ECON 204 Midterm
Risk Aversion, Game Theory, and Welfare

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at or a little after 18:40 and last 100 minutes.

1. *(10 points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I further promise neither to help others nor use electronic devices. In specific I promise not to use a calculator.

   Name and Surname: _

   Student ID: _

   Signature:

   _

2. *(15 points total)* About Pareto efficiency and Nash equilibrium.

   (a) *(3 points)* Define what it means for an allocation A to Pareto dominate an allocation B.

   Definition 1 *This means that everyone prefers A to B and some people like A strictly better than B.*

   (b) *(3 points)* Define what it means for an allocation to be Pareto efficient.

   Definition 2 *It means that their is no (feasible) allocation that Pareto dominates it.*

   In other words, it means any change that makes some strictly better off makes others strictly worse off.

   (c) *(3 points)* Define a best response.

   Definition 3 *For any given (mixed) strategy of the other players, this is the set of optimal responses for a given player.*

   For a less technical definition, this means given what you expect others to do you are doing the best for yourself you can.

   (d) *(3 points)* Define a Nash equilibrium.

   Definition 4 *First of all, you can simply say it is any strategy at the intersection of Best Responses.*

   Alternatively it can be a (mixed) strategy σ^* such that for all i and $\sigma_i \in \Sigma_i = \Delta(S_i)$ $u_i(\sigma^*) \geq u_i(\sigma_i, \sigma^*_{-i})$

   Finally it can be considered as the intersection of two concepts:

i. Rationality—*there is a* $\beta^*_{-i} \in \times_{j \neq i} \Sigma_i$ *such that* $\sigma_i^* \in \arg\max_{\sigma_i \in \Sigma_i} u_i\left(\sigma_i, \beta^*_{-i}\right)$

ii. Correct Expectations—$u_i\left(\sigma^*\right) = u_i\left(\sigma_i^*, \beta^*_{-i}\right)$

(e) *(3 points)* Why is a Nash equilibrium only Pareto efficient by coincidence?

Solution 5 *I hate to say, but I don't really understand the question. Pareto efficiency analyze the decisions of a kind hearted social planner (she likes people to be happy) while Nash equilibrium is about the intersection of individual incentives.*

They are radically different problems, so why should they have the same solution?

To explain this better, the social choice problem is unconstrained. Basically the planner can choose anything she likes, while Nash equilibrium is all about individual incentives. The only coincidence is both like more utility, but best response (and thus Nash equilibrium) is about getting the most *utility given what others are doing—among a small feasible set. It would be more surprising if they were the same.*

And yes, I didn't explain this in class. So? "Questions don't have to make sense, answers do."

Remark 6 *And for those of you who answered "but the game below has a Pareto efficient Nash equilibrium and therefore the two concepts are the same." Wow, not only do you not know how to find a best response but you also missed the obvious clue that the Nash equilibria should not be Pareto efficient. You really are not good at Game Theory.*

Remark 7 *Your performance on question 3 is critical. If you did well on this question you probably will do well in topics using Game Theory in the future, if you did not then you really should avoid those classes.*

3. *(26 points total)* Consider the following strategic form game, like always player 1 chooses the row and player 2 chooses the column:

	α	β	δ	χ
A	$a+b+c+d; a+b+c+d^1$	$0; a+b+c+d+e^2$	$-d; a+b$	$d; c+d$
B	$a; b$	$a+b; b+c^{12}$	$0; b$	$a; c$
C	$c; 0$	$a; a$	$b; a+c^{12}$	$a+b+c+d; c^1$
D	$b; -e$	$b; b+c$	$-a; a+b$	$a+b+d; a+b+c^2\,(SE)$

	α	β	δ	χ
A	$4; 2$	$0; 2$	$6; 5^{12}$	$4; 3$
B	$3; 0$	$2; 7^{12}$	$4; 4$	$14; 3^1$
C	$14; 14^1$	$-5; 6$	$0; 15^2$	$5; 8$
D	$2; -1$	$-4; 6$	$2; 5$	$11; 9^2\,(SE)$

