Bertrand-Edgeworth Equilibrium in a Cash-in-Advance Economy

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Abstract

We introduce a dynamic general equilibrium model that exhibits two unusual features. The first is the presence of positive profits enjoyed by the producers in a stationary monetary competitive equilibrium despite constant returns technologies. The second is the supportability of such a competitive equilibrium as pure strategy Nash equilibrium of a Bertrand-Edgeworth price competition game if the number of firms is sufficiently large. This nice result is obtained under wage taking behavior. Under wage competition, however, the usual non-existence problems associated with Bertrand-Edgeworth models enter into play.

Keywords: Stationary competitive equilibrium, Bertrand-Edgeworth model, Nash equilibrium, cash-in-advance, price competition, wage competition, reserve supply, residual consumption.

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1 Introduction

Most widely studied models of price competition assume that product prices are set before production costs are incurred by the oligopolists. In these models, competitive equilibrium can be sustained as a Nash equilibrium if firms have to (Chamberlin 1933, Dastidar 1997), or choose to (Dixon, 1990), meet all their demand. As a result, a *reserve supply* from rival firms prevails. The availability of such a reserve supply, voluntary or enforced, eliminates the incentives of a firm to charge above the competitive price.

A similar reserve supply appears in equilibria of a discrete price setting game (Dixon, 1993) and of a carefully designed market mechanism (Dixon, 1992), where firms report their prices and 'capacities' simultaneously. If reserve supply of this type or the other is absent, it is well known that Bertrand-Edgeworth equilibrium fails to exist (Edgeworth, 1897), unless marginal costs are constant (Bertrand, 1883). If, however, firms first determine their capacities by producing output at a nonzero cost and enter, at the next stage, a round of price competition under the given capacities, then the Cournot production levels prevail in equilibrium (Kreps and Scheinkman, 1983).

In the present paper, we study, in a dynamic monetary model, the same sequencing of events as in Kreps and Scheinkman (1983); but obtain results similar to those in Dixon (1990, 1992), where competitive equilibrium is sustained as a Nash equilibrium in the Bertrand-Edgeworth price setting game. The key reason is that as in Dastidar (1995, 1997) firms have to meet all demand up to their capacity and that sales are costly in terms of foregone utility from current consumption. Higher sales help building up higher levels of working capital in the following periods. Nevertheless, there is a trade-off between current consumption and tomorrow's capacity as determined by tomorrow's working capital. This trade-off, together with the competition from the *reserve supply* of rivals, is sufficient to keep prices down at competitive equilibrium levels. *Undercutting* is bad because it yields too little current consumption while *charging more* is bad because it leads to too little sales and hence too little working capital for the future.

In the context of a cash-in-advance model, we relate the stationary monetary competitive equilibria of a financially constrained monetary economy to the Nash equilibria of a Bertrand-Edgeworth model of competition. By doing so, we attempt to partially fill the gap between dynamic general equilibrium models with price taking behavior on the one hand and the one shot Bertrand-Edgeworth price competition models or their repeated versions, on the other. We concentrate on the model of a monetary economy examined by Başçı and Sağlam (2001). This is a deterministic and simplistic version of the broader class of limited participation models introduced by Fuerst (1992), where firms face finance constraints in their short term factor payments. In the version we consider, the economy consists of infinitely lived agents and the markets operate via fiat money. There are two types of producer-consumers with constant returns technologies; we distinguish them with the high-tech and low-tech labels, depending on their productivities. To identify the high-tech type of agents, we also use the terms "firms" and "entrepreneurs" interchangeably. Similarly, for the low-tech types, we sometimes use the term workers.

The agents have type-specific initial labor endowments and money holdings and are allowed to produce as well as to transact in the spot labor and good markets under given market prices. They face cash-in-advance constraints in all markets. As an equilibrium concept, it is natural to study the stationary monetary competitive equilibrium (SMCE) in which the production, consumption, trading and saving decisions are, in each time period, feasible, optimal, and time invariant under the stationary competitive prices. Başçı and Sağlam (2001) show that the sequencing of the markets matters for the equilibrium of such a financially constrained monetary economy. In the case where good market opens before the labor market, SMCE exhibits the well celebrated equality of real wage and marginal product of labor and equivalently the equality of price and its marginal cost. However, in the "labor market first" case the equilibrium real wage is found to be below the marginal product of labor as a consequence of the cash-in-advance constraints imposed in the labor market, and therefore firms (the high-tech agents) obtain positive profits.

In this paper, we focus on SMCE of the "labor market first" case, to address the question as to whether monetary competitive equilibrium with positive producers' gains can be formalized as an equilibrium of a genuine model of price competition. We deal with Bertrand-Edgeworth model since in our model firms have endogeneous capacity constraints resulting from finance constraints in their labor market transactions. We prove that when all firms are wage takers, SMCE is a Nash equilibrium of the Bertrand-Edgeworth model if either (i) the number of firms in the market is sufficiently high, or (ii) the time preferences of firms are sufficiently small.

A result stating that every SMCE can be achieved through price competition if the economy is very "large" sounds obvious at first, since Walrasian equilibrium has always been associated with the idea that there are many traders in the market or that the size of any trader is very small with respect to the market size.[1] However, the literature on Bertrand-Edgeworth price competition aimed at establishing such a result typically faces problems of non-existence of pure strategy equilibria in the price setting game.[2] The problem in static setups was solved by Dixon (1990) and Dastidar (1995) when price competition takes place before production decisions. We show that in a dynamic model with consumption and working capital, Bertrand-Edgeworth equilibrium exists even if price competition takes place after production,

The remainder of the paper is organized as follows: Section 2 introduces the basic structure. Section 3 builds up a Walrasian framework for the monetary economy under consideration. Section 4 introduces the Bertrand-Edgeworth competition model associated with our financially constrained production economy and then presents the results. Finally, Section 5 concludes by reflecting on the results and their possible extensions.

