) more importantly, the fractional root declines in every country with the adoption of inflation targeting. We can illustrate these features by taking a closer look at Australia. GPH and GSP methods find a fractional root of 0.15 (stationary) before the regime switch while the maximum likelihood methods' results range from 0.5 to 0.8 (nonstationary). These estimates decline to lower and stationary values after the adoption of inflation targeting in all methods; to 0 in GPH and GSP, and to around 0.2-0.3 with MPL and FDML. Investigation of the other countries results in similar findings. These results corroborate the claim of our paper that once the heterogeneity in individual inflation expectations is reduced, the persistence of the aggregate series will become decrease.

(Insert Table 3 Here)

Since the sample sizes corresponding to the post-targeting periods are relatively small (in some cases a little over 6 years), it is necessary to run Monte Carlo simulations to test the validity of our findings. We try to replicate the sample sizes in our analysis by using a sample of five hundred observations<sup>16</sup> and dividing it into two sections of 425 and 75 observations. We then select Fox-Taqqu estimator out of the four, due to its better performance with unknown means (Cheung & Diebold, 1994), to estimate the fractional differencing parameter for both parts of the sample for 11 different levels of  $d$ . Investigation of the results in Table 4 show that the smaller sample estimations contain

larger standard errors. The estimates of  $d$ , however, are not very different (well within one standard deviation) from either the actual value of  $d$  or the estimated value of the part with the larger number of observations. These simulations indicate clearly that the small sample sizes in the latter parts of our dataset were not the determining factors of our results. In other words, the notable drop in  $\hat{d}$  that comes with a switch to inflation targeting is the result of a convergence in expectations, leading to less persistence in inflation.

(Insert Table 4 Here)

## **5 Concluding Remarks**

By exploring further the issue of cross-sectional aggregation of individual series leading to long memory in the aggregate series, we reach two important conclusions. First, by deriving the analytical relation between the moments of the individual AR(1) coefficients and the long memory parameter, we demonstrate the significance of the heterogeneity of AR(1) coefficients and the proximity of their mean to 1. If these decline in value, so should the fractional differencing parameter decreasing the persistence of the series. Second, we establish that the aggregation of heterogeneous inflation expectations is the most likely cause of long memory in inflation. Since the adoption of inflation targeting has little to no effect in the aggregation of prices and money supply persistence, the moderation of inflation expectations is a plausible explanation of the changes in the time series properties of inflation toward shorter memory.

## Appendix

### A1

The aggregate series  $x_t = \sum_{i=1}^N x_{it}$  will have  $k$ -th autocovariance of  $A_1 k^{1-q}$  where  $A_1$  is an appropriate constant (i.e., significant dependence in  $x_t$  even at large lags). Comparing this with the slowly decaying autocovariance approximation for fractionally integrated processes of order  $d$  (i.e.,  $A_2 j^{2d-1}$ ), it can be deduced that  $x \sim I(d)$  where  $d \geq 1 - q/2$ . Granger shows that for  $0 \leq q \leq 1$  ( $d \geq 0.5$ ), the process will not have a finite variance. Values of  $q \in (1, 2)$  (or  $0 < d < 0.5$ ) will be equal to stationary long memory processes, and higher values will have intermediate memory or anti-persistence ( $-1 < d < 0$ ). It is also noted in Granger that the order of integration depends only on  $q$  which determines the slope of the approach of  $f(\alpha)$  to  $\alpha = 1$ . Shortening the range of  $\alpha$  from above results in disappearance of long memory feature (i.e., when  $\alpha$  is not allowed to be close to 1), and that the conclusions don't change when  $b < \alpha \leq 1$  (where  $b > 0$ ).

### A2

$$K_{it}^* = \left( \frac{N}{N - \gamma \theta_i \omega_i} \right) \left[ \theta_i K_{it} + \frac{\gamma \theta_i}{N} \sum_{k=1}^{N/i} \pi_{t+1}^{ke} \right] \cong \left[ \theta_i K_{it} + \frac{\gamma \theta_i}{N} \sum_{k=1}^{N/i} \pi_{t+1}^{ke} \right]$$

$$\eta_{t+1}^* = \left( \frac{N}{N - \gamma \theta_i \omega_i} \right) (\eta_{t+1}^i + \theta_i z_t) \cong (\eta_{t+1}^i + \theta_i z_t)$$