	α	β	δ	χ
A	$0;5$	$7;6^{12}$	$2;5$	$2;1$
B	$-2;7$	$5;6$	$5;-3$	$11;8^{2}\,(SE)$
C	$5;3^{12}$	$2;2$	$1;0$	$12;1^{1}$
D	$-4;7$	$0;15^{2}$	$12;12^{1}$	$4;5$

	α	β	δ	χ
A	$0;3$	$5;2$	$8;5^{12}$	$5;3$
B	$3;7^{12}$	$11;2^{1}$	$5;5$	$2;0$
C	$-5;8$	$9;10^{2}\,(SE)$	$3;5$	$3;-4$
D	$-1;8$	$1;3$	$0;15^{2}$	$11;11^{1}$

	α	β	δ	χ
A	$-3;4$	$1;-4$	$6;9^{2}\,(SE)$	$1;6$
B	$0;1$	$3;1$	$3;5$	$4;6^{12}$
C	$1;8^{12}$	$5;0$	$11;5^{1}$	$3;3$
D	$-2;4$	$11;11^{1}$	$2;7$	$0;15^{2}$

(a) *(10 points)* Carefully explain player two's best response to C. Also find all the pure strategy best responses, you may mark them on the table above but you will loose two points if you do not explain your notation below.

Solution 8 *I will base my answers off of the abstract game:*

	α	β	δ	χ
A	$a+b+c+d;a+b+c+d^{1}$	$0;a+b+c+d+e^{2}$	$-d;a+b$	$d;c+d$
B	$a;b$	$a+b;b+c^{12}$	$0;b$	$a;c$
C	$c;0$	$a;a$	$b;a+c^{12}$	$a+b+c+d;c^{1}$
D	$b;-e$	$b;b+c$	$-a;a+b$	$a+b+d;a+b+c^{2}$ (SE

all the other games are this game with permuted rows and columns and the abstract coefficients are replaced with numbers.

To find the best response to C we look for the maximum value of the second payoff in the third row. Since $u_2(C,\delta) = a+c > \max\{a,c\} = \max\{u_2(C,\beta), u_2(C,\chi)\} > \min\{a,c\} = \min\{u_2(C,\beta), u_2(C,\chi)\} > 0 = u_2(C,\alpha)$ *this is δ.*

In all cases, I mark a 1 in the upper right hand corner if it is best response for player 1 and a 2 for player 1.

Remark 9 *So many of you missed one of the two subquestions here. If you got 8 of 10 you should be embarrassed about your reading skills, but you did fine.*

(b) *(3 points)* Find the pure strategy Nash equilibria.

Solution 10 *They are the boxes in each game with both a 1 and a 2 in the corner.*

For either one it is a Nash equilibrium because it is the intersection of best responses, or in other words for both people given what they other is doing they would not change their choice if they could.

Notice—as was obviously hinted at in the last question—neither is Pareto efficient. In the abstract game:

$$
\begin{aligned}
u(B,\beta) &= (a+b;b+c) << (a+b+c+d;a+b+c+d) = u(A,\alpha) \\
u(C,\delta) &= (a+b;b+c) << (b;a+c) = u(A,\alpha)
\end{aligned}
$$

Sequential Version Now consider the variation of this game where player 1 chooses first and then player 2 chooses after observing what player 1 has chosen.

(c) *(6 points)* Write down player 2's optimal strategy—be sure to be clear about which action is taken down after which strategy by player 1.

Solution 11 *In the game:*

	α	β	δ	χ
A	$a+b+c+d;a+b+c+d^1$	$0;a+b+c+d+e^2$	$-d;a+b$	$d;c+d$
B	$a;b$	$a+b;b+c^{12}$	$0;b$	$a;c$
C	$c;0$	$a;a$	$b;a+c^{12}$	$a+b+c+d;c^1$
D	$b;-e$	$b;b+c$	$-a;a+b$	$a+b+d;a+b+c^2$ $(SE$

it is $(\beta(A), \beta(B), \delta(C), \chi(D))$

The most important criteria is that you recognize player 2's strategy has four different actions in it, and that you clarify which action is taken after which strategy of player 1.

(d) *(3 points)* Find the subgame perfect equilibrium.

