ECON 439

Quiz 05—Repeated Games.

Kevin Hasker

1. *(1 Point)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, calculators or other electronic devices. I further promise to neither help other students nor accept help from them.

   Name and Surname: _
   Student ID: _
   Signature:
   _

2. *(19 points total)* Consider the following game as a stage game of an infinitely repeated game with discount factor $\delta \in (0, 1)$.

   Remark 1 *These are all variations of the following games with rows and columns permuted and different values for* (a, b, c, d).

Player 2

Player 1

	L	C	R
U	$a+b+c+d; a+b+c+d^{12}$	$a+c+d; c+d^{1}$	$-c; a+b$
M	$a+b+c; a+b+c^{2}$	$c+d; c+b$	$-a; -a-c$
D	$b+c+d; -b$	$-b-d; -c$	$d; b^{12}$

$(\chi, D) = (M, C)$ $\quad \frac{a}{a+c} = \underline{\delta}$

Player 2

Player 1

	L	C	R
U	$7; 5^{1}$	$-4; 5$	$10; 10^{12}$
M	$-4; -4$	$1; 3^{12}$	$8; -3$
D	$5; 7$	$-2; -6$	$9; 9^{2}$

$(\chi, D) = (D, L)$ $\quad \frac{1}{3} = \underline{\delta}$

Player 2

Player 1

	L	C	R
U	$10; -2$	$-5; -5$	$3; 2^{12}$
M	$11; 11^{12}$	$9; 8^{1}$	$-5; 3$
D	$8; 8^{2}$	$8; 7$	$-1; -6$

$(\chi, D) = (D, C)$ $\quad \frac{1}{6} = \underline{\delta}$

Player 2

Player 1

	L	C	R
U	$9; 9^{2}$	$-3; -4$	$3; 6$
M	$8; -5$	$2; 5^{12}$	$-7; -1$
D	$11; 11^{12}$	$-1; 8$	$6; 3^{1}$

$(\chi, D) = (U, R)$ $\quad \frac{3}{4} = \underline{\delta}$

Player 2

		L	C	R
Player 1	U	$12; 12^2$	$4; 7$	$-5; -8$
	M	$13; 13^{12}$	$9; 4^1$	$-3; 9$
	D	$8; -4$	$-5; -3$	$1; 4^{12}$

$(\chi, D) = (U, C)$ $\frac{5}{8} = \underline{\delta}$

(a) *(8 points)* Find the pure strategy best responses in this game and the pure strategy Nash equilibria. *You may mark the best responses on the table above but you will loose two points if you do not explain your notation below.*

Solution 2 *I will mark a 1 for the BR of player 1 in the upper right hand corner, a 2 for 2, and Pure strategy NE will have both a 1 and 2 in them. However the students need to write down their strategies in the space provided for example for the game:*

Player 2

		L	C	R
Player 1	U	$a+b+c+d; a+b+c+d^{12}$	$a+c+d; c+d^1$	$-c; a+b$
	M	$a+b+c; a+b+c^2$	$c+d; c+b$	$-a; -a-c$
	D	$b+c+d; -b$	$-b-d; -c$	$d; b^{12}$

$(\chi, D) = (M, C)$

They would need to write down (U, L) *and* (D, R)

(b) *(2 points)* Find the (pure) minimax strategy in this game, it is one of the Nash equilibria but you need to show it is minimax.

Solution 3 *For my answers I will use the abstract game all of these are based on:*

Player 2

		L	C	R
Player 1	U	$a+b+c+d; a+b+c+d^{12}$	$a+c+d; c+d^1$	$-c; a+b$
	M	$a+b+c; a+b+c^2$	$c+d; c+b$	$-a; -a-c$
	D	$b+c+d; -b$	$-b-d; -c$	$d; b^{12}$

$(\chi, D) = (M, C)$

Notice that it is clearly (D, R)*, but to verify this is true we need to notice that all pure strategy best response for player 1 are strictly higher than d, and for player 2 are strictly higher than b. Thus* (D, R) *is the joint mutual minimax strategy.*

(c) *(3 points)* Write down a strategy that will support people playing (χ, D) forever if the discount factor (δ) is high enough.

Solution 4 *Using the game:*

		Player 2		
		L	C	R
Player 1	U	$a+b+c+d; a+b+c+d^{12}$	$a+c+d; c+d^{1}$	$-c; a+b$
	M	$a+b+c; a+b+c^{2}$	$c+d; c+b$	$-a; -a-c$
	D	$b+c+d; -b$	$-b-d; -c$	$d; b^{12}$

$(\chi, D) = (M, C)$

an excellent strategy is

$$s^t = \begin{cases} (M,C) & \text{if } t=1 \text{ or } (M,C) \text{ last period} \\ (D,R) & \text{else} \end{cases}$$

(d) *(6 points)* Prove that it is an equilibrium **in every subgame** and find the minimal δ such that it is an equilibrium.

Solution 5 *First of all, their are two subgames:*

i. (2 points) *Expect (D,R) forever this is an equilbrium because no player can change what will happen in the future by changing their strategy, and the strategy in the current period is a best response.*

ii. (2 points for each player) *Expect (M,C) in the future:*

$$\begin{aligned} V_1^*(M,C) &= (c+d) + \frac{\delta}{1-\delta}(c+d) \\ V_1'(M,C) &= u_1(U,C) + \frac{\delta}{1-\delta} u_1(D,R) \\ &= (a+c+d) + \frac{\delta}{1-\delta}(d) \end{aligned}$$

$$\begin{aligned} V_1^*(M,C) &\geq V_1'(M,C) \\ (c+d) + \frac{\delta}{1-\delta}(c+d) &\geq (a+c+d) + \frac{\delta}{1-\delta}(d) \\ \frac{\delta}{1-\delta}(c+d-d) &\geq (a+c+d) - (c+d) \\ \delta c &\geq (1-\delta)a \\ \delta(c+a) &\geq a \\ \delta &\geq \frac{a}{a+c} = \underline{\delta} \end{aligned}$$

We also need to check this for player 2:

$$\begin{aligned} V_2^*(M,C) &= (c+b) + \frac{\delta}{1-\delta}(c+b) \\ V_2'(M,C) &= u_2(M,L) + \frac{\delta}{1-\delta} u_2(D,R) \\ &= (a+b+c) + \frac{\delta}{1-\delta}(b) \end{aligned}$$

$$
\begin{aligned}
V_2^*(M,C) &\geq V_2'(M,C) \\
(c+b)+\frac{\delta}{1-\delta}(c+b) &\geq (a+b+c)+\frac{\delta}{1-\delta}(b) \\
\frac{\delta}{1-\delta}(c+b-b) &\geq (a+b+c)-(c+b) \\
\frac{\delta}{1-\delta}c &\geq a \\
\delta &\geq \frac{a}{a+c}=\underline{\delta}
\end{aligned}
$$

you should not give them any points for finding the minimal δ *since their weren't enough points to assign to it. While* $\underline{\delta}$ *does not depend on i they still need to check both players.*