ECON 204 Sec 01

Quiz 5

Dr. Kevin Hasker

1. *(2 Points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I also promise not to use calculators or other electronic devices.

   Name and Surname: _

   Student ID: _

   Signature:

   _

2. *(18 points)* Consider the following stage game.

	L	C	R
U	$8;6$	$0;3$	$8;2$
M	$4;6$	$-2;1$	$5;3$
D	$11;-4$	$4;1$	$1;0$

$$(x,y) = (M,R), NE = (D,C), \delta_1^* = \frac{3}{4}, \delta_2^* = \frac{3}{5}, \delta^* = \frac{3}{4}$$

: : :

	L	C	R
U	$9;-4^1$	$4;3^{12}$	$3;0$
M	$4;6^2$	$-2;3$	$7;5$
D	$8;6^2$	$0;5$	$8;2^1$

$$(x,y) = (M,R), NE = (U,C), \delta_1^* = \frac{1}{4}, \delta_2^* = \frac{1}{3}, \delta^* = \frac{1}{3}$$

: : :

	L	C	R
U	$4;6^2$	$7;5$	$-2;3$
M	$9;-4^1$	$3;0$	$4;3^{12}$
D	$8;6^2$	$8;2^1$	$0;5$

$$(x,y) = (U,C), NE = (M,R), \delta_1^* = \frac{1}{4}, \delta_2^* = \frac{1}{3}, \delta^* = \frac{1}{3}$$

: : :

	L	C	R
U	$-3;4$	$1;9^2$	$5;7$
M	$1;4^{12}$	$9;-1^1$	$4;0$
D	$0;7$	$7;9^2$	$7;3^1$

$$(x,y) = (U,R), NE = (M,L), \delta_1^* = \frac{1}{3}, \delta_2^* = \frac{2}{5}, \delta^* = \frac{2}{5}$$

: : :

(a) *(6 points)* Find the pure strategy best responses, you may mark them on the table above but you will loose two points if you do not explain your notation below.

Solution 1 *Like usual I have marked them with a 1 in the upper right hand corner if they are a best response for the row player who is player 1. I use a 2 for the column player, who is player 2.*

(b) *(2 points)* Find the Nash equilibrium, explain why it is a Nash equilibrium.

Solution 2 (1 point) *The Nash equilibrium is the box with both a 1 and a 2 in it. It is marked underneath the game as the NE.*

(1 point) *It is a Nash equilibrium because both parties are best responding given what the other will do.*

Now consider the infinitely repeated game with the game above as the stage game. Remember payoffs t periods in the future are discounted by δ^{t-1} for $0 < \delta < 1$.

(c) *(3 points)* Find a Grimm or trigger strategy that supports the strategy pair (x, y) as the equilibrium path when players are patient enough (δ is high).

Solution 3 *These games are all variations of:*

	L	C	R
U	$a+b+d; b+c+d$	$a+b+d; c$	$0; b+c$
M	$a; b+c+d$	$a+b; b+c$	$-c; b$
D	$a+b+d+d; -a$	$b; 0$	$a; b$

where $(x, y) = (M, C)$, the other strategy we care about is the NE pair, which is marked below each game above. In this game it is (D, R) given this the strategy can be written many different ways, two of which are:

$$s_t = \begin{cases} (M,C) & \textit{if } (M,C) \textit{ in } t-1 \textit{ or } t=1 \\ (D,R) & \textit{else} \end{cases}$$

$$s_t = \begin{cases} (M,C) & \textit{if } t=1 \textit{ or } (M,C) \textit{ in every previous period} \\ (D,R) & \textit{else} \end{cases}$$

(d) *(7 points)* Prove that if δ is high enough this is a subgame perfect equilibrium of the infinitely repeated game. Be sure to check all subgames, find the minimal δ for each player, and then the minimal δ such that this is an equilibrium.

Solution 4 *The subgames are (M, C) forever or (D, R) forever.*

(2 points) *In the (D, R) forever subgame we notice first that no action by either player today can change the subgame in the future. Second*

we notice that each person is playing a best response of the stage game. Thus it is an equilibrium.

(2 points) *In the (M,C) subgame the value for person one from co-operating is:*

$$\begin{aligned} V_1^*(M,C) &= \frac{1}{1-\delta}u_1(M,C) = u_1(M,C) + \frac{\delta}{1-\delta}u_1(M,C) \\ &= (a+b) + \frac{\delta}{1-\delta}(a+b) \end{aligned}$$

if they deviate their best strategy is to play U, thus they get:

$$\begin{aligned} \hat{V}_1(M,C) &= u_1(U,C) + \frac{\delta}{1-\delta}u_1(D,R) \\ &= (a+b+d) + \frac{\delta}{1-\delta}(a) \end{aligned}$$

In order for this to be an equilibrium we need:

$$\begin{aligned} V_1^*(M,C) - \hat{V}_1(M,C) &\geq 0 \\ (a+b) - (a+b+d) + \frac{\delta}{1-\delta}((a+b) - (a)) &\geq 0 \\ -d + \frac{\delta}{1-\delta}b &\geq 0 \\ \frac{\delta}{1-\delta}b &\geq d \\ \delta b &\geq (1-\delta)d \\ \delta &\geq \frac{d}{b+d} = \delta_1^* \end{aligned}$$

(2 points) *We also need to check the same things for player 2, whose best deviation is to play L*

$$\begin{aligned} V_2^*(M,C) &= u_1(M,C) + \frac{\delta}{1-\delta}u_1(M,C) \\ &= (b+c) + \frac{\delta}{1-\delta}(b+c) \\ \hat{V}_2(M,C) &= u_1(M,L) + \frac{\delta}{1-\delta}u_1(D,R) \\ &= (b+c+d) + \frac{\delta}{1-\delta}b \end{aligned}$$

$$\begin{aligned} V_2^*(M,C) - \hat{V}_2(M,C) &\geq 0 \\ ((b+c) - (b+c+d)) + \frac{\delta}{1-\delta}((b+c) - b) &\geq 0 \\ -d + \frac{\delta}{1-\delta}c &\geq 0 \\ \delta &\geq \frac{d}{c+d} = \delta_2^* \end{aligned}$$

(1 point) *Thus this will be an equilibrium when* $\delta \geq \delta^* = \max\left(\frac{d}{c+d}, \frac{d}{b+d}\right)$