

# Should equilibria require that players are uninformed? Arguing against the Trigger Strategy and Simple Optimal Penal Code in Repeated Matching Games.\*

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## Abstract

In repeated matching games trigger strategies and simple optimal penal codes are not equilibria unless strong informational assumptions are made. If only local information is used by the strategy and other weak information flows are allowed both strategies fail.

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## 1 Introduction

When can equilibria depend on players being uninformed? The general answer is when nobody who has the information will reveal it. This criterion is not met in repeated matching games and thus under reasonable informational assumptions trigger strategies are never equilibria and the simple optimal penal code sometimes fails to exist.

Trigger strategies were the first method used to enforce cooperation in the repeated game, and are still the most common. Friedman [4] used these strategies to establish the first folk theorem, and in the modern literature many papers analyze these strategies alone. The simple optimal penal code (Abreu [1]) is undoubtedly the most important strategy in the literature. With the simple

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optimal penal code, all equilibrium paths of a game can easily be found—a large and complicated space. In fact much of the appeal of trigger strategies is that they are often simple optimal penal codes. Both strategies were first developed for the standard repeated game under full information, and like most models this one is most useful as an abstraction of more realistic models. If these strategies fail in more realistic models then their usefulness is questionable.

One more realistic model is the repeated matching game, in which people can interact with anyone in a (large) group rather than always the same people. In extending the standard model the most important question is how much information players know. Assuming full information is only reasonable if the group is small, and this assumption is rarely met even in a group the size of most academic departments. In a group the size of a small town the assumption is untenable, and a more restrictive assumption must be found.

A viable alternative is to assume that players always check references, or use *local information* (Okuno-Fujiwara and Postlewaite [11]). Not only is this technique common in empirical examples, but Hasker [5] shows it can be an equilibrium using costly messages (allowing for lies.) Under this assumption, trigger strategies and simple optimal penal codes do exist—indeed Okuno-Fujiwara and Postlewaite [11] use trigger strategies to prove a folk theorem—however one has to assume that players have absolutely no other information sources. Global information sources like rumors and newspapers must be ruled out. If one allows for weak global information flows then trigger strategies and the simple optimal penal code are no longer equilibria.

As Kandori [7] first noted, the desirability of local information is that players can get this information, not that they never get more. While this other information might not be reliable enough to depend on assuming players dismiss the information is strong. Thus for his folk theorem Kandori required that people only use local information even if they have full information. While this restriction is good for a folk theorem, it seems too strong to analyze equilibria. Thus in this paper I consider a weaker model of information flows—one that replicates rumors—in which players might receive one bit of information about one other player in each period. Most global information sources have more of an impact than I am proposing, thus this is a weak test of an equilibria’s viability.

This paper is also a test of whether these equilibria are viable with weaker assumptions on information. For example, the equilibria that use no information processing in Kandori [7] and Ellison [3] are questionable based on my analysis. Also Ahn and Suominen [2] consider the supplier’s quality provision problem where consumers are clustered into informational neighborhoods, and only consider trigger strategies. The effect of global information flows is to allow a player to “see into his future,” while weak informational assumptions require the strategy to be only affected by a player’s past. Both trigger strategies and simple optimal penal codes fail under this type of introspection.

Under full information, related research suggests that trigger strategies are, again, not viable.

Mailath and Morris [9] show this in an environment of almost complete information—where there is a small and independent probability of people being misinformed about the past. In a repeated matching game ignoring this problem implies that thousands of messages never have an error. The intuition behind this result is the opposite of that here, in their paper the problem is that “looking into his past” a player can be convinced they were mistaken. This result implies, for example, that Klein and Leffler’s [8] seminal work on the quality provision problem should be reassessed.

To illustrate that these results are not sensitive to the matching rule I show that they hold in two extreme cases. The first matching rule is the uniform—the most diffused—and the second is the “Looped Townsend Turnpike,” the least diffused. In the Townsend Turnpike (Townsend [12]) one of the populations is arranged in a line, and each player in the other population moves one player forward each period. I loop this turnpike to remove the endgame effect. With the uniform matching rule players are the most uncertain about who they will interact with, and with the Looped Townsend Turnpike players are completely certain.