2 Basic Structure

We consider an economy involving two commodities at each time t, labor and a nonstorable consumption good, apple. There are two types of agents indexed by $i = 1, 2$. There exist finite numbers, N_1 and N_2, of the first and second types in the economy. We assume $N_1, N_2 \geq 2$. Let $\mathcal{N}_i = (1, ..., N_i)$ be the set of agents of type i. Neither of the types value leisure, and their preferences over the lifetime consumption are in the same additively separable form given by $\sum_{t=0}^{\infty} \beta_i^t U_{ij}(c_{ijt})$, for all $j \in \mathcal{N}_i$ of type i. Here, $\beta_i \in (0, 1)$ is the common discount factor for all agents of type i, c_{ijt} the apple consumption and $U_{ij}(.)$ the instantaneous utility function of agent j of type i. We assume that U_{ij} is twice continuously differentiable, $U'_{ij}(.) > 0$, $U''_{ij}(.) < 0$ and $U_{ij}(0) = 0$ for all $j \in \mathcal{N}_i$ of type i.

Each agent of type 1 has a labor endowment $\bar{L}_1 > 0$ whereas each agent

[1] In fact the statement of Arrow (1959) saying that if it exists, a Bertrand equilibrium has to coincide with a Walrasian equilibrium forms a justification for this belief.

[2] Shubik (1959), Benassy (1989), Allen and Hellwig (1986a,b), Vives (1986), Börgers (1992).

of type 2 has a labor endowment $\bar{L}_2 = 0$. All agents of type i have a constant returns technology $f_i(L) = \gamma_i L$ to convert labor into apples with marginal product of labor equal to γ_i. We assume that type 2 agents own a superior know-how, and let $\gamma_2 = \gamma > 1$ and $\gamma_1 = 1$ without loss of generality. Since $\gamma > 1$, we identify type 1 and type 2 agents by the "low-tech" and the "high-tech" labels, respectively. Other than these production possibilities, there are no endowment of apples.

We will also assume that agent $j \in \mathcal{N}_i$ of type i is endowed with the money balance, $M_{ij,0} = \bar{M}_i$ at time zero. For obvious reasons, we require that $\bar{M}_1 + \bar{M}_2 > 0$, i.e. the total quantity of money initially held is strictly positive. Moreover, there is no further monetary intervention to the economy; so the total money stock remains constant.

We denote and describe a **society** by $\mathcal{S} = \langle N_i, \bar{L}_i, \bar{M}_i, U_{ij}, \beta_i, f_i | i = 1, 2$ and $j \in \mathcal{N}_i\rangle$, provided that all parameters listed obey the stated assumptions above.

Now given a society, a **trade institution** for that society is the description of choice variables for each type of agents, constraints on the given choice variables determined by given prices, and a feasibility requirement (market clearing) for the collective choices of agents. The choice variables of agent j of type i for the period t are listed below.

Choice Variables:

c_{ijt} : Consumption,
L_{ijt} : Labor demand ((+) demand, (-) supply),
q_{ijt} : Apple demand ((+) demand, (-) supply),
$M_{ij,t}$: Money balances in the beginning of the period,
w_{ijt} : Nominal wage rate,
p_{ijt} : Nominal apple price.

The timing of transactions is as follows: Agent j of type i starts period t with an initial money balance $M_{ij,t}$. First the labor market opens where labor can be purchased at the market wages $\langle w_{ijt} | i \in \{1, 2\}$ and $j \in \mathcal{N}_i\rangle$. All wage bills must have been paid before the good market opens. Then apple production takes place with the purchased and unsold labor. After the harvest of apples, good market opens where apple can be purchased at the market prices $\langle p_{ijt} | i \in \{1, 2\}$ and $j \in \mathcal{N}_i\rangle$. These transactions will determine the next period's money balance of each agent.

3 Walrasian Framework

We shall here consider that all agents take the wage and price as given. Let w_t and p_t be the market wage and apple price respectively, at time t.

Given the endowment structure assumed in the previous section and a sequence of strictly positive prices $\langle w_t, p_t \rangle_{t=0}^{\infty}$, we can write the lifetime utility maximization problem, (P_{ij}), of agent j of type i as follows:

$$(P_{ij}) \qquad \max \sum_{t=0}^{\infty} \beta_i^t U_{ij}(c_{ijt})$$

subject to, for all t,

$$c_{ijt} = f_i(\bar{L}_i + L_{ijt}) + q_{ijt},$$

$$-\bar{L}_i \leq L_{ijt} \leq \frac{M_{ij,t}}{w_t},$$

$$-f_i(\bar{L}_i + L_{ijt}) \leq q_{ijt} \leq \frac{M_{ij,t} - w_t L_{ijt}}{p_t},$$

$$M_{ij,t+1} = M_{ij,t} - w_t L_{ijt} - p_t q_{ijt},$$

$$M_{ij,0} = \bar{M}_i \geq 0.$$

The upper bound on labor purchases, L_{ijt}, comes from the cash-in-advance requirement and the assumption that the labor market opens first. The lower bound shows the maximum amount of labor that agent j of type i can sell. The constraints on apple purchases need to be similarly read, taking into account that the payments/receipts in apple market come after those in labor market.

We will call the trade institution that lets the labor market open first **financially constrained** by virtue of the fact that a producer is restricted in its labor purchases with the amount of money he holds in the beginning of each period. By a **financially constrained production economy** we mean a society $\mathcal{S}$ operating under a financially constrained trade institution, and denote it by $\mathcal{FCE}$. Now we can define our equilibrium concepts.

3.1 The Monetary Competitive Equilibrium

We say that $\langle p_t, w_t, c_{ijt}, L_{ijt}, , q_{ijt}, M_{ij,t+1} | i \in \{1,2\}$ and $j \in \mathcal{N}_i\rangle_{t=0}^{\infty}$ is a **Monetary Competitive Equilibrium (MCE)** of the financially constrained production economy $\mathcal{FCE}$, if $w_t, p_t > 0$ for all t, and

(i) $\langle c_{ijt}, L_{ijt}, q_{ijt}, M_{ij,t+1} | \, i \in \{1,2\}$ and $j \in \mathcal{N}_i\rangle_{t=0}^{\infty}$ solves (P_{ij})

(ii) $\sum_{i=1}^{2}\sum_{j=1}^{N_i} L_{ijt} = 0$ and $\sum_{i=1}^{2}\sum_{j=1}^{N_i} q_{ijt} = 0$ for all t

(iii) $\sum_{i=1}^{2}\sum_{j=1}^{N_i} M_{ij,t} = N_1\bar{M}_1 + N_2\bar{M}_2$ for all t.

Since the institution has the equal treatment property and since all agents of the same type i start with the same money balance $\bar{M}_i$, the above definition is stated in terms of the consumption, labor demand, good demand and money demand per representative agent within each type. The first condition is lifetime utility maximization under perfect foresight of future prices and price taking behavior. The second states the clearing of commodity markets. The third is the money market clearing.

