ECON 439

Quiz 2

Dr. Kevin Hasker

1. *(3 Points)* **Honor Code:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. As well, I will not assist others nor use a calculator or other electronic device.

   Name and Surname: _
   Student ID: _
   Signature:
   _

α	β	τ	χ	μ	$BR_1(p_2)$	$BR_2(p_1)$	p_1^*	p_2^*
15	3	2	9	1	$\frac{1}{3}p_2+7$	$\frac{1}{3}p_1+3$	9	6
24	3	2	2	10	$\frac{1}{3}p_2+5$	$\frac{1}{3}p_1+9$	9	12
14	2	1	1	6	$\frac{1}{4}p_2+4$	$\frac{1}{4}p_1+\frac{13}{2}$	6	8
12	4	2	1	11	$\frac{1}{4}p_2+2$	$\frac{1}{4}p_1+7$	4	8

Remark 1 *Please notice how much simpler the solutions are when the abstract coefficients are replaced with numbers. For example finding the equilibrium when* $\alpha = 14$ *is:*

$$\begin{aligned}
p_1 &= \frac{1}{4}\left(\frac{1}{4}p_1+\frac{13}{2}\right)+4 \\
p_1 &= \frac{1}{16}p_1+\frac{45}{8} \\
16p_1 &= 16\left(\frac{1}{16}p_1+\frac{45}{8}\right)=p_1+90 \\
15p_1 &= 90 \\
p_1 &= 6 \\
p_2 &= \frac{1}{4}(6)+\frac{13}{2}=\frac{3}{2}+\frac{13}{2}=\frac{16}{2}=8
\end{aligned}$$

2. Consider a market where firms compete by choosing price. They have symmetric demand curves, for $i \in \{1,2\}$ and $j = \{1,2\} \setminus i$ their demand curve is:

$$q_i = \alpha - \beta p_i + \tau p_j \ .$$

   They have different cost functions, $c_1(q) = \chi q_1$ and $c_2(q) = \mu q_2$.

   WARNING: *Changing this problem so that firms choose quantity instead of price will result in you getting no credit.*

Remark 2 *Be very strict about this rule, either solving for the wrong variable* ***or*** *solving using abstract coefficients* ($\{\alpha, \beta, \tau, \chi, \mu\}$ *in this quiz)* *should result in no credit. After all solving for the wrong variable will simply result in a different equilibrium.*

(a) *(1 point)* Show that $\pi_i = (p_i - c_i)\, q_i$ in this problem, where c_i is the firm's constant marginal cost.

Solution 3 *In general* $\pi_i = p_i q_i - c_i(q)$, *but since* $c_i(q) = c_i q_i$, $\pi_i = p_i q_i - c_i q_i = (p_i - c_i)\, q_i$

(b) *(2 points)* Set up each firm's objective function, be sure not have any abstract coefficients in it.

$$\begin{aligned}\pi_1 &= (p_1 - \chi)(\alpha - \beta p_1 + \tau p_2) \\ \pi_2 &= (p_2 - \mu)(\alpha - \beta p_2 + \tau p_1)\end{aligned}$$

(c) *(4 points)* Find the first order conditions of both firms.

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_1} &= (\alpha - \beta p_1 + \tau p_2) - \beta(p_1 - \chi) = 0 \\ \frac{\partial \pi_2}{\partial p_2} &= (\alpha - \beta p_2 + \tau p_1) - \beta(p_2 - \mu) = 0\end{aligned}$$

(d) *(6 points)* Find the best responses of both firms.

$$\begin{aligned}\alpha + \beta\chi + \tau p_2 &= 2\beta p_1 \\ p_1 &= \frac{\alpha + \beta\chi + \tau p_2}{2\beta} \\ p_1 &= \frac{1}{2}\chi + \frac{1}{2}\frac{\alpha}{\beta} + \frac{1}{2\beta}\tau p_2\end{aligned}$$

$$\begin{aligned}\alpha + \beta\mu + \tau p_1 &= 2\beta p_2 \\ p_2 &= \frac{\alpha + \beta\mu + \tau p_1}{2\beta} \\ p_2 &= \frac{1}{2}\mu + \frac{1}{2}\frac{\alpha}{\beta} + \frac{1}{2\beta}\tau p_1\end{aligned}$$