**Table 1: Descriptive statistics of monthly inflation before and after adoption of inflation targeting**

	<i>Pre-Targeting</i>				<i>Post-Targeting</i>			
	<i>Mean</i>	<i>Variance</i>	<i>Kurtosis</i>	<i>Count</i>	<i>Mean</i>	<i>Variance</i>	<i>Kurtosis</i>	<i>Count</i>
<i>Australia</i>	5.30	4.42	3.63	386	1.38	1.25	2.71	64
<i>Canada</i>	4.97	2.91	2.80	374	1.53	1.19	6.97	94
<i>Finland</i>	6.41	3.73	3.53	397	1.12	0.69	1.95	71
<i>Israel</i>	31.28	36.70	5.56	383	8.79	2.58	3.19	85
<i>Spain</i>	8.38	4.74	3.28	420	2.72	1.13	1.79	48
<i>Sweden</i>	6.07	2.93	2.73	395	1.45	1.63	2.50	72
<i>UK</i>	6.89	4.54	4.26	394	2.46	0.79	2.39	74

Notes: Displayed values are percentages.

**Table 2: Results of Lewbel test on inflation expectations in UK and Australia**

	<i>Pre-Targeting</i>		<i>Post-Targeting</i>	
	<i>UK</i>	<i>Australia</i>	<i>UK</i>	<i>Australia</i>
$\rho_1$ ( <i>mean</i> )	0.77	0.94	0.91	0.90
$\rho_2$ ( <i>variance</i> )	0.036	0.014	0.016	0.043

Notes: ARIMA (2,1,0) was used to derive the estimates in the above table. GARCH specifications were added when necessary.

<b>Table 3: Estimates of the fractional root from four estimation techniques</b>					
		<i>GPH</i>	<i>GSP</i>	<i>MPL</i> <sup>§</sup>	<i>FDML</i>
<i>Country</i>	<i>Period</i>	$\hat{d}$	$\hat{d}$	$\hat{d}$	$\hat{d}$
<i>Australia</i>	<i>Pre</i>	0.15* (1.67)	0.15** (2.50)	0.49** (6.00)	0.77** (5.50)
	<i>Post</i>	-0.08 (0.53)	-0.03 (0.33)	0.19** (7.71)	0.30** (3.75)
<i>Canada</i>	<i>Pre</i>	0.32** (3.55)	0.38** (6.33)	0.93 (0.70)	1.00** (14.3)
	<i>Post</i>	0.06 (0.43)	-0.01 (0.11)	0.02** (12.3)	0.23** (3.83)
<i>Finland</i>	<i>Pre</i>	0.43** (5.38)	0.37** (7.40)	0.57** (2.44)	0.87** (9.67)
	<i>Post</i>	0.19 (1.19)	0.05 (0.50)	0.19** (8.21)	0.30** (4.29)
<i>Israel</i>	<i>Pre</i>	0.66** (7.33)	0.52** (8.67)	0.83 (1.29)	0.89** (8.09)
	<i>Post</i>	0.05 (0.29)	0.03 (0.33)	0.04** (7.40)	0.47** (5.88)
<i>Spain</i>	<i>Pre</i>	0.73** (4.87)	0.20** (3.33)	1.20 (1.65)	0.85** (10.6)
	<i>Post</i>	-0.09 (0.26)	-0.01 (0.10)	0.29** (6.36)	0.49** (8.17)
<i>Sweden</i>	<i>Pre</i>	0.15* (1.87)	0.18** (3.00)	0.56** (2.20)	0.97** (6.47)
	<i>Post</i>	-0.13 (0.81)	-0.14 (1.40)	0.29** (2.87)	-0.05 (0.56)
<i>UK</i>	<i>Pre</i>	0.38** (4.75)	0.33** (5.50)	0.70 (1.48)	0.970** (5.39)
	<i>Post</i>	-0.18 (1.13)	-0.29** (2.90)	0.66** (2.46)	0.21** (2.98)

Notes: Estimation methods are log periodogram regression of Geweke & Porter-Hudak (GPH), Gaussian semiparametric estimation described in Robinson & Henry (GSP), frequency domain approximate MLE by Fox & Taqu (FDML), and time domain Modified Profile Likelihood suggested by Cox & Reid (MPL) and implemented by An & Bloomfield (1993). Values in the parentheses represent the t-statistics. \*\* (\*) indicates 95% (90%) significance. Significance in every test but MPL implies being significantly different than 0. Schwartz-Bayesian and Hannan-Quinn Criteria are used to determine the orders of AR and MA parameters in all the tests. We also add 1 to  $d$  estimate when data was differenced for stationarity prior to estimation.

