ECON 439
Quiz 5
Dr. Kevin Hasker

1. *(3 points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, calculator or other electronic device. I will also neither give nor receive assistance from any other student.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(19 points total)* Consider the following extensive form game.

   Round 1 Firm 1 chooses their location, $l_1 \in \{1, 2, 3, 4, 5\}$

   Round 2 Firm 2 chooses whether to enter or not, if they enter it will cost them F.

   Round 3 Firm 2 chooses their location, $l_2 \in \{1, 2, 3, 4, 5\}$

   At each location $l \in \{1, 2, 3, 4, 5\}$ their are c_l customers, the distribution is given in the table below:

l:	1	2	3	4	5	$\sum_{l=1}^{5} c_l$	$\equiv$	$C = 30, l_m = 2$	If $l_1 =$	1	2	3	4	5
c_l:	12	4	2	10	2				$BR_2 =$	2	2	2	3	4

l:	1	2	3	4	5	$\sum_{l=1}^{5} c_l$	$\equiv$	$C = 30, l_m = 4$	If $l_1 =$	1	2	3	4	5
c_l:	2	10	2	4	12				$BR_2 =$	2	3	4	4	4

l:	1	2	3	4	5	$\sum_{l=1}^{5} c_l$	$\equiv$	$C = 30, l_m = 2$	If $l_1 =$	1	2	3	4	5
c_l:	10	6	2	2	10				$BR_2 =$	2	2	2	3	4

l:	1	2	3	4	5	$\sum_{l=1}^{5} c_l$	$\equiv$	$C = 30, l_m = 2$	If $l_1 =$	1	2	3	4	5
c_l:	10	2	2	6	10				$BR_2 =$	2	2	2	3	4

   If firm 2 enters, the customers will go to the firm closest to their location, if firm 2 does not they will go to firm 1. Firms seek to maximize their number of customers. (You may assume firm 2 makes one unit of money for each customer.)

   (a) *(5 points)* Find firm 2's best response for each location of firm 1—be sure to show some work explaining your answer. Fill it into the table below:

Solution 1 *The answer is in the key above, they should do some work explaining the answer. My explanation is the in this class of problems if l_m is the median location then the best response is always*

$$BR_2(l_1) = \begin{cases} l_1 + 1 & l_1 < l_m \\ l_m & l_1 = l_m \\ l_1 - 1 & l_1 > l_m \end{cases}$$

but for the students I expect some work in at least some cases. Though if they find the median location and use the explanation I did, that is enough.

(b) *(3 points)* Assuming firm 2 enters, find the optimal location for firm 1.

Solution 2 *It is $l_1 = l_m$ if they locate at l_m firm two will as well, and they will have $C/2$ customers. If they locate at any other location the other firm will be strictly closer to l_m, and by definition (and uniqueness) of l_m they will earn strictly less than half the customers. I told them I expect some work here, so let me show you the work I would use to prove this statement. For the economy:*

l:	1	2	3	4	5
c_l:	2	10	2	4	12

If $l_1 =$	1	2	3	4	5
$BR_2 =$	2	3	4	4	4
$\pi_1 = D_1 =$	2	12	14	15	12

it is clear that $l_1 = l_m = 4$ is the option that gives the highest profit.

(c) *(8 points)* Write down *all* pure strategy equilibrium strategies below, it should be a function of F. Warning: I expect precision in your answers, if you miss an element you will not get credit for it.

Solution 3 *Each element is worth one point.*

$F \geq \frac{C}{2}$	$l_1 \in \{1,2,3,4,5\}$	*Not Enter*	$BR_2(l_1)$ *(which is written above)*
$F \leq \frac{C}{2}$	$l_1 = l_m$	*Enter*	$BR_2(l_1)$ *(which is written above)*

*note that it is important that when F is high they point out that firm 1's optimal location is unspecified, and that $BR_2(l_1)$ is **still** part of firm 2's strategy.*

(d) *(3 points)* Firm 1 has a weakly dominant strategy, what is it? Explain.

Solution 4 *$l_1 = l_m$ is a weakly dominant strategy, it is the unique best response if firm 2 enters, and when firm 2 does not enter it is as good as any other location.*