## 2 The Model

In a *two player repeated matching game* there are two population— $I_r$  and  $I_c$ —with  $N \in [3, \infty)$  members. Let  $I_r = \{0, 1, 2, \dots, N-1\}$  and  $I_c = \{N, N+1, N+2, \dots, 2N-1\}$ . Players are matched in each period  $t \in \{1, 2, 3, \dots\}$ . For all  $i \in I_r, j \in I_c$  the uniform matching rule is:

$$P(i \text{ and } j \text{ interact in period } t) = \frac{1}{N} \quad (1)$$

the Looped Townsend Turnpike matching rule is:<sup>1</sup>

$$P(i \text{ and } j \text{ interact in period } t) = \begin{cases} 1 & \text{if } i = (t+j) \bmod N \\ 0 & \text{else} \end{cases} \quad (2)$$

Let  $j$  be the person interacting with  $i$  in period  $t$ . When matched  $l \in P_k$  takes an action from  $A_k$  for  $l \in \{i, j\}$   $k \in \{r, c\}$ . The payoff from  $a \in A_r \times A_c$  is written  $\pi_k(a)$  with population  $k$ ’s action written first. The value for a player of population  $k$  from the sequence of action profiles  $\{a^t\}$  will be  $\sum_{t=1}^{\infty} \delta^t \pi_k(a^t)$  for  $\delta \in (0, 1)$ .

We write strategies as *social norms*  $\{Z, \sigma, \tau\}$ . The social status of player  $i \in I_r \times I_c$  in period  $t$  is denoted  $z_i^t \in Z$  where  $Z = \{0, 1, 2\}$  or  $Z = \{0, 1\}$  is the *set of social statuses*. The *social standard of behavior* is  $\sigma : Z^2 \rightarrow A_r \times A_c$ . The *transition rule*,  $\tau$ , takes a player’s social status— $z_i^{t-1}$ , action she took last period— $a_i^{t-1}$ , and the action she was supposed to take— $\sigma_i^{t-1}$ ; and the same information for her opponent in period  $t$  and returns  $z_i^t$ . Social norms use *local information processing* since  $\tau$  is only affected by  $\{z_i^{t-1}, a_i^{t-1}, \sigma_i^{t-1}\}_{l \in \{i, j\}}$ .

We will be interested in sequential equilibria which are *durable to informational trembles*.

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<sup>1</sup>where  $(t+j) \bmod N$  is the remainder of  $\frac{t+j}{N}$ .

**Definition 1** An informational tremble is when player  $i \in I_r \times I_c$  observes the social status of  $l \in I_r \times I_c$ , where the distribution of observations in period  $t$  satisfies  $P(i \text{ observes } z_l^t) > 0$  for all  $l \in I_r \times I_c$   $\tau \leq t$ .

**Definition 2** A strategy is durable to informational trembles if there exists an  $\varepsilon > 0$  such that if the probability of an informational tremble is less than  $\varepsilon$  the strategy is a sequential equilibrium.

### 3 The Failure of Trigger Strategies.

Undeniably the most popular strategies in the analyst’s toolbox are the trigger strategies; they are even referred to as Grimm strategies because of their “folktale nature.” Okuno-Fujiwara and Postlewaite [11] used these strategies since durability to informational trembles was not an issue—with a continuum population and uniform matching any countable number of deviations has probability zero. Due to their unappealing nature Kandori [7] required equilibria to be *globally stable* in order to rule out trigger strategies, however this was unnecessary since he required a strong version of durability to informational trembles. The problem with trigger strategies is that “bad news is permanent,” once a player deviates he plays a static Nash equilibrium forever. At the same time local information requires that players who have not met this person still cooperate. Thus if there is any weak information flow this information must spread, and cooperation will collapse. To illustrate this consider the following prisoner’s dilemma:

$$\begin{array}{cc}
 & \begin{array}{c} I_c \\ C \quad D \end{array} \\
 \begin{array}{c} I_r \\ C \\ D \end{array} & \begin{array}{|c|c|} \hline C & \begin{array}{c} 1, 1 \\ -1, 2 \end{array} \\ \hline D & \begin{array}{c} 2, -1 \\ 0, 0 \end{array} \\ \hline \end{array}
 \end{array} \tag{3}$$

Let  $Z = \{0, 1\}$ , write the transition rule as a two step function  $\tau = \{\tau^n, \tau^x\}$ . The first step is:

$$\zeta_i = \tau^n(z_i^{t-1}, a_i^{t-1}, \sigma_i^{t-1}) = \begin{cases} 1 & \text{if } a_i^{t-1} \neq \sigma_i^{t-1} \\ z_i^{t-1} & \text{otherwise} \end{cases} \tag{4}$$

the second step is

$$z_i^t = \tau^x(\zeta_i, \zeta_j) = \begin{cases} 0 & \zeta_i = \zeta_j = 0 \\ 1 & \text{otherwise} \end{cases} \tag{5}$$