In this paper, we will focus on the following monetary competitive equilibrium.

3.2 The Stationary Monetary Competitive Equilibrium

A **stationary monetary competitive equilibrium (SMCE)** of the financially constrained economy $\mathcal{FCE}$, is a MCE where the prices, wages, consumption levels, demands, supplies, money holdings are all constant over time. These constant values will be represented by the list $\langle p^*, w^*, c^*_{ij}, L^*_{ij}, q^*_{ij}, M^*_{ij} | \, i \in \{1,2\}$ and $j \in \mathcal{N}_i\rangle$.

The above definition can also be called a steady state equilibrium. It should be noted that the initial money distribution over the two types matters for the existence of a stationary equilibrium. So, in looking for SMCE, one should search not only for prices, but also for initial money distributions $(\bar{M}_1, \bar{M}_2)$ over types that will yield an allocation consistent with stationarity as well as conditions (i)-(iii) of monetary competitive equilibrium.

It is not very difficult to establish that SMCE exists only if $(\bar{M}_1, \bar{M}_2) = (0, M)$, where M denotes the quantity of nominal money stock per firm. Since all prices are constant over time, if a worker-type agent was to start with a positive initial money balance, she would bring it down to zero in finite time.[3] Using this reasoning, the following proposition characterizes the set of SMCE over the parameter space of (β_2, γ).

Proposition 1: *(Başçı and Sağlam (2001)) SMCE of a $\mathcal{FCE}$ exists if and only if $(\bar{M}_1, \bar{M}_2) = (0, M)$ and $\beta_2\gamma \geq 1$. Moreover, the set of SMCE is characterized by (1)-(10):*

$$w^* = \begin{cases} N_2 M/(N_1 \bar{L}_1) & \text{if } \beta_2\gamma > 1 \\ \bar{w} \in [N_2 M/(N_1 \bar{L}_1), \infty) & \text{if } \beta_2\gamma = 1 \end{cases} \tag{1}$$

$$p^* = \frac{w^*}{\beta_2\gamma} \tag{2}$$

$$L_{1j}^* = -N_2 M/(N_1 w^*), \quad j \in \mathcal{N}_1 \tag{3}$$

$$L_{2j}^* = M/w^*, \quad j \in \mathcal{N}_2 \tag{4}$$

$$q_{1j}^* = -\frac{w^*}{p^*} L_{1j}^*, \quad j \in \mathcal{N}_1 \tag{5}$$

$$q_{2j}^* = -\frac{w^*}{p^*} L_{2j}^*, \quad j \in \mathcal{N}_2 \tag{6}$$

$$c_{1j}^* = \bar{L}_1 + (1 - \frac{w^*}{p^*}) L_{1j}^*, \quad j \in \mathcal{N}_1 \tag{7}$$

$$c_{2j}^* = (\gamma - \frac{w^*}{p^*}) L_{2j}^*, \quad j \in \mathcal{N}_2 \tag{8}$$

$$M_{1j}^* = 0, \quad j \in \mathcal{N}_1 \tag{9}$$

$$M_{2j}^* = M, \quad j \in \mathcal{N}_2 \tag{10}$$

It is immediate to observe that whenever SMCE exists, the real wage is strictly below the marginal product of labor in the "high-tech" production

[3] A sketch of the proof of this statement is given by Stokey and Lucas (with Prescott, 1989, Exercise 5.17).

plant, and equals $w^*/p^* = \beta_2\gamma < \gamma$. This is due to the fact that money earned by the producers can only be spent in the following period. If $\beta_2\gamma > 1$ then SMCE exists and is unique. If $\beta_2\gamma = 1$ then there exists a continuum of SMCE that can be Pareto ranked, and SMCE does not exist if $\beta_2\gamma < 1$.

By using (2) and (4), we can rewrite (8) as

$$c_{2j}^* = (1 - \beta_2) f(M/w^*), \; j \in \mathcal{N}_2.$$

The above equation shows that the per period consumption (hence the life-time utility) of any producer is positive in equilibrium. ¿From (1) it then follows that whenever w^* is determined endogeneously, i.e., $\beta\gamma > 1$ case, each firm's per period consumption is positive if and only if the number of firms per workers, N_2/N_1, is finite. However, the aggregate consumption of the firms in the market is independent of their number and equals $(1 - \beta_2)\gamma \sum_{j \in \mathcal{N}_1} (\bar{L}_{1j})$. Low-tech agents, overall, consume the fraction β_2 of the total output produced in the high-tech plants while high-tech agents are left, for consumption, with whatever remains. It should also be noted that, the lower the time preference of firms (β_2) is, the higher is their total profits in equilibrium.

4 Competition under Bertrand-Edgeworth Model

We will characterize the conditions under which every stationary monetary competitive equilibrium of our economy is a Nash equilibrium of a Bertrand-Edgeworth model of competition.

We would like to argue first why we deal with Bertrand-Edgeworth model which assumes finite capacities. The firms in our economy are endowed with constant returns technologies so that they have no capacity constraints coming from the technology side. However, inability to meet demand may arise due to cash-in-advance constraints in the factor markets. Since the labor market opens before the good market, production level of each firm is determined before firms enter competition in the good market. Therefore, a firm, which aims to undercut its rivals in the good market by reducing its price in some period, may not cover the market demand in that period unless it has just produced a sufficiently large quantity of apples. A similar situation arises when firms compete in the labor market. The winning firm of the wage competition may not be able to hire the amount of labor that is needed to meet the good market demand, since firms face cash-in-advance constraints and have finite money holdings.

We assume that all agents in the economy have a recursive reasoning. They believe competition may occur only in the current period while they expect prices and wages to be in SMCE from the next period onwards. In addition, we restrict ourselves to the case where price and/or wage can be set by only type 2 agents. Type 1 agents are assumed to be both price and wage takers.

If high-tech agents charge different prices in any time period, consumers start buying from the firm(s) with the lowest price offer. Following the "equal shares" rationing rule, considered for example by Deneckere and Kovenock (1992), we assume that consumers with equal demands receive equal shares of the output available whenever excess demand exists at a certain price. With their residual money, consumers continue to buy from the lowest of the remaining price offers.

Likewise, workers first supply their labor to the firm(s) that bid the highest wage. Firms with the same wage offerings will employ equal shares of labor supply, and workers with the same supply share the market demand equally. If workers have excess labor supply, they offer it to the firm(s) with the second highest wage, and so on.

