ECON 204 Sec 01
Quiz 4
Dr. Kevin Hasker

1. *(3 Points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I also promise neither to help others nor to use calculators or other electronic devices.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(17 points)* Consider the following stage game.

   Remark 1 *Just to be clear, all of these games where based on the same abstract game with rows and columns changed and different values for the constants. (x, y) depends on the game (it is always the pair of strategies that has a utility of $(a + b, a + b)$).*

	L	C	R
U	$a+b; a+b$	$0; b$	$a; a+b+c$
M	$a+b+c; a+c$	$-b; a+b+c+d$	$a+b+c; a+b+c$
D	$b; 0$	$a; a$	$a+b+c+d; -b$

$$(x, y) = (U, L)$$

a	b	c	d
2	4	3	5

	L	C	R
U	$6; 6$	$0; 4$	$2; 9^2$
M	$9; 5^1$	$-4; 14^2$	$9; 9$
D	$4; 0$	$2; 2^{12}$	$14; -4^1$

$$\delta^* = \frac{3}{7}$$

	L	C	R
U	$0; b$	$a+b; a+b$	$a; a+b+c$
M	$-b; a+b+c+d$	$a+b+c; a+c$	$a+b+c; a+b+c$
D	$a; a$	$b; 0$	$a+b+c+d; -b$

$$(x, y) = (U, C)$$

a	b	c	d
3	2	4	6

	L	C	R
U	$0; 2$	$5; 5$	$3; 9^2$
M	$-2; 15^2$	$9; 7^1$	$9; 9$
D	$3; 3^{12}$	$2; 0$	$15; -2^1$

$$\delta^* = \frac{2}{3}$$

:

	L	C	R
U	$0; b$	$a; a+b+c$	$a+b; a+b$
M	$a; a$	$a+b+c+d; -b$	$b; 0$
D	$-b; a+b+c+d$	$a+b+c; a+b+c$	$a+b+c; a+c$

$$(x, y) = (U, R)$$

a	b	c	d
6	3	2	4

	L	C	R
U	$0; 3$	$6; 11^2$	$9; 9$
M	$6; 6^{12}$	$15; -3^1$	$3; 0$
D	$-3; 15^2$	$11; 11$	$11; 8^1$

$$\delta^* = \frac{2}{5}$$

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	L	C	R
U	$a; a$	$b; 0$	$a+b+c+d; -b$
M	$-b; a+b+c+d$	$a+b+c; a+c$	$a+b+c; a+b+c$
D	$0; b$	$a+b; a+b$	$a; a+b+c$

$$(x, y) = (D, C)$$

a	b	c	d
4	5	1	3

	L	C	R
U	$4; 4^{12}$	$5; 0$	$13; -5^{1}$
M	$-5; 13^{2}$	$10; 5^{1}$	$10; 10$
D	$0; 5$	$9; 9$	$4; 10^{2}$

$$\delta^* = \frac{1}{6}$$

(a) *(6 points)* Find the pure strategy best responses, you may mark them on the table above but you will loose two points if you do not explain your notation below.

Solution 2 *I mark them with a 1 in the upper right hand corner for player 1, and a 2 for player 2.*

(b) *(2 points)* Find the Nash equilibrium, explain why it is a Nash equilibrium.

Solution 3 *It is the unique square with both a 1 and 2 in the upper right hand corner. It is a Nash equilibrium because for both of them given what the other player is doing it is the best this player can do for themselves.*

Now consider the infinitely repeated game with the game above as the stage game. Remember payoffs t periods in the future are discounted by δ^{t-1} for $0 < \delta < 1$.

(c) *(3 points)* Find a Grimm or trigger strategy that supports the strategy pair (x, y) as the equilibrium path when players are patient enough (δ is high).

Solution 4 *In abstract the strategy is:*

$$s_t = \begin{cases} (x, y) & \text{if } (x, y) \text{ last period or } t = 1 \\ NE & \text{else} \end{cases}$$

where (x, y) is given above the numeric values of the game, and NE is the Nash equilibrium found in part b.

(d) *(6 points)* Prove that if δ is high enough this is a subgame perfect equilibrium of the infinitely repeated game. Be sure to check all subgames, find the minimal δ such that this is an equilibrium. NOTE: You may use symmetry to simplify your argument.

Solution 5 *We need to check the NE forever subgame (1 point) for recognizing this). In this subgame it is an equilibrium because no one person can change the future (1 point) and both parties are best responding to their stage game incentives (1 point) (3 points total)*

To analyze the (x, y) forever subgame I will use the game:

	L	C	R
U	$0; b$	$a; a+b+c$	$a+b; a+b$
M	$a; a$	$a+b+c+d; -b$	$b; 0$
D	$-b; a+b+c+d$	$a+b+c; a+b+c$	$a+b+c; a+c$

where $(x, y) = (U, R)$ and $NE = (M, L)$ thus the strategy is:

$$s_t = \begin{cases} (U,R) & \text{if } (U,R) \text{ last period or } t=1 \\ (M,L) & \text{else} \end{cases}$$

Noticing that all the relevant payoffs are symmetric I can see that:

$$\begin{aligned} V^* &= u(U,R) + \frac{\delta}{1-\delta} u(U,R) \\ &= (a+b) + \frac{\delta}{1-\delta}(a+b) \end{aligned}$$

$$\begin{aligned} V' &= u_1(D,R) + \frac{\delta}{1-\delta} u_1(M,L) \\ &= (a+b+c) + \frac{\delta}{1-\delta} a \end{aligned}$$

$$\begin{aligned} V^* &\geq V' \\ (a+b) + \frac{\delta}{1-\delta}(a+b) &\geq (a+b+c) + \frac{\delta}{1-\delta} a \\ \frac{\delta}{1-\delta}(a+b-a) &\geq (a+b+c) - (a+b) \\ \frac{\delta}{1-\delta} b &\geq c \\ \delta b &\geq c(1-\delta) \\ (b+c)\delta &\geq c \\ \delta &\geq \frac{c}{b+c} = \delta^* \end{aligned}$$

the value for each game is found above.