The social standard of behavior is

$$\sigma(z_i^t, z_j^t) = \begin{cases} \{C, C\} & \text{if } z_i^t = z_j^t = 0 \\ \{D, D\} & \text{otherwise} \end{cases} \tag{6}$$

In this strategy a player is “good” if her status is zero. If her status is good she cooperates with all players she meets, otherwise she always defects. Notice this is certainly an equilibria with local

information since every player will be “surprised” with the news their current partner has defected and cooperate up to that moment. But weak information flows overturn this peculiar result, as I now prove in proposition 1.

**Proposition 1** *The trigger strategy is not durable to informational trembles.*

**Proof.** In the Looped Townsend Turnpike assume player 1 deviates in period one. Then for player  $2N - 2$  independent of her action in period two her value from period three on will be zero, thus she will defect in period two. With uniform matching assume all players but 1 and  $N$  deviate in the first period, and that  $N$  is matched with 1 for at least the first  $N + 1$  periods. Then with strictly positive probability in period  $N$  player  $N$  will realize that all other  $N - 1$  players have defected and it will be her best response to defect since  $N \geq 3$  implies:

$$\pi_c(D, C) - \pi_c(C, C) > \frac{\frac{\delta}{N}}{1 - \frac{\delta}{N}} \pi_c(C, C) \quad (7)$$

■

First notice that this is not a case in which no one who has the information that  $2N - 2$  or  $N$  should defect will not want to reveal it. In fact everyone who knows that these players should defect will have no reason to withhold it. I do not submit a mechanism by which this information can be transmitted, but rumors are frequently circulated. A player with a “good heart” would tell others to deviate while they can benefit from it, and if the payment is small enough can be paid for the information. Note as well that simultaneous deviations were not necessary, in fact only one deviation is needed in either case.

One intuitive criticism of trigger strategies is that they are too “unforgiving.” Why should one small error ruin a player’s future? In repeated matching games players must be forgiven not for themselves but for others, to give others a reason to cooperate.

## 4 A Stage Game without a Simple Optimal Penal Code.

The simple optimal penal code (Abreu [1]) is one of the seminal results in standard repeated games. It simplifies the task of finding all of the equilibria from that of finding a potentially countable set to that of finding one. The simple optimal penal code is the set of all players’ worst equilibrium paths, one for each player. If any path is an equilibrium it is an equilibrium when after any deviation the continuation path is that player’s worst.

Since many trigger strategies are simple optimal penal codes I have already shown that simple optimal penal codes are not always equilibria. However the proof above is based on “bad news” accumulating, and there is a second dimension to the problem. This problem involves what can be done about simultaneous deviations. Under full information simultaneous deviations can be

ignored immediately, under local information they cannot, and weak information flows makes this a problem. Simultaneous deviators can learn of each other before they interact, and as will be shown this can cause them to defect again. Consider the stage game:

		$I_c$			
		$H_r$	$H_c$	$M_r$	$M_c$
$I_r$	$H_r$	4, 2	3, 3	0, -2	0, -2
	$H_c$	3, 3	2, 4	0, -2	0, -2
	$M_r$	-2, 0	-2, 0	-4, 0	-5, -5
	$M_c$	-2, 0	-2, 0	-5, -5	0, -4

(8)

Let  $\delta = \frac{1}{2}$ , the form of the stage game means that the unique worst equilibrium path for each population ( $k \in \{r, c\}$ ) has the simple form (from any period  $t$  on):

$$\underline{w}(k) = \begin{cases} \{M_k, M_k\} & \text{in period } t \\ \{H_k, H_k\} & \text{else} \end{cases} \quad (9)$$

It can be easily verified that if  $\underline{w}(k)$  is the initial path and the path after a deviation by either player, then this is an equilibrium. It must be the worst since its discounted value is zero. Finally it is unique since only  $\{M_r, M_r\}$ ,  $\{M_c, M_c\}$  and  $\{H_k, H_l\}$   $k, l \in \{r, c\}$  can be part of any equilibrium path when  $\delta = \frac{1}{2}$ ,  $\{M_k, M_k\}$  must be part of the punishment path in order to get the discounted payoff to zero, and after  $\{M_k, M_k\}$  is played in any period in all future periods  $\{H_k, H_k\}$  must be played.