<sup>§</sup>Statistics for the modified profile likelihood estimation are testing the null of  $d-1$  being significantly different than 0. Therefore, insignificance indicates that the coefficient is not significantly different than 1.

**Table 4: Monte Carlo simulation for small sample properties of Fox-Taqqu MLE**

$d$	Mean <i>(full sample)</i>	Std. Dev. <i>(full sample)</i>	Mean <i>(pre-target)</i>	Std. Dev. <i>(pre-target)</i>	Mean <i>(post-target)</i>	Std. Dev. <i>(post-target)</i>
0	-0.008	0.037	-0.009	0.039	-0.016	0.102
0.1	0.091	0.037	0.090	0.039	0.087	0.101
0.2	0.192	0.036	0.192	0.037	0.191	0.106
0.3	0.294	0.036	0.293	0.040	0.303	0.099
0.4	0.396	0.039	0.394	0.042	0.413	0.106
0.5	0.500	0.039	0.498	0.042	0.532	0.100
0.6	0.602	0.039	0.600	0.042	0.643	0.114
0.7	0.707	0.040	0.704	0.042	0.756	0.114
0.8	0.812	0.042	0.808	0.044	0.876	0.106
0.9	0.911	0.038	0.906	0.040	0.970	0.090
1.0	0.995	0.031	0.992	0.034	1.029	0.073

Notes: Sample of 500 is split into 425 for the 1<sup>st</sup> part and 75 for the 2<sup>nd</sup>. Results are from 3000 iterations. ARFIMA model  $(0, d, 0)$  is chosen for ease of display.

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## Endnotes

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<sup>1</sup> Long memory refers to the persistence of shocks that is caused by either a unit or a fractional root. In a fractionally integrated process, the differencing operator,  $d$ , in the lag polynomial,  $(1-L)^d$ , is allowed to assume fractional values.

<sup>2</sup> Sowell (1992) shows existence of long memory in aggregate economic activity, while Hassler & Wolters (1995), Baillie, Chung & Tieslau (1996) analyze fractional integration in aggregate prices. Andersen & Bollerslev (1997) and Liu (2000) provide evidence of long memory in asset price volatility.

<sup>3</sup> Recently Parke (1999) showed that a sequence of shocks with stochastic magnitude and duration can lead to long memory while Liu (2000) and Diebold & Inoue (2001) demonstrated that regime-switching processes can produce series that are observationally equivalent to fractional integration.

<sup>4</sup> Chambers (1998) extends this analysis by incorporating temporal aggregation to cross-sectional aggregation. Lippi & Zaffaroni (2000) broadens Granger's results by using a milder semiparametric specification.

<sup>5</sup> Information on individual-agent dynamic behavior can be derived from aggregate dynamics by utilizing a method as in Lewbel (1994) where he shows that the moments of koyck lag individual coefficients are going to equal the autoregression coefficients of the aggregate data.

<sup>6</sup> Granger chooses the beta distribution due to its mathematical convenience and adds that the choice of the distribution does not affect the results. Beta distribution is also flexible in terms of mimicking the normal and uniform distributions for particular values of  $p$  and  $q$ .

<sup>7</sup> Further details can be found in Granger (1980) and the appendix A1 of this paper.

<sup>8</sup> It is sufficient to concentrate on just the mean and variance of  $\alpha$  since the beta distribution has the convenient property of having recurrent central moments. Higher central moments contain the

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same information as the variance, so finding the relation of the degree of fractional differencing to higher moments would not alter our conclusions.

<sup>9</sup>  $\mu_n = [nC/(1+b)(1+n+b)]\mu_{n-1}$ . Consequently, the variance of  $\alpha$  is  $\sigma_\alpha^2 = \mu C/(b+2)(b+3)$

<sup>10</sup> These authors have found that forecast errors are not only serially correlated, but also correlated with past information.

<sup>11</sup> Such a system can be derived from the Phillips Curve equation. For such models  $\gamma$  equals 1. See Mankiw (2001) for an example.

<sup>12</sup> Values for  $K_{it}^*$  and  $\eta_{t+1}^{i*}$  can be found in appendix A2.

<sup>13</sup> This relation requires that the aggregate inflation expectations be cointegrated with the variable  $K_t$ , which is plausible since  $K_t$  represents variables like the output gap or money growth rate.

<sup>14</sup> The pioneer inflation targeting country, New Zealand, is not included in the analysis since it does not report a monthly price index.

<sup>15</sup> Data on index-linked bonds prior to their adoption of inflation exists only in these two countries.

<sup>16</sup> We have 498 observations for the countries used in the analysis.