ECON 439
## Quiz 4
Dr. Kevin Hasker

1. *(3 points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(17 points total)* Consider the following sequential game, where player 1 makes the first decision, chosing between a and b.
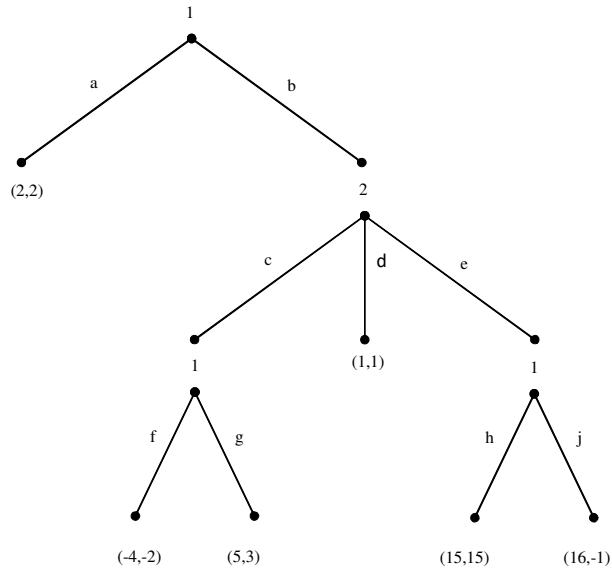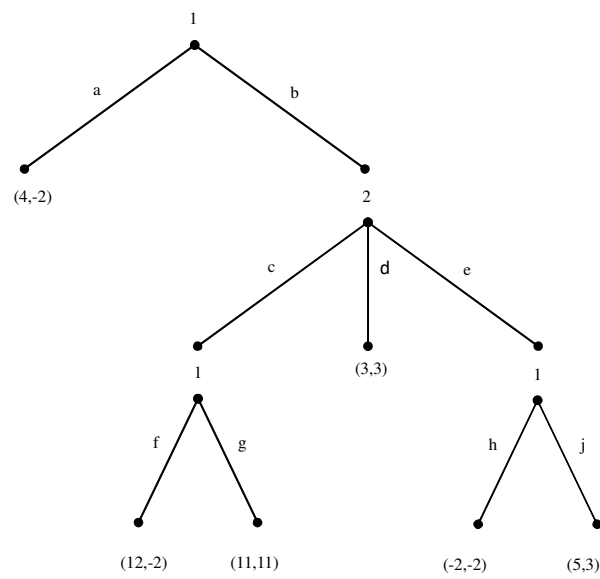

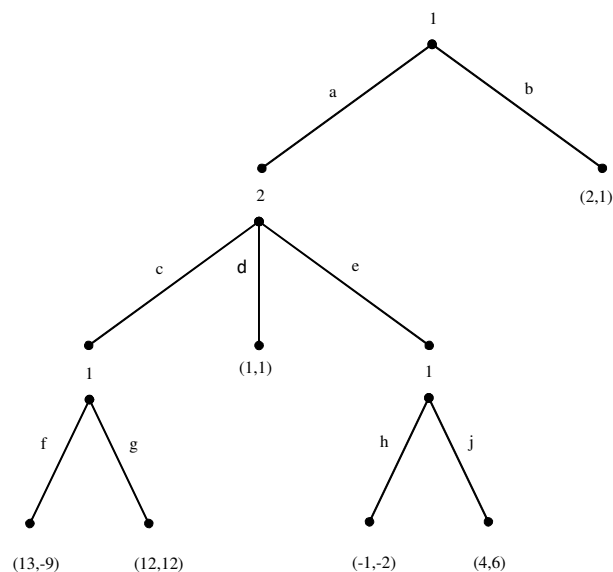SPE $(b, g, j)\,(c)$

ETNE $(a, h, j)\,(d)$
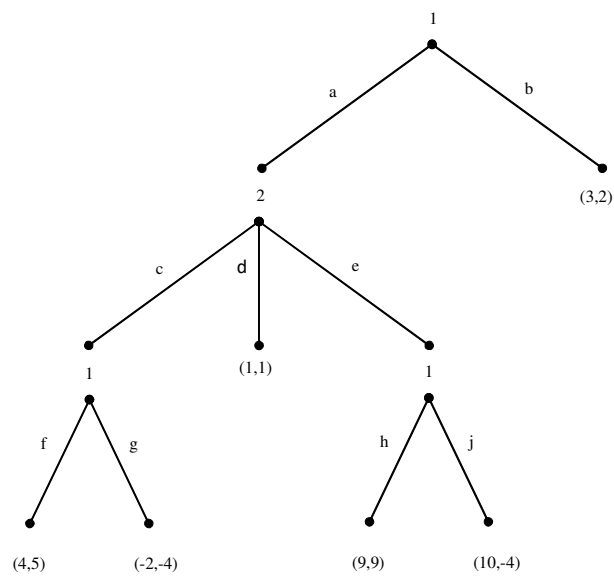
SPE $(a, f, j) (c)$
ETNE $(b, g, j) (d)$

SPE $(b, f, j) (e)$
ETNE $(a, f, h) (d)$

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SPE $(a, f, j)(e)$

ETNE $(b, f, h)(d)$

(a) *(8 points)* Write down all the strategies of player 1.