To make this into a social norm let the set of social statuses be  $Z = \{0, 1, 2\}$ , and write the transition rule as  $\tau = \{\tau^n, \tau^x\}$ .

$$\zeta_i = \tau_i^n(z_i^{t-1}, a_i^{t-1}, \sigma_i^{t-1}) = \begin{cases} 2 & \text{if } a_i^{t-1} \neq \sigma_i^{t-1} \\ 1 & \text{else if } z_i^{t-1} = 2 \\ z_i^{t-1} & \text{else} \end{cases} \quad (10)$$

$$z_i^t = \tau_i^x(\zeta_i, \zeta_j) = \begin{cases} 0 & \text{if } \zeta_i \leq \zeta_j \\ \zeta_i & \text{else} \end{cases} \quad (11)$$

Notice that simultaneous deviations are ignored once players interact but cannot be until that time. The social standard of behavior for any action profile  $a \in A_r \times A_c$  is (for  $k \in \{r, c\}$ ):

$$\sigma(z_i^t, z_j^t) = \begin{cases} \{M_k, M_k\} & \text{if } z_i^t = 2, i \in I_k \\ \{H_k, H_k\} & \text{if } z_i^t = 1, i \in I_k \\ a & \text{if } z_i^t = z_j^t = 0 \end{cases} \quad (12)$$

But this strategy is not an equilibrium independent of  $a$ .

**Proposition 2** *The simple optimal penal code is not an equilibrium in the looped Townsend turnpike or uniform repeated matching game.*

**Proof.** Note that  $\pi_k(M_k, M_k) + \frac{\delta}{1-\delta}\pi_k(H_k, H_k) = 0$  for  $k \in \{r, c\}$ . Thus if a player of population  $k$  expects to receive  $\pi_k(M_k, M_k)$  today he *must* expect to receive  $\pi_k(H_k, H_k)$  forever

in the future. Now consider the following sequence of play in the Looped Townsend Turnpike. In period one player 1 and player  $2N - 2$  deviate. In period three 1 will interact with player  $2N - 2$ . If both 1 and  $2N - 2$  are informed of the other's social status in period two, then both of them can not expect to receive  $\pi_k(H_k, H_k)$  with probability one in period three. Thus one or both will deviate.

With uniform matching if they are not matched in period two then both know with probability  $\frac{1}{N}$  they can not both receive  $\pi_k(H_k, H_k)$  in period three thus one or both will deviate. ■

Here only the original deviators have the information and would not mind sharing it, and also they can only profit if there is another person who deviated simultaneously. Thus the argument for the exchange is weaker than above, but still assuming local information does not rule out other information sources. For example, if the players are husband and wife they should share this information. More generally this could cause players to form informational “neighborhoods” (Ahn and Suominen [2]), and it could make great news for the evening broadcast. Repeated matching games without some global information flows are too rare to be interesting.

## 5 Conclusion

In repeated matching games how much information players have is an important issue. The analyst clearly should not assume that they have too much, but neither should she ignore that they might have more. If players might have more information than the strategy uses, trigger strategies and simple optimal penal codes are not equilibria. While a weak folk theorem can be proven in this environment (Hasker [6]), the set of equilibria is critically different from standard repeated games.

While trigger strategies are not equilibria in Mailath and Morris [9] as well, the reason is diametrically different. In Mailath and Morris [10] they show rule out *separating* strategies—a strategy which has two paths with an infinite number of different actions. Such strategies might be equilibria here, the simple optimal penal code above is both separating and an equilibrium if  $\delta \geq \frac{2}{3}$ . In Mailath and Morris a long enough sequence of “bad” observations can convince players that they were mistaken about the past since no one is ever sure of what has happened. Thus the problem is backward looking, while here the problem is forward looking.

However I conjecture that the results of both papers on trigger strategies are not coincidental. While these strategies are analytically simple that simplicity also means they are sensitive to underlying assumptions. There is a set of strategies that satisfies both almost common knowledge and weak information flows but trigger strategies are not a subset. Without a doubt other important refinements also rule out these strategies.

A natural question at this point is how to find equilibria that are not sensitive to global information flows. The easiest methodology would be to require, like Kandori [7], that the strategy is

still an equilibrium if players have full information. Any equilibria ruled out by this refinement is essentially sensitive to “informational arbitrage.” In a repeated matching game should equilibria require that players are uninformed?

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