Products in the economy are homogeneous and firms have no relative advantage with respect to each other in marketing their products.

These last four assumptions are made to ensure that all firms using the same wage (product price) strategy will get (face) the same labor supply (commodity demand).

Let w_j and p_j be respectively the wage and the product (apple) price set by firm $j \in \mathcal{N}_2$ in the current period, and $\mathbf{w}/\mathbf{p} \equiv \langle w_j/p_j \rangle_{j \in \mathcal{N}_2}$ denote the vector of real wages charged by the firms in the economy. We define $\mathbf{w}_{-j}/\mathbf{p}_{-j} \equiv \langle w_i/p_i \rangle_{i \neq j}$, for notational convenience. Moreover, $\mathbf{w}^*/\mathbf{p}^* = \langle w^*/p^* \rangle_{j \in \mathcal{N}_2}$ denotes N_2-tuple of SMCE wages.

Let $\bar{U}_{2j}(\mathbf{w}/\mathbf{p})$ denote the lifetime utility of firm j when the market real wages are given by $\mathbf{w}/\mathbf{p}$ in the current period.

We say that $\mathbf{w}/\mathbf{p}$ is a Nash equilibrium if for all $j \in \mathcal{N}_2$

$$\bar{U}_{2j}(\mathbf{w}/\mathbf{p}) \geq \bar{U}_{2j}(\hat{w}_j/\hat{p}_j, \mathbf{w}_{-j}/\mathbf{p}_{-j}) \quad \text{for all } \hat{w}_j/\hat{p}_j \geq 0.$$

Proposition 2: *Assume that all firms are wage takers. If* $N_2 > 1/(1-\beta_2)$*, then every SMCE is a Nash equilibrium of the Bertrand-Edgeworth model.*

Proof: Since firms are wage takers by assumption, we have to check only for the absence of an incentive for any of the firms to deviate from SMCE good price p^*.

The lifetime utility of firm $j \in \mathcal{N}_2$ if every firm sells at price p^* is:

$$\begin{aligned} \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*) &= \sum_{t=0}^{\infty} \beta_2^t U_{2j}(c_{2j}^*) \\ &= \frac{1}{1-\beta_2} U_j(c_{2j}^*) \end{aligned}$$

Take and fix any $j \in \mathcal{N}_2$. Let $\bar{U}_{2j}(w^*/p_j, \mathbf{w}_{-j}^*/\mathbf{p}_{-j}^*)$ denote the lifetime utility of firm j when it unilaterally deviates from SMCE by setting a price $p_j \neq p^*$ in the current period while all other firms stick to SMCE price p^*. We have to consider two cases:

Case 1 – Cutting the price: $p_j < p^*$.

All consumers first buy from firm j in period $t = 0$. The revenue of firm j depends on its supply, which has been produced just before competition in the good market takes place. We have the following two subcases:

(i) $f_2(L_{2j}^*) \geq N_2 M/p_j$.

The current supply, $f_2(L_{2j}^*)$, of firm j is at least as high as its demand, which is $N_2 M/p_j$. The deviating firm, by receiving the whole money stock in the economy, expects to hire the market labor supply, $N_1 \bar{L}_1$, at SMCE real wages w^*/p^*, from period $t = 1$ onwards. The lifetime utility of firm j is given by

$$\bar{U}_{2j}(w^*/p_j, \mathbf{w}_{-j}^*/\mathbf{p}_{-j}^*) = U_{2j}\left(f_2\left(\frac{M}{w^*}\right) - \frac{N_2 M}{p_j}\right)$$

$$
\begin{aligned}
& + \sum_{t=1}^{\infty} \beta_2^t U_{2j} \left(f_2 \left(\frac{N_2 M}{w^*} \right) - \frac{N_2 M}{p^*} \right) \\
< \; & U_{2j} \left(f_2 \left(\frac{M}{w^*} \right) - \frac{N_2 M}{p^*} \right) \\
& + \frac{\beta_2}{1-\beta_2} U_{2j} \left((1-\beta) f_2 \left(\frac{N_2 M}{w^*} \right) \right).
\end{aligned}
$$

Using (1), (4) and (8) yield $c_{2j}^* = f_2(N_2 M/w^*) - M/p^* = (1-\beta) f_2(N_2 M/w^*)$. Inserting c_{2j}^* into the above inequality, we obtain

$$
\begin{aligned}
\bar{U}_{2j}(w^*/p_j, \mathbf{w}_{-j}^*/\mathbf{p}_{-j}^*) &< U_{2j}\left(c_{2j}^* - (N_2-1)M/p^*\right) + \frac{\beta_2}{1-\beta_2} U_{2j}(N_2 c_{2j}^*) \\
&< \frac{1}{1-\beta_2} U_{2j}\Big((1-\beta_2)(c_{2j}^* - (N_2-1)M/p^*) \\
&\quad + \beta_2 (N_2 c_{2j}^*)\Big) \\
&= \frac{1}{1-\beta_2} U_{2j}(c_{2j}^*) \\
&= \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*).
\end{aligned}
$$

Thus $\bar{U}_{2j}(w^*/p_j, \mathbf{w}_{-j}^*/\mathbf{p}_{-j}^*) < \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*)$.

(ii) $f_2(L_{2j}^*) < N_2 M/p_j$.

Firm j cannot meet the market demand in period $t = 0$ if it unilaterally reduces its price below the SMCE level. By selling all its output, firm j obtains a revenue $f_2(L_{2j}^*)p_j$, but then consumes nothing, hence derives no utility in period $t = 0$. Firm j believes that it will produce, sell and earn as much as in SMCE from period $t = 1$ onwards. The lifetime utility of firm j is then given by

$$
\bar{U}_{2j}(w^*/p_j, \mathbf{w}_{-j}^*/\mathbf{p}_{-j}^*) = \sum_{t=1}^{\infty} \beta_2^t U_{2j} \left(f \left(\frac{f_2(L_{2j}^*)p_j}{w^*} \right) - \frac{f_2(L_{2j}^*)p_j}{p^*} \right)
$$

$$
\begin{aligned}
&= \frac{\beta_2}{1-\beta_2} U_{2j}\left(f\left(\frac{f_2(L_{2j}^*)p_j}{\beta_2\gamma p^*}\right) - \frac{f_2(L_{2j}^*)p_j}{p^*}\right) \\
&< \frac{1}{1-\beta_2} U_{2j}\left((1-\beta_2)\frac{f_2(L_{2j}^*)p_j}{p^*}\right) \\
&< \frac{1}{1-\beta_2} U_{2j}\left((1-\beta_2)f_2(L_{2j}^*)\right) \\
&= \frac{1}{1-\beta_2} U_{2j}(c_{2j}^*) \\
&= \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*).
\end{aligned}
$$

Thus, $\bar{U}_{2j}(w^*/p_j, \mathbf{w}_{-j}^*/\mathbf{p}_{-j}^*) < \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*)$.

