ECON 204

Quiz 10: Monopoly

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1. *(5 Points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, calculators or other electronic devices. I further promise to neither help other students nor accept help from them.

   Name and Surname: _
   Student ID: _
   Signature: _

χ	α	β	MR	MR_P	Q^m	P^m	Π^m	DWL
6	64	4	$16-\frac{1}{2}Q$	$64-8P$	20	11	100	50
10	48	2	$24-Q$	$48-4P$	14	17	98	49
15	54	2	$27-Q$	$54-4P$	12	21	72	36
7	78	6	$13-\frac{1}{3}Q$	$78-12P$	18	10	54	27
11	76	4	$19-\frac{1}{2}Q$	$76-8P$	16	15	64	32
10	84	6	$14-\frac{1}{3}Q$	$84-12P$	12	12	24	12

2. *(15 points total)* A monopolist has the cost function $C(q)=\chi q$ and the demand curve $Q=\alpha-\beta P$.

   (a) *(1 points)* What is their revenue and marginal revenue?

   Solution 1 *Marginal revenue is supposed to be in terms of quantity, so:*

   $$\begin{aligned} P &= \frac{\alpha}{\beta}-\frac{1}{\beta}Q \\ R(Q) &= PQ=\left(\frac{\alpha}{\beta}-\frac{1}{\beta}Q\right)Q \\ MR &= \frac{\alpha}{\beta}-\frac{2}{\beta}Q \end{aligned}$$

   but I am sure many of you wrote it in terms of price, thus I will also accept that answer:

   $$\begin{aligned} R(P) &= P(\alpha-\beta P) \\ MR_P &= \alpha-2\beta P \end{aligned}$$

   (b) *(4 points)* What is the profit maximizing price and quantity and what is their profit?

Solution 2 *Their are three ways to do this, if you write MR in terms of quantity you can solve*

$$\begin{aligned} MR &= \frac{\alpha}{\beta} - \frac{2}{\beta}Q = \chi = MC \\ Q^m &= \frac{1}{2}\alpha - \frac{1}{2}\beta\chi \end{aligned}$$

or you can simply set up the objective function and solve for either price or quantity:

$$\Pi = (P - \chi)\, Q$$

$$\begin{aligned} \Pi(Q) &= \left(\frac{\alpha}{\beta} - \frac{1}{\beta}Q - \chi\right) Q \\ \frac{d\Pi}{dQ} &= \frac{\alpha}{\beta} - \frac{1}{\beta}Q - \chi - \frac{1}{\beta}Q = 0 \\ Q^m &= \frac{1}{2}\alpha - \frac{1}{2}\beta\chi \end{aligned}$$

$$\begin{aligned} \Pi(P) &= (P - \chi)(\alpha - \beta P) \\ \frac{d\Pi}{dP} &= (\alpha - \beta P) - \beta(P - \chi) = 0 \\ P^m &= \frac{1}{2}\chi + \frac{1}{2}\frac{\alpha}{\beta} \end{aligned}$$

If we are correct then the two should be equivalent.

$$\begin{aligned} Q(P^m) &= \alpha - \beta P^m \\ &= \alpha - \beta\left(\frac{1}{2}\chi + \frac{1}{2}\frac{\alpha}{\beta}\right) \\ &= \frac{1}{2}\alpha - \frac{1}{2}\beta\chi = Q^m \end{aligned}$$

which is indeed exactly what we wrote. No matter how you solve it you should find the same profit:

$$\begin{aligned} \Pi^m &= \left(\frac{1}{2}\chi + \frac{1}{2}\frac{\alpha}{\beta} - \chi\right)\left(\frac{1}{2}\alpha - \frac{1}{2}\beta\chi\right) \\ &= \frac{1}{4\beta}(\alpha - \beta\chi)^2 \end{aligned}$$

(c) *(3 points)* What is the efficient price and quantity? What will be the firm's profit then?

Solution 3 *The efficient per unit price is marginal cost, or $P^e = \chi$. Thus the efficient quantity is $Q^e = \alpha - \beta\chi$. However, as should be obvious, if the price is marginal cost the firm's profit will be zero.*

(d) *(4 points)* What is the dead weight loss from the monopolist producing the profit maximizing level of output?

Solution 4 *In this problem the deadweight loss is a triangle, with the area:*

$$\begin{aligned} DWL &= \frac{1}{2}\left(P^m - P^e\right)\left(Q^e - Q^m\right) \\ &= \frac{1}{2}\left(\frac{1}{2}\chi + \frac{1}{2}\frac{\alpha}{\beta} - \chi\right)\left(\alpha - \beta\chi - \left(\frac{1}{2}\alpha - \frac{1}{2}\beta\chi\right)\right) \\ &= \frac{1}{8\beta}\left(\alpha - \beta\chi\right)^2 \end{aligned}$$

(e) *(3 points)* Someone says that it is Pareto Dominant to force the monopolist to produce the welfare maximizing level of output. Why is this not correct and what can you do to make it Pareto Improving to have the Monopolist produce the welfare maximizing level of output?

Solution 5 *If we do this then the monopolist will make zero profit. In order to make this change a Pareto improvement the monopolist has to agree to it, or we must offer them a subsidy* $S \geq \Pi^m$*. The deadweight loss shows that this is possible and for society to benefit as well, but without this offer it is not Pareto dominant.*