ECON 204

Quiz 8: Externalities and Public Goods.

Kevin Hasker

1. *(4 Points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, calculators or other electronic devices. I further promise to neither help other students nor accept help from them.

   Name and Surname: _

   Student ID: _

   Signature:

   _

P	a	b	c	d	$\frac{\partial U_\alpha}{\partial q_\alpha}$	$\frac{\partial U_\beta}{\partial q_\beta}$	q_α^s	q_β^s
14	32	1	38	2	$18-2q_\beta-2q_\alpha$	$24-4q_\beta-4q_\alpha$	9	6
24	34	1	38	1	$10-2q_\beta-2q_\alpha$	$14-2q_\beta-2q_\alpha$	5	7
12	40	2	22	1	$28-4q_\beta-4q_\alpha$	$10-2q_\beta-2q_\alpha$	7	5
30	36	$\frac{1}{2}$	40	$\frac{1}{2}$	$6-q_\beta-q_\alpha$	$10-q_\beta-q_\alpha$	6	10
14	34	1	42	2	$20-2q_\beta-2q_\alpha$	$28-4q_\beta-4q_\alpha$	10	7
20	36	1	30	1	$16-2q_\beta-2q_\alpha$	$10-2q_\beta-2q_\alpha$	8	5
16	36	2	32	1	$20-4q_\beta-4q_\alpha$	$16-2q_\beta-2q_\alpha$	5	8
12	48	2	20	$\frac{1}{2}$	$36-4q_\beta-4q_\alpha$	$8-q_\beta-q_\alpha$	9	8

2. *(16 points)* Consider two citizens (α and β) who are privately contributing to the pulbic good. Person α buys q_α units at a price of P, person β buys q_β units at the same cost per unit, thus the total amount of the public good is $Q = q_\alpha + q_\beta$.

   Since this is a public good it is non-rival not excludable and thus they both benefit from the total amount provided regardless of whether they contribute or not. Their total benefit is:

$$\begin{aligned} B_\alpha(Q) &= (a - bQ)\,Q \\ B_\beta(Q) &= (c - dQ)\,Q \end{aligned}$$

   (a) *(2 points)* Assuming they maximize their benefit minus their cost, write down each person's objective function—do not use abstract coefficients.

$$\begin{aligned} U_\alpha(q_\alpha, q_\beta) &= (a - bQ)\,Q - Pq_\alpha \\ &= (a - b(q_\alpha + q_\beta))(q_\alpha + q_\beta) - Pq_\alpha \end{aligned}$$

$$\begin{aligned} U_\beta(q_\alpha, q_\beta) &= (c - dQ)\,Q - Pq_\beta \\ &= (c - d(q_\alpha + q_\beta))(q_\alpha + q_\beta) - Pq_\beta \end{aligned}$$

(b) *(6 points)* Write down all the first order conditions for this problem with care to consider all cases.

Solution 1 *They should get one point for the FOC* $\frac{\partial U}{\partial q} = 0$ *and two points for each FOC where* $\frac{\partial U}{\partial q} < 0$.

$$
\begin{aligned}
\frac{\partial U_\alpha}{\partial q_\alpha} &= a - 2b\left(q_\alpha + q_\beta\right) - P \\
\frac{\partial U_\alpha}{\partial q_\alpha} &= 0, q_\alpha \geq 0 \\
\frac{\partial U_\alpha}{\partial q_\alpha} &< 0, q_\alpha = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial U_\beta}{\partial q_\beta} &= c - 2d\left(q_\alpha + q_\beta\right) - P \\
\frac{\partial U_\beta}{\partial q_\beta} &= 0, q_\beta \geq 0 \\
\frac{\partial U_\beta}{\partial q_\beta} &< 0, q_\beta = 0
\end{aligned}
$$

(c) *(4 points)* Assuming the other person provides nothing, find out how much each person would provide if they were the only person in this society.

Solution 2 *I will call these the "stand alone quantities" and denote them* q_α^s *and* q_β^s.

$$
\begin{aligned}
\frac{\partial U_\alpha}{\partial q_\alpha} &= a - 2b\left(q_\alpha + 0\right) - P = 0 \\
q_\alpha^s &= \frac{1}{2b}\left(a - P\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial U_\beta}{\partial q_\beta} &= c - 2d\left(0 + q_\beta\right) - P \\
q_\beta^s &= \frac{1}{2d}\left(c - P\right)
\end{aligned}
$$

(d) *(4 points)* Find the (unique) Nash equilibrium and show that it is an equilibrium.

Solution 3 *If* $q_\alpha^s > q_\beta^s$ *then* $q_\alpha = q_\alpha^s$, $q_\beta = 0$ *if* $q_\alpha^s < q_\beta^s$ *then* $q_\alpha = 0$, $q_\beta = q_\beta^s$.
Assume that $q_\alpha^s > q_\beta^s$, *then to verify this is a Nash equilibrium they should establish that* $\frac{\partial U_\beta}{\partial q_\beta} < 0$ *when* $q_\alpha = q_\alpha^s$ *and then mention that*

clearly in this case their answer in part c verifies that α *is providing the right amount.*

Alternatively, they could start with a random guess at either q_α *or* q_β *and in a few steps they will arrive at this solution. That should be worth full credit.*