Case 2 – Raising the price: $p_j > p^*$.

In period $t = 0$, all consumers first buy from the remaining $(N_2 - 1)$ firms who stick to SMCE price p^*. The assumption that $N_2 > 1/(1-\beta_2)$ implies $(N_2 - 1)f_2(L_{2j}^*) > \beta_2 N_2 f_2(L_{2j}^*) = N_2 M/p^*$, that is, the aggregate supply of the remaining $N_2 - 1$ firms who has not deviated from SMCE covers all of the market demand, $N_2 M/p^*$, in period $t = 0$. Thus, firm j is not able to sell in that period and must consume its whole supply $f_2(L_{2j}^*)$, as no storage is possible. Receiving no cash in the good market in period $t = 0$, which cash would be needed to finance the required wage payments in period $t = 1$, firm j stops producing and hence consuming after period $t = 0$ (since $\bar{L}_2 = 0$). Firm j's lifetime utility then satisfies

$$
\begin{aligned}
\bar{U}_{2j}(w^*/p_j, \mathbf{w}_{-j}^*/\mathbf{p}_{-j}^*) &= U_{2j}\left(f_2(L_{2j}^*)\right) \\
&\le \frac{1}{1-\beta_2} U_{2j}\left((1-\beta_2)f_2(L_{2j}^*)\right) \\
&= \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*).
\end{aligned}
$$

Thus, $\bar{U}_{2j}(w^*/p_j, \mathbf{w}_{-j}^*/\mathbf{p}_{-j}^*) \le \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*)$. *Q.E.D.*

It is interesting that a firm cannot not become better off by selling at a lower price in the current period than SMCE price p^*, even though at such a price, which may still exceed the marginal cost of producing apples, the firm would face the whole market demand. To understand this result, note that firms enter the good market by having already produced as much as they could (in proportion to their money balances at the beginning of each period) irrespective of the apple price which will emerge in the market. When a firm undercuts its rivals and gets the whole market demand, it cannot increase its supply accordingly within the same period, and therefore it sells more than what the optimal (lifetime) trading plan suggests. So the first period consumption of such a deviating firm is less than that would be chosen in SMCE. The assumptions that utility functions are concave and β_2 is less than one yield the result that the decrease in consumption leads to a fall in the utility of the deviating firm in period zero, which more than offsets the increase in the overall utility thereafter, implying that price cutting is never optimal for the firms.

Proposition 2 also shows that in an economy involving more than $1/(1-\beta_2)$ firms, no firm can ever become better off by selling at a price higher than p^* in the current period.

In Bertrand-Edgeworth models, the reason for the failure of a competitive equilibrium to be supported as a pure strategy Nash equilibrium is the presence of an incentive to increase the price above the competitive equilibrium level. In our model, we have showed, price cutting is never optimal. So if SMCE ever fails to be supported as a Nash equilibrium in the price competition game, it must be due to the profitability of increasing the price.

If a firm unilaterally raises its product price above p^* in the current period, all consumers first buy from the remaining $N_2 - 1$ firms that stick to p^*. The deviating firm cannot sell anything in the current period if the number of firms is sufficiently large, i.e., $N_2 > 1/(1-\beta_2)$, for in this case the supplies of any $N_2 - 1$ out of N_2 firms will cover the market demand at SMCE prices. Receiving no money from the good market, the deviating firm can neither produce nor consume from the next period onwards, and must consume all of its current supply in the same period as there is no storage available. The concavity of the utility functions implies that the deviating firm then becomes worse off in the overall, which shows that no wage taking firm will unilaterally deviate from SMCE.

An immediate corollary of Proposition 2 is that when firms' common time preference β_2 is very close to zero, the market demand for apples, which is linearly increasing in β_2, becomes so small that it can be covered by the supply of any single producer in the market. Thus, in markets with "very myopic" producers, price competition under Bertrand-Edgeworth model may lead to SMCE if the number of firms is at least two, i.e., "two" is enough for competition.[4]

Another direct corollary would state that in a market involving arbitrarily large number of firms the market share of any single firm becomes so small that the aggregate supply of any $N_2 - 1$ of N_2 firms can cover the market demand unless β_2 is equal to one, implying that no firm in a "large" economy has an incentive to deviate from SMCE.

However, if the number of firms in the market is not sufficiently large, any firm j who unilaterally raises its price above the SMCE level may still receive demand as consumers may be left with residual money after buying from the firms in $\mathcal{N}_2 \backslash \{j\}$ that have not deviated from SMCE. In fact, due to the concavity of the utility functions, the time preference β_1 being less than one and the absence of a credit market, consumers may find it optimal to spend all of their residual money on the good supplied by firm j in the current period if the price differential $p_j - p^*$ is not too large. But even in that case, the revenue firm j obtains is less than that in SMCE, since the remaining firms sell and hence earn more than they do in SMCE, due to a multiplied demand (by $N_2/(N_2-1)$) for their products. A firm which unilaterally raises its current price will then be facing a trade-off between the increase in the first period consumption (utility) and decrease in the consumption (utility) thereafter. The next proposition considers this trade-off so as to characterize some additional conditions under which every SMCE is Nash equilibrium.

Proposition 3: *Assume all firms are wage takers, and there is a firm $j \in \mathcal{N}_2$ such that $U'_{2,j}(0) < \infty$. Then there exists some $\beta_2^a, \beta_2^b \in ((N_2 - 1)/N_2, 1)$ satisfying $\beta_2^a \leq \beta_2^b$ such that (i) for any given $\beta_2 \in ((N_2 - 1)/N_2, \beta_2^a)$, every SMCE is a Nash equilibrium of the Bertrand-Edgeworth model, (ii) for any given $\beta_2 \in (\beta_2^b, 1)$, no SMCE is a Nash equilibrium of Bertrand-Edgeworth model.*

[4] Of course, too small β_2 may lead to the nonexistence of SMCE itself. However, if labor is assumed to be supplied inelastically at all real wage levels by the workers, then this problem does not appear.

