ECON 204

Quiz 10: Monopoly
Kevin Hasker

1. *(3 Points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, calculators or other electronic devices. I further promise to neither help other students nor accept help from them.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(6 points)* When studying second degree price discrimination, we characterize it as an $(q(\theta), F(\theta))$ where $F(\theta)$ is the total fee type θ gets, and $q(\theta)$ is the amount of the good he receives. We also say that $B(\theta, q)$ is the benefit this person receives. We require $\frac{\partial^2 B}{\partial q \partial \theta} > 0$ which implies $\frac{\partial B}{\partial \theta} > 0$.

   One of the constraints is that for $\theta > \tilde{\theta}$:

   $$B(\theta, q(\theta)) - F(\theta) \geq B\left(\theta, q\left(\tilde{\theta}\right)\right) - F\left(\tilde{\theta}\right)$$

   or that it is incentive compatible for player θ to admit they are type θ rather than claim they are type $\tilde{\theta}$. Show that this implies:

   $$B(\theta, q(\theta)) - F(\theta) > B\left(\tilde{\theta}, q\left(\tilde{\theta}\right)\right) - F\left(\tilde{\theta}\right)$$

   or that people's net utility is increasing in type. (Yes, in class I said this was hard, but I was wrong.)

   Proof. Since $\frac{\partial B}{\partial \theta} > 0$ and $\theta > \tilde{\theta}$ we know that $B\left(\theta, q\left(\tilde{\theta}\right)\right) > B\left(\tilde{\theta}, q\left(\tilde{\theta}\right)\right)$ and thus:

   $$B(\theta, q(\theta)) - F(\theta) \geq B\left(\theta, q\left(\tilde{\theta}\right)\right) - F\left(\tilde{\theta}\right) > B\left(\tilde{\theta}, q\left(\tilde{\theta}\right)\right) - F\left(\tilde{\theta}\right)$$

   or the conclusion. ■

   Remark 1 *Did I ask for a formal proof? No, I did not. Someone can more or less say what I did and get full credit, probably for most you will find* something *to give some partial credit to... except for those who leave it blank.*

3. *(4 points)* For a firm with an inverse demand curve $P(Q)$, show that

   $$MR = P\left(1 - \frac{1}{|\varepsilon|}\right)$$

   where $\varepsilon = \frac{dQ}{dP}\frac{P}{Q} < 0$ is the elasticity of demand.

Solution 2

$$\begin{aligned} R(Q) &= P(Q)\,Q \\ MR &= P(Q) + \frac{dP}{dQ}Q \\ MR &= P + \frac{dP}{dQ}Q\frac{P}{P} \\ &= P\left(1 + \frac{dP}{dQ}\frac{Q}{P}\right) \end{aligned}$$

since $\frac{dP}{dQ}\frac{Q}{P} = \frac{1}{\frac{dQ}{dP}\frac{P}{Q}} = \frac{1}{\varepsilon}$ *we know that:*

$$MR = P\left(1 + \frac{1}{\varepsilon}\right)$$

and since $\varepsilon < 0$ *we know that* $-|\varepsilon| = \varepsilon$ *thus:*

$$MR = P\left(1 - \frac{1}{|\varepsilon|}\right)$$

4. *(3 points)* Consider a third degree price discriminator selling to two markets, A and B. We observe that $P_A < P_B$, what does this tell us about $|\varepsilon_A|$ and $|\varepsilon_B|$ where ε_A is the elasticity of demand in market A and ε_B is the same in market B?

Solution 3 *We know that*

$$\begin{aligned} MR_A &= MR_B \\ P_A\left(1 - \frac{1}{|\varepsilon_A|}\right) &= P_B\left(1 - \frac{1}{|\varepsilon_B|}\right) \end{aligned}$$

Since $P_B > P_A$ *we must have*

$$\left(1 - \frac{1}{|\varepsilon_A|}\right) > \left(1 - \frac{1}{|\varepsilon_B|}\right)$$

to make this equality true, or

$$\begin{aligned} -\frac{1}{|\varepsilon_A|} &> -\frac{1}{|\varepsilon_B|} \\ \frac{1}{|\varepsilon_A|} &< \frac{1}{|\varepsilon_B|} \\ |\varepsilon_B| &< |\varepsilon_A| \end{aligned}$$

which means that the elasticity of demand in market A is higher.

5. *(4 points)* In first degree price discrimination, each person is provided their Pareto efficient quantity. Why does the monopolist decide to do this?

Solution 4 *Because the monopolist sucks up every bit of excess happiness you have from the good through a fixed fee. This turns the monopolist into a welfare maximizer, but only because all the welfare goes into their greedy little pocket.*