Solution 12 *Player 1's part of the strategy is marked with* (SE) *in each game above. In the game:*

	α	β	δ	χ
A	$a+b+c+d;a+b+c+d^1$	$0;a+b+c+d+e^2$	$-d;a+b$	$d;c+d$
B	$a;b$	$a+b;b+c^{12}$	$0;b$	$a;c$
C	$c;0$	$a;a$	$b;a+c^{12}$	$a+b+c+d;c^1$
D	$b;-e$	$b;b+c$	$-a;a+b$	$a+b+d;a+b+c^2$ $(SE$

we would show this by adding another column, denoted $BR_2(X)$ *where we enter the payoffs of players 1 and 2 if* X *is used.*

	α	β	δ	χ
A	$a+b+c+d;a+b+c+d^1$	$0;a+b+c+d+e^2$	$-d;a+b$	$d;c+d$
B	$a;b$	$a+b;b+c^{12}$	$0;b$	$a;c$
C	$c;0$	$a;a$	$b;a+c^{12}$	$a+b+c+d;c^1$
D	$b;-e$	$b;b+c$	$-a;a+b$	$a+b+d;a+b+c^2$ $(SE$

and we can see that $a+b+d$ *gives the highest payoff in this column, so player 1 will play* D.

(e) *(4 points)* In a general game, will player 1's payoff go up in the sequential version? Go down? Or can we not tell? Also for player 2 will it go up, down or can we not tell?

Solution 13 *"First Mover's Advantage" tells us that player 1's payoff will (weakly) go up (* in comparison to pure strategy Nash equilibria*), and that we have no idea what will happen to player 2's payoff.*

Remark 14 *Many of you answered this for the game in this question. It is disappointing that you clearly did not understand what "general" means and you did not ask. I found this to be true so many times throughout this exam.*

Remark 15 *The order of the next several questions is different on different exams. I didn't find anyone who had copied from their neighbors.*

4. *(16 points total)* Consider an alternating offer bargaining game with two periods.

In period 1, player 1 makes an offer, $f_1 \in \{0, 1, 2, 3\}$. Player two can accept the offer (A_1) or reject the offer (R_1) and then play will continue to period two.

In period 2, player 2 makes an offer, $f_2 \in \{0, 1, 2\}$ and then player one can either accept it (A_2) or reject it (R_2).

If f_1 is accepted player 1 gets $3 - f_1$ and player 2 gets f_1. If f_1 is rejected but f_2 is accepted player 2 gets $2 - f_2$ and player 1 gets f_1. If both f_1 and f_2 are rejected both players get zero.

Period 2 analysis First analyze the period two interaction.

(a) *(4 points)* Write down all of the best responses of player 1 (the person who accepts or rejects the offer.) Be sure your notation is clear.

Solution 16 *We will need to respond to the offer of* $\{0, 1, 2\}$ *For* $x \in \{1, 2\}$ *we notice that* $x > 0$ *so accepting is the best response, but player 1 is indifferent between accepting and rejecting a payoff of zero. Thus the BR's are:*

$$(A(0), A(1), A(2))$$
$$(R(0), A(1), A(2))$$

(b) *(4 points)* For each of the best response of player 1, find the optimal offer of player 2 (f_2). Write down the equilibrium utilities of player 2.

$$BR_2((A(0), A(1), A(2))) = 0,\ u_2 = 2$$
$$BR_2((R(0), A(1), A(2))) = 1,\ u_2 = 1$$

Period 1 Analysis Now find the subgame perfect equilibria.

(c) *(4 points)* For each possible utility that player 2 can get in the second period, write down all the best responses.

If $(R(0), A(1), A(2))$ then $u_2 = 1$ and thus player 2 will reject f_1 if $f_1 < 1$, accept $f_1$if $f_1 > 1$ and be indifferent if $f_2 = 1$, thus their best responses can be:

$$(R(0), A(1), A(2), A(3))$$
$$(R(0), R(1), A(2), A(3))$$

If $(A(0), A(1), A(2))$, $u_2 = 2$ so they will reject any offer strictly below 2 and accept any offer strictly above 2 or:

$$(R(0), R(1), A(2), A(3))$$
$$(R(0), R(1), R(2), A(3))$$

(d) *(4 points)* Find the subgame perfect equilibria of the entire game.

