A Unified Treatment of Pricing in Monopoly

1 Demand and Costs.

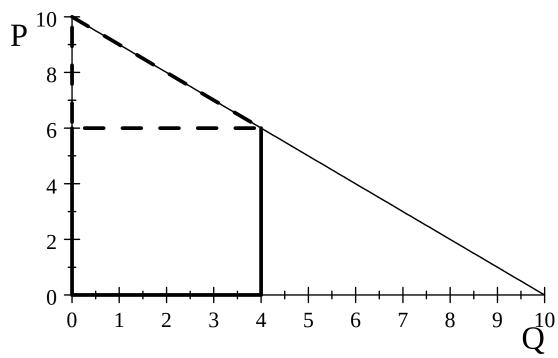
The demand function of our (representative) consumer is:

$$Q = a - bP$$

We assume that this implies that their benefit of the good is their consumer surplus plus their expenditure on the good, or

$$B(Q) = \int_0^Q \left(\frac{a}{b} - \frac{1}{b}z\right) dz = \frac{a}{b}Q - \frac{1}{2b}Q^2$$

To understand how we find this area graphically assume that $a = 10$ and $b = 1$ then this is the triangle and the square in the area below.



The area of the triangle is:

$$\frac{1}{2}h_1 b = \frac{1}{2}\left(\frac{a}{b} - \left(\frac{a}{b} - \frac{1}{b}Q\right)\right)Q = \frac{1}{2b}Q^2$$

The area of the square is:

$$h_2 b = \left(\frac{a}{b} - \frac{1}{b}Q\right)Q = \frac{a}{b}Q - \frac{1}{b}Q^2$$

Then $B(Q) = \frac{a}{b}Q - \frac{1}{b}Q^2 + \frac{1}{2b}Q^2 = \frac{a}{b}Q - \frac{1}{2b}Q^2$. In this handout we must think of this function as the "utility" of the consumers. Of course this is not a reasonable utility function in general, but everything we say in this handout will be precisely true if we treat it as if it is.[1] Now if we let $E(Q)$ be the amount of expenditure on the good then we can define

$$CS(Q) = B(Q) - E(Q)$$

as the consumer surplus, or the net benefit. In general we have assumed that $E(Q) = PQ$, but this handout is about alternative pricing models. Even though it is about alternative pricing models, the most general pricing we will consider is a fixed fee and constant per-unit pricing:

$$\begin{aligned} E(Q) &= T + PQ \\ CS(Q) &= B(Q) - (T + PQ) \\ &= \frac{a}{b}Q - \frac{1}{2b}Q^2 - (T + PQ) \\ &= \left(\left(\frac{a}{b} - P\right) - \frac{1}{2b}Q\right) Q - T \end{aligned}$$

Throughout this handout we will assume that the costs are:

$$C(Q) = cQ + F$$

or constant marginal cost pricing. I will also ignore the fixed costs, since this handout is about pricing not when to produce.

2 Standard Monopoly, Constant per-unit Pricing.

In the standard model the Monopolist sets a price, P, and meets all the demand at that price. So in that case the Monopolists objective function is:

$$\max_Q \left(\frac{a}{b} - \frac{1}{b}Q\right) Q - cQ$$

with the first order condition:

$$\frac{a}{b} - \frac{2}{b}Q - c = 0 \ .$$

Using this we can easily solve for the *monopoly* price and quantity:

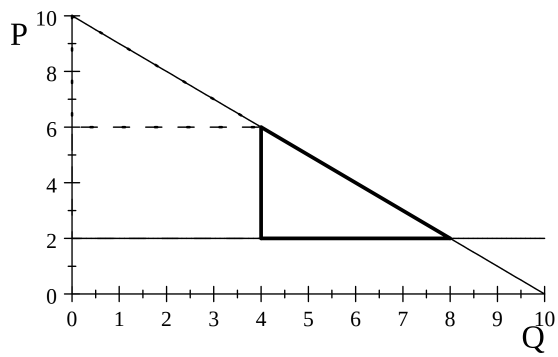
$$\begin{aligned} Q_m &= \frac{1}{2}(a - bc) \\ P_m &= \frac{a}{b} - \frac{1}{b}\left(\frac{1}{2}(a - bc)\right) \\ &= \frac{1}{2}\frac{a}{b} + \frac{1}{2}c \ . \end{aligned}$$

[1] The primary problem with this as a utility function it that it is satiated, and thus does not satisfy monotonicitiy (more is better). To be precise if we maximize this over Q then the optimal Q is a, if we can achieve this point (which corresponds to $P = 0$) then this consumer wants nothing else.

The efficient price is, of course, c because that is marginal cost and thus the *efficient* quantity is:

$$\begin{aligned} Q_e &= a - bc \\ P_e &= c \end{aligned}$$

Let $a = 10$, $b = 1$, $c = 2$ then $Q_m = 4$, $P_m = 6$, and $P_e = 2$, $Q_e = 8$. Then in the graph below the upper triangle (in dots) is the Consumer surplus, the box below it (with a dashed border) is the profit, and the Dead Weight Loss is the lower triangle with thick edges.