(e) *(4 points)* Find the equilibrium prices of both firms.

$$\begin{aligned}p_1 &= \frac{1}{2}\chi + \frac{1}{2}\frac{\alpha}{\beta} + \frac{1}{2\beta}\tau\left(\frac{1}{2}\mu + \frac{1}{2}\frac{\alpha}{\beta} + \frac{1}{2\beta}\tau p_1\right) \\ p_1 &= \frac{1}{2}\chi + \frac{1}{2}\frac{\alpha}{\beta} + \frac{1}{4}\frac{\alpha}{\beta^2}\tau + \frac{1}{4\beta}\tau\mu + \frac{1}{4\beta^2}\tau^2 p_1 \\ \left(1 - \frac{1}{4\beta^2}\tau^2\right) p_1 &= \frac{1}{2}\chi + \frac{1}{2}\frac{\alpha}{\beta} + \frac{1}{4}\frac{\alpha}{\beta^2}\tau + \frac{1}{4\beta}\tau\mu \\ p_1 &= \frac{\frac{1}{2}\chi + \frac{1}{2}\frac{\alpha}{\beta} + \frac{1}{4}\frac{\alpha}{\beta^2}\tau + \frac{1}{4\beta}\tau\mu}{\left(1 - \frac{1}{4\beta^2}\tau^2\right)} \\ &= \frac{1}{4\beta^2 - \tau^2}\left(2\alpha\beta + \alpha\tau + 2\beta^2\chi + \beta\tau\mu\right)\end{aligned}$$

$$\begin{aligned} p_2 &= \frac{1}{2}\mu + \frac{1}{2}\frac{\alpha}{\beta} + \frac{1}{2\beta}\tau\left(\frac{1}{4\beta^2 - \tau^2}\left(2\alpha\beta + \alpha\tau + 2\beta^2\chi + \beta\tau\mu\right)\right) \\ &= \frac{1}{4\beta^2 - \tau^2}\left(2\alpha\beta + \alpha\tau + 2\beta^2\mu + \beta\tau\chi\right) \end{aligned}$$

Solution 4 *As a bonus I would like to show that solving for the wrong variable when the paramaters are:*

$$q_i = 14 - 2p_i + p_j,\ c_1(q) = q_1,\ c_2(q) = 6q_2$$

Will not result in the correct equilibrium. First we have to solve for the inverse demand curves:

$$\begin{aligned} p_1 &= \frac{14 - q_1 + p_2}{2} \\ q_2 &= 14 - 2p_2 + p_1 \\ &= 14 - 2p_2 + \frac{14 - q_1 + p_2}{2} \\ &= 21 - \frac{1}{2}q_1 - \frac{3}{2}p_2 \\ p_2 &= \frac{21 - \frac{1}{2}q_1 - q_2}{\frac{3}{2}} = 14 - \frac{2}{3}q_2 - \frac{1}{3}q_1 \end{aligned}$$

$$\pi_i = \left(14 - \frac{2}{3}q_i - \frac{1}{3}q_j\right)q_i - c_i q_i$$

$$\left(14 - \frac{2}{3}q_i - \frac{1}{3}q_j\right) - \frac{2}{3}q_i - c_i = 0$$

$$q_i = \frac{21}{2} - \frac{1}{4}q_j - \frac{3}{4}c_i$$

$$\begin{aligned} q_1 &= \frac{39}{4} - \frac{1}{4}q_2 \\ q_2 &= 6 - \frac{1}{4}q_1 \end{aligned}$$

$$\begin{aligned} q_1 &= \frac{39}{4} - \frac{1}{4}\left(6 - \frac{1}{4}q_1\right) \\ q_1 &= \frac{44}{5} \\ q_2 &= 6 - \frac{1}{4}\left(\frac{44}{5}\right) = \frac{19}{5} \end{aligned}$$

$$p_2 = 14 - \frac{2}{3}\left(\frac{44}{5}\right) - \frac{1}{3}\left(\frac{19}{5}\right) = \frac{103}{15} = 6.8667$$

whereas in the orginal problem $p_2 = 8$.