Solution 17 *Can you guess why this is so few points? Because it's so hard. Here is the list of all the equilibria.*

Period 2 strategy	*Period 1 strategy*	f_1^*	f_2^*	u_1	u_2
$(A(0), A(1), A(2))$	$(R(0), R(1), A(2), A(3))$	2	0	1	2
$(A(0), A(1), A(2))$	$(R(0), R(1), R(2), A(3))$	3	0	0	3
$(R(0), A(1), A(2))$	$(R(0), A(1), A(2), A(3))$	1	1	2	1
$(R(0), A(1), A(2))$	$(R(0), R(1), A(2), A(3))$	2	1	1	2

5. *(16 points total)* About Risk Aversion.

(a) *(4 points)* Define what it means to be risk averse.

Definition 18 *A person is risk averse if they prefer the expected outcome of any lottery to the lottery itself.*

(b) *(5 points)* Explain why consumer's risk aversion will motivate stores to have a return policy.

Solution 19 *In short it is a form of insurance, and like any insurance we are willing to pay more than the expected cost.*

(c) *(3 points)* Define the coefficient of relative risk aversion.

Solution 20

$$rr(w) = -\frac{u''(w)}{u'(w)\,w}$$

Remark 21 *You know what really angered me? People who derived it's value for CRRA utility functions. I did not ask that and frankly I almost marked you down for making a hard job (grading exams) that much harder.*

(d) *(2 points)* What does it mean for person a to be more risk averse than person b?

Solution 22 *It means that any lottery a is willing to choose b will be willing to choose.*

(e) *(2 points)* If person a is more risk averse than person b is a's coefficient of relative risk aversion higher than b's? Lower? Or can we not tell?

Solution 23 *It should be higher.*

6. *(17 points total)* About adverse selection and signalling.

(a) *(4 points)* Define adverse selection.

Definition 24 *Adverse selection is the tendency for the people you do not want to do business wanting the most to do business with you. This is only a problem in a model of asymmetric information—where you can not tell which sellers are high or low quality.*

(b) *(3 points)* Explain how adverse selection might cause markets to collapse.

Solution 25 *The price always has to be based on the average worth, and the highest worth being sold will always be strictly higher than the average. Thus if buyers do not value a good sufficiently over the sellers, they will not be willing to pay enough to keep the maximal quality in the market. This can cause a vicious spiral, where the best drop out, drive down the average, and then this repeats.*

Remark 26 *For full credit I was looking for you to explain the spiral—i.e. how having some high quality goods dropping out can force medium quality goods to drop out.*

(c) *(4 points)* Define a signal—be sure that your definition is for economics, general answers will get no credit.

Definition 27 *A signal is an action for which the (marginal) benefit is lower than the (marginal) cost, but it is taken to hide or reveal information about the person taking the action.*

(d) *(3 points)* Give an example of a signal that is used to reveal something about the person signalling.

Solution 28 *There are so many, of course one can claim that a University education can be like this.*

Certification can be another example, trustworthy certification is intended to establish a quality level for a good—but of course it has to be trustworthy.

Warranties are another excellent example. Since the cost of the warranty is basically the cost of repairs, an extended warranty is a very loud signal that you expect the cost of repairs to be low. Hyundai, for example, started offering a seven year warranty for this reason.

Remark 29 *Some people tried to use the same example for both parts d and e. For example one person tried to use raising in poker as an example for both. While an argument can be made for it being both, using it for both lessens its value for either.*

I know that—for example—university education has both separating (revealing info) and pooling (hiding info) in it. In that case I talk about the group that is being separated from (high school degrees) or a characteristic of the individual that makes it pooling (flunking a lot of classes).

(e) *(3 points)* Given an example of a signal that is used to hide something about the person signalling.

Solution 30 *An example I keep returning to is how all students graduate from high school even though they didn't need to in my hometown.*

Remark 31 *An interesting example of pooling that* ***so*** *many of you used was people without work still getting up every morning to "go to work." I assume this spread from one student to all of you (or from a tutor, in which case congratulations to that tutor). It is an excellent example.*