Proof: Let $j \in (1, ..., N_2)$ and $N_2 < 1/(1 - \beta_2)$. Suppose that firm j unilaterally deviates from SMCE in the current period by setting $p_j > p^*$.

Then all consumers will first buy from the remaining $N_2 - 1$ firms who stick to price p^*. The demand for the products priced at p^* is $N_2 M/p^*$ while $(N_2 - 1) f_2(L_{2j}^*)$ is the supply. The assumption that $N_2 < 1/(1 - \beta_2)$ implies an excess demand of the amount $N_2 M/p^* - (N_2 - 1) f_2(L_{2j}^*) > 0$. According to the equal shares rationing rule, each consumer $i \in \mathcal{N}_1$ will then get

$$c^s = \frac{N_2 - 1}{N_1} f_2(L_{2j}^*)$$

after transacting with $(N_2 - 1)$ firms which sell at price p^*. Moreover, each consumer $i \in \mathcal{N}_1$ will be left with a residual money M^r, where

$$M^r = \frac{1}{N_1}[N_2 M - (N_2 - 1) f_2(L_{2j}^*) p^*].$$

Consumers face the problem of how to optimally spend their residual money M^r over time. Let $m_{i,t}$ denote the money each consumer $i \in \mathcal{N}_1$ holds at time t out of her residual money at time zero, $m_{i,0} = M^r$.

Let $\tilde{p}$ denote the sequence of product prices consumers will face over time after they buy from the $(N_2 - 1)$ firms in period $t = 0$:

$$\tilde{p} = \begin{cases} p_j & if \;\; t = 0 \\ p^* & if \;\; t > 0 \end{cases}$$

Without spending any part of her residual money, every consumer can consume at least c^s in the current period. ¿From the next period onwards, each consumer $i \in \mathcal{N}_1$ expects to consume c_{1i}^* even if she transfers no residual money from period zero to the future periods, since she, being boundedly rational, believes that prices and wages will be as in SMCE from $t = 1$ onwards. Let $\tilde{c}$ denote the endowment, i.e., the sequence of "status-quo" quantities of apples that agent $i \in \mathcal{N}_1$ expects to consume over time:

$$\tilde{c}_i = \begin{cases} c^s & if \;\; t = 0 \\ c_{1i}^* & if \;\; t > 0 \end{cases}$$

Each agent $i \in \mathcal{N}_1$ has then the following problem to solve:

$$\max_{\{m_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta_1^t U_{1,i}(c_{it})$$

subject to, for all t

$$m_{i,t+1} = m_{i,t} - \tilde{p}_t(c_{i,t} - \tilde{c}_{i,t}),$$

$$m_{i,t+1} \geq 0,$$

$$m_{i,0} = M^r > 0,$$

where $c_{i,t}$ is the consumption of agent i in period t. Solving for $c_{i,t}$ from the first constraint we can rewrite the objective function of each agent $i \in \mathcal{N}_1$ as

$$\max_{\{m_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta_1^t U_{1,i}\left(\tilde{c}_{i,t} + \frac{1}{\tilde{p}_t}(m_{i,t} - m_{i,t+1})\right).$$

We know that for $\tilde{p}_0 = p^*$, type 1 consumer will choose the SMCE consumption, c_{1i}^*, which satisfies the below Euler condition for all t:

$$U'_{1,i}(c_{1i}^*) > \beta_1 U'_{1,i}(c_{1i}^*)$$

This condition, together with the corresponding $m_{i,t} = 0$ for all t and the transversality condition, which is automatically satisfied due to stationarity of the consumption plan, is necessary and sufficient for maximization (Başçı and Sağlam, 2001).

Now, suppose that $\tilde{p}_0 > p^*$ is a price at which consumers strictly prefer to spend all of their residual money in period $t = 0$, i.e., $m_{i,t} = 0$ for all $t > 0$. In this case, too, the Euler condition must be satisfied as a strict inequality in period $t = 0$, that is

$$U'_{1,i}(c_{i,0}) \quad > \quad \beta_1 \frac{\tilde{p}_0}{\tilde{p}_1} U'_{1,i}(c_{i,1}),$$

which can be rewritten as

$$U'_{1,i}\left(c_{1i}^* - M^r(\frac{1}{p^*} - \frac{1}{p_j})\right) \quad > \quad \beta_1 \frac{p_j}{p^*} U'_{1,i}(c_{1i}^*),$$

using the fact that $c_{i,0} = c^s + M^r/p_j$ and $c_{i,1} = \tilde{c}_{i,1} = c^*_{1i}$ under the presumption $m_{i,t} = 0$ for all $t > 0$. To show that there exists p_j satisfying the above inequality, let us define $p_\epsilon \equiv p^* + \epsilon$, where $\epsilon > 0$. Inserting $p_j = p_\epsilon$ into the above inequality and calculating the limits of both sides as ϵ goes to zero, we get

$$\lim_{\epsilon \to 0} U'_{1,i}\left(c^*_{1i} - M^r \left(\frac{1}{p^*} - \frac{1}{p_\epsilon}\right)\right) = U'_{1,i}(c^*_{1i}) > \beta_1 U'_{1,i}(c^*_{1i}) = \lim_{\epsilon \to 0} \beta_1 \frac{p_\epsilon}{p^*} U'_{1,i}(c^*_{1i}).$$

It follows from the continuity of $U_{1,j}$'s that there exists an interval $[p^*, p^c)$ such that $U'_{1,i}\left(c^*_{1i} - M^r((p^*)^{-1} - (p_j)^{-1})\right) > \beta_1 (p_j/p^*) U'_{1,i}(c^*_{1i})$ for all $p_j \in [p^*, p^c)$.