Solution 1 $S_1 = \{a, b\} \times \{f, g\} \times \{h, j\} = \left\{ \begin{array}{l} (a, f, h), (a, f, j), (a, g, h), (a, g, j), \\ (b, f, h), (b, f, j), (b, g, h), (b, g, j) \end{array} \right\}$

(b) *(6 points)* Find the subgame perfect equilibrium strategies using backward induction, and write them down below.

Solution 2 *It is written by the game above. If they write down the strategy (i.e. all four actions in any order) then give them full credit.*

If they do not and it is fairly clear from the table how they are marking their best responses, feel free to give them a point of credit for each best response. Of course if you can't figure out their notation give them zero.

(c) *(3 points)* Find an empty threat Nash equilibrium where player 1 makes an empty threat. How do the payoffs in this equilibrium compare to their payoffs in the subgame perfect equilibrium? Do they increase or decrease?

Solution 3 *The answer for each quiz is above, and it always results in both parties doing worse than in the SPE. As in the empty threat*

hurts both players. I will go through the logic for the following game.

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a   b

(2,2)   2

c   d   e

1   (1,1)   1

f   g   h   j

(-4,-2)   (5,3)   (15,15)   (16,-1)
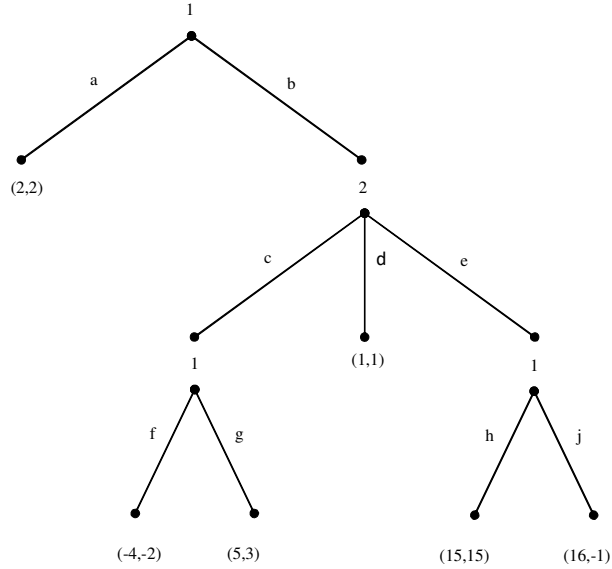
here the SPE is $(b, g, j) (c)$.

If player one changes from a to b this is not an equilibrium without other changes.

If player one changes from j to h, player 2 must choose c, and then player 1 will have to carry out their threat by playing h, but obviously they must switch to j and this cannot be an equilibrium.

If player one changes from g to f, player 2 will then choose d, and then player 1 will choose a. Thus the ETNE is $(a, f, j) (d)$

Just to be careful, I will convert the game to the strategic form and find all pure strategy equilibria:

	c	d	e
(a, f, h)	$2; 2^2$	$2; 2^{12}$	$2; 2^2$
(a, f, j)	$2; 2^2$	$2; 2^{12}$	$2; 2^2$
(a, g, h)	$2; 2^2$	$2; 2^{12}$	$2; 2^2$
(a, g, j)	$2; 2^2$	$2; 2^{12}$	$2; 2^2$
(b, f, h)	$-4; -2$	$1; 1$	$15; 15^2$
(b, f, j)	$-4; -2$	$1; 1^2$	$16; -1^1$
(b, g, h)	$5; 3^1$	$1; 1$	$15; 15^2$
(b, g, j)	$5; 3^{12}$	$1; 1$	$16; -1^1$

Note that I did not ask them to do this and they do not get any more points for it.

As you can see, there are actually four empty threat Nash equilibria with the same payoff, the pair (a, d) is sufficient. However if d is

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not a best response then it is not one where player 1 made the empty threat. For d to be a best response we have to have (f, j), so in the others player 2 makes an empty threat.