Suppose now firm j sets $p_j \in (p^*, p^c)$. It then gets the whole residual money $N_1 M^r$ of consumers in the current period. We can write firm j's lifetime utility as

$$\begin{aligned}
\bar{U}_{2j}(w^*/p_j, \mathbf{w}^*_{-j}/\mathbf{p}^*_{-j}) &= U_{2j}\left(f_2(L^*_{2j}) - \frac{N_1 M^r}{p_j}\right) \\
&\quad + \sum_{t=1}^{\infty} \beta_2^t U_{2,j}\left((1-\beta_2) f_2\left(\frac{N_1 M^r}{w^*}\right)\right) \\
&= U_{2j}\left(f_2(L^*_{2j}) - \frac{N_1 M^r}{p_j}\right) \\
&\quad + \frac{\beta_2}{1-\beta_2} U_{2,j}\left((1-\beta_2) f_2\left(\frac{N_1 M^r}{w^*}\right)\right).
\end{aligned}$$

Notice first that we have

$$\lim_{\beta_2 \to \frac{N_2-1}{N_2}} \bar{U}_{2j}(w^*/p_j, \mathbf{w}^*_{-j}/\mathbf{p}^*_{-j}) = U_{2j}(f_2(L^*_{2j})) < \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*)$$

by the fact that

$$\begin{aligned}
\lim_{\beta_2 \to \frac{N_2-1}{N_2}} M^r &= \lim_{\beta_2 \to \frac{N_2-1}{N_2}} M - c^s p^*, \\
&= \lim_{\beta_2 \to \frac{N_2-1}{N_2}} (M/N_1)[N_2 - (N_2-1)/\beta_2] = 0.
\end{aligned}$$

Since $U_{2,j}$'s are continuous by assumption, it follows that there exists $\beta_2^a \in ((N_2-1)/N_2, 1)$ such that (i) holds.

Now let j be such that $U'_{2,j}(0) < \infty$. We will show that for β_2 sufficiently close to one, firm j has an incentive to deviate from SMCE price. Note that we obtain

$$\begin{aligned}
\lim_{\beta_2 \to 1} \bar{U}_{2j}(w^*/p_j, \mathbf{w}^*_{-j}/\mathbf{p}^*_{-j}) &= \lim_{\beta_2 \to 1} U_{2j}\left(f_2(L^*_{2j}) - \frac{N_1 M^r}{p_j}\right) \\
&\quad + \lim_{\beta_2 \to 1} f_2\left(\frac{N_1 M^r}{w^*}\right) U'_{2,j}(0), \\
&= U_{2j}\left(f_2(L^*_{2j}) - \frac{M}{p_j}\right) + f_2\left(\frac{M}{w^*}\right) U'_{2,j}(0), \\
&> f_2\left(\frac{M}{w^*}\right) U'_{2,j}(0), \\
&= \lim_{\beta_2 \to 1} \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*),
\end{aligned}$$

using $\lim_{\beta_2 \to 1} M^r = \lim_{\beta_2 \to 1}(M/N_1)[N_2-(N_2-1)/\beta_2] = M/N_1$ and $f_2(L^*_{2j}) - M/p_j > f_2(L^*_{2j}) - M/p^* = c^*_2 > 0$.

Since $U_{2,j}$ is continuous for all $j \in \mathcal{N}_2$, it follows that there exists $\beta_2^b \in (N_2-1)/N_2, 1)$ such that (ii) holds, which completes the proof. *Q.E.D.*

Now we turn to the wage competition case, under which SMCE, generically, is not supported as a Nash equilibrium.

Proposition 4: *Assume that firms can set wages in the labor market. Then SMCE is a Nash equilibrium of the Bertrand-Edgeworth model if and only if* $w^*/p^* = 1$.

Proof: Recall first from Proposition 1 that SMCE exists if and only if $\beta\gamma \geq 1$ and $w^*/p^* = \beta\gamma$. Now suppose that firm j unilaterally deviates from SMCE wage in period $t = 0$. Let us consider two cases:

(i) $\beta\gamma > 1$.

In this case, $w^*/p^* = \beta\gamma > 1$ and there exists a unique SMCE, by Proposition 1. Suppose that firm j sets $w_j \in [p^*, w^*)$. When the remaining (N_2-1) firms stick to w^*, they will get the whole demand in the labor market; but they can hire no more labor than they do in SMCE, for there exist cash-in-advance constraints in all markets. The deviating firm j will then get the residual labor supply, which is L^*_{2j}. After the labor market transactions in the current period, firm j will be left with the residual money $M^r = M - w_j L^*_{2j}$. Suppose that firm j spends $(N_2-1)/N_2 M^r$ of its residual money on apples in the current period, while saving M^r/N_2 for the next period. The workers will spend all their wage earnings, $(N_2-1)M + w_j L^*_{2j}$, in the good market in period $t=0$. Firm j will then obtain $(1/N_2)$th of the total sales after the good market closes in the current period. Adding up its sales revenue and saving out of its residual money, we can calculate the money holding of firm j from $t=1$ onwards as

$$\frac{1}{N_2}[(N_2-1)M + w_j L^*_{2j}] + \frac{M^r}{N_2} = M,$$

which implies that every other firm will continue to hold M as well, so SMCE prices are still feasible after the current period. The lifetime utility of firm j can then be written as

$$\begin{aligned}
\bar{U}_{2j}(w_j/p^*, \mathbf{w}^*_{-j}/\mathbf{p}^*_{-j}) &= U_{2j}\left(f_2(L^*_{2j}) - \frac{(N_2-1)M + w_j L^*_{2j}}{N_2 p^*} + M^r \frac{N_2-1}{N_2 p^*}\right) \\
&\quad + \sum_{t=1}^{\infty} \beta_2^t U_{2,j}(c^*_{2j}) \\
&= U_{2j}\left(f_2(L^*_{2j}) - \frac{w_j L^*_{2j}}{p^*}\right) + \frac{\beta_2}{1-\beta_2} U_{2,j}(c^*_{2j}) \\
&= U_{2j}\left(f_2(L^*_{2j})\left(1 - \beta_2 \frac{w_j}{w^*}\right)\right) + \frac{\beta_2}{1-\beta_2} U_{2,j}(c^*_{2j}), \\
&> U_{2j}\left(c^*_{2j}\right) + \frac{\beta_2}{1-\beta_2} U_{2,j}(c^*_{2j}), \\
&= \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*).
\end{aligned}$$

Thus, for any given β_2 satisfying $\beta_2\gamma > 1$, there exists $w_j/p^* \in [1, \beta_2\gamma)$ such that $\bar{U}_{2j}(w_j/p^*, \mathbf{w}^*_{-j}/\mathbf{p}^*_{-j}) > \bar{U}_{2j}(\mathbf{w}^*/\mathbf{p}^*)$, implying that SMCE is not Nash equilibrium of the Bertrand-Edgeworth model if $\beta_2\gamma > 1$.

(ii) $\beta\gamma = 1$.

In this case, $w^*/p^* = 1$ and there exist infinitely many SMCE, by Proposition 1. Let us show that $w_j/p_j = 1$ is a Nash equilibrium strategy for every firm $j \in \mathcal{N}_2$.

Take and fix any $j \in \mathcal{N}_2$. Note that firm j would choose $w_j/p_j \geq 1$ in a price competition game, for otherwise it could not attract any labor and would get zero lifetime utility. Let us first suppose that firm j sets $w_j/p_j > 1$ such that $w_j > w^*$. In that case, firm j can hire less labor than in SMCE. Hence by producing and consuming less, it obtains smaller lifetime utility. Next suppose $p_j < p^*$. In this case, firm j would not be better off as was explained in the proof of Proposition 2.

Therefore, unilateral deviation from SMCE is not profitable if and only if w^*/p^* is equal to one. *Q.E.D.*

Notice that regardless what SMCE wage rate is, no firm can ever become better off by unilaterally raising its nominal wage rate in the current period above the SMCE level due to the cash-in-advance constraints. A profitable deviation from SMCE in (nominal) wage competition may then arise only through wage cutting. When the common time preference of firms, β_2, is equal to their common marginal labor cost of production $1/\gamma$, SMCE wage rate turns out to be "one", the critical wage below which low-tech agents refuse to work for the high-tech agents. Therefore, SMCE is a Nash equilibrium when $w^*/p^* = 1$. On the other hand, when $w^*/p^* = \beta\gamma > 1$, there exists a room for profitable deviations from SMCE by the high-tech agents, since in that case low-tech agents can be made to accept working at any real wage rate in the interval $[1, \beta\gamma)$.

5 Concluding Remarks

A widespread belief is that under CRTS technologies, competition wipes out pure profits. In our version of a limited participation model, equilibrium good prices are above the marginal costs in every period and hence firms make positive profits. It is a natural question to ask whether price competition among sufficiently large number of wage taking firms can always eliminate those profits. Our answer to this question is no. The presence of pure profits is not due to the lack of price competition but is merely due to the presence of financial constraints. Such positive profits can be attributed to the fiat money that firms hold as working capital.

A second widespread belief is that price competition with a *large* number of small firms leads to the price taking behavior and hence to a Walrasian equilibrium. In this paper, we have given a justification for this belief in a dynamic economy with residual consumption of sellers.

A 'good market first' scenario would mean the elimination of financial constraints of firms. It was shown by Başçı and Sağlam (2001) that under such a scenario residual consumption for firms would not be present since pure profits would be swept out in a competitive equilibrium. Obviously, in such an economy, due to the constant marginal cost structure, the classical Bertrand result follows: Competitive equilibrium is supported by price competition even with two firms.

The literature relating Walrasian equilibria to the Nash equilibria of Bertrand-Edgeworth models has extensively studied single-shot markets. Shubik (1959) established that a Bertrand-Edgeworth model with homogeneous goods has no equilibrium in pure strategies when costs are strictly convex. It was proved by Benassy (1989) that the same result holds in heterogeneous goods markets as well if products are sufficiently similar. Allen-Hellwig (1986a,b) and Vives (1986) showed that the mixed strategy Nash equilibria of a Bertrand-Edgeworth model converge in distribution to the Walrasian equilibrium when the number of competitors is very large. More recently, Börgers (1992) found that the price strategies in a Bertrand-Edgeworth model which survive the iterated elimination of dominant strategies are very close to the Walrasian equilibrium price (i) if any $n-1$ out of n firms in the market have sufficient capacity to cover demand at marginal costs, (ii) if any given total capacity is owned by a large number of very small firms.

In our dynamic model, we established both the existence of a Nash equilibrium in a Bertrand-Edgeworth price competition game and that it coincides with the corresponding price taking equilibrium (SMCE) for a rich set of parameter values. This result, which is obtained under wage taking behavior and hence under sole price competition, hinges upon the presence of a positive *residual consumption* for the 'high-tech' agents, a factor that has been missing in the static formulations.

It is also interesting to observe that when time preferences of firms are very small, SMCE turns out to be Nash equilibrium for every market with at least two firms. At the other extreme, we find that for any given finite number of firms, SMCE is not Nash equilibrium if time preferences of firms are sufficiently close to one.

In the case where wage competition is allowed, the usual non-existence problems of Bertrand competition models with capacity constraints show themselves. In this case there is no counterpart of the residual consumption observed under price competition. Therefore, in line with the classical remark of Edgeworth, the only possible case, for supporting a 'Walrasian' equilibrium as a 'Bertrand' equilibrium, would be a highly elastic labor supply. In our model this can happen for parameter values that make the labor supply curve infinitely elastic in equilibrium.

The dynamic nature of the problem is needed to make fiat money valued and to justify, on reputational grounds, the behavioral assumption that a producer aims to meet all the demand it faces at its quoted price, even when it is suboptimal to do so (cf. Dixon (1990), Dastidar (1997)). Such a reputational story is not described in detail here. Nevertheless, it may be realistic to assume that a seller would in fact suffer more by turning down a customer, than by fulfilling this extra demand and consume less in this period. In fact the inventory theory models in the literature are built upon this assumption. These models study a trade-off between holding less inventories and turning down less consumers under a stochastic demand.

Another implicit assumption in our model is the 'recursive reasoning' used by our agents. More precisely, we have assumed that our workers and entrepreneurs take forecasts of future prices as given and take for granted that they will be able to buy and sell any amount that their budget and production technology allows, at these prices. In other words, agents act in a Walrasian fashion regarding future prices and only choose current prices. This 'recursive reasoning', though fulfilled in equilibrium as any 'conjectural equilibrium' would be, is, in fact, a boundedly rational way of thinking. To see this, one may assume that a single producer regards the whole sequence of prices as a choice. Then, especially with a small number of firms, a policy of undercutting the rivals for a sufficiently long period of time and in that way collecting all the cash in the economy, and afterwards following the monopolistic pricing rule to extract the whole social surplus, is definitely a different way of reasoning that may indeed be optimal if all others follow a recursive reasoning. A full strategic analysis where the whole path of future prices are taken as choice variables requires a more sophisticated investigation and is outside the scope of this paper (see Benoit and Krishna, 1987, for a related model with multiple equilibria).

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