The profit of the firm is:

$$\begin{aligned} \Pi_m &= (P_m - c)\, Q_m \\ &= \left(\frac{1}{2}\frac{a}{b} + \frac{1}{2}c - c\right)\frac{1}{2}(a - bc) \\ &= \frac{1}{4b}(a - bc)^2 \end{aligned}$$

And the dead weight loss is:

$$\begin{aligned} DWL &= \frac{1}{2}(P_m - P_e)(Q_e - Q_m) \\ &= \frac{1}{2}(P_m - c)\, Q_m \end{aligned}$$

where the $(Q_e - Q_m) = Q_m$ only because $Q_m = \frac{1}{2}Q_e$ in this example. This doesn't hold in general, but it makes it obvious that the DWL is half the profit, weird ehh?

$$DWL = \frac{\Pi}{2} = \frac{1}{8b}(a - bc)^2 \ .$$

The Consumer Surplus is:

$$\begin{aligned}
CS &= \frac{a}{b}Q_m - \frac{1}{2b}Q_m^2 - P_m Q_m \\
&= \frac{a}{b}Q_m - \frac{1}{2b}Q_m^2 - \left(\frac{1}{2}\frac{a}{b} + \frac{1}{2}c\right) Q_m \\
&= \frac{1}{2b}Q_m \left(a - bc - Q_m\right) \\
&= \frac{1}{2b}\left(\frac{1}{2}\left(a - bc\right)\right)\left(a - bc - \left(\frac{1}{2}\left(a - bc\right)\right)\right) \\
&= \frac{1}{8b}\left(a - bc\right)^2
\end{aligned}$$

which is the same as the DWL. Yep, its a weird example all right.

3 First Degree or Perfect Price Discrimination.

Like I said above we will only need to look at:

$$E\left(Q\right) = T + PQ$$

however in this section we need to assume *explicitly* that there is only one consumer with the given demand curve. Then we need to think about this consumer's decision better. Their net utility will be:

$$\begin{aligned}
CS\left(Q\right) &= B\left(Q\right) - E\left(Q\right) \\
&= \frac{a}{b}Q - \frac{1}{2b}Q^2 - \left(T + PQ\right)
\end{aligned}$$

So we can see that maximizing this implies that

$$\begin{aligned}
\frac{a}{b} - \frac{1}{b}Q - P &= 0 \\
Q &= a - bP
\end{aligned}$$

which gives us the demand curve above. However, we haven't really considered the second option the consumer needs to think about—that they may not want to consume at all. They will consume if

$$\begin{aligned}
\frac{a}{b}Q - \frac{1}{2b}Q^2 - \left(T + PQ\right) &\geq 0 \\
\frac{a}{b}Q - \frac{1}{2b}Q^2 - PQ &\geq T\ .
\end{aligned}$$

Now what is the firm's profits?

$$\max_{T,P} T + PQ - cQ \text{ such that } Q = a - bP$$

obviously it is optimal to make T as large as it possibly can be, in which case it should be $\frac{a}{b}Q - \frac{1}{2b}Q^2 - PQ = T$. Thus their modified profit function is:

$$\begin{aligned}\max_P \frac{a}{b}Q - \frac{1}{2b}Q^2 - PQ + PQ - cQ \text{ such that } Q &= a - bP \\ \max_Q \frac{a}{b}Q - \frac{1}{2b}Q^2 - cQ \text{ such that } Q &= a - bP\end{aligned}$$

Remembering that $B(Q) = \frac{a}{b}Q - \frac{1}{2b}Q^2$ and doing the substitution gives us:

$$\max_P B(Q) - cQ \text{ such that } Q = a - bP$$

But this is exactly the social welfare function! When we solve for the *first degree price discrimination* quantity we find that it is:

$$\begin{aligned}\frac{a}{b} - \frac{1}{b}Q - c &= 0 \\ Q_f &= a - bc\end{aligned}$$

and by checking the constraint we can see that $P_f = c$. So the Monopolist produces the optimal quantity. Of course he also extracts all of the consumer's surplus, but since efficiency does not care about the distribution of surplus—only the quantity of surplus—this is not important for efficiency. Their profit is:

$$\begin{aligned}\Pi_f &= \frac{a}{b}(a - bc) - \frac{1}{2b}(a - bc)^2 - c(a - bc) \\ \Pi_f &= \frac{1}{2b}(a - bc)^2 \ .\end{aligned}$$

4 Second Degree or Quantity Based Price Discrimination.

For the rest of the discussion we will assume that there are two types of consumers. High types who are willing to pay a lot for the good and low types who are not.

$$\begin{aligned}Q_h &= a_h - b_h P_h \\ Q_h &= a_l - b_l P_l \\ a_h &> a_l \\ \frac{a_h}{b_h} &> \frac{a_l}{b_l}\end{aligned}$$

and that there are N_h of the high demand people and N_l of the low demand people. Now just like before the fixed fee will be just equal to the net benefit of *either* the high or the low demand types, but which? We can't tell.

$$T = B_l(Q_l) - PQ_l \text{ or } B_h(Q_h) - PQ_h$$

On the other hand if $T = B_h(Q_h) - PQ_h$ then the low types will not buy and we are just solving the first degree profit maximization problem above, so let us assume $T = B_l(Q_l) - PQ_l$. Then the profit function will be:

$$\max_P (N_h + N_l)(B_l(Q_l) - PQ_l) + N_l PQ_l + N_h PQ_h - cN_l Q_l - cN_h Q_h$$

$$\max_P (N_h + N_l) B_l(Q_l) - cN_l Q_l - N_h PQ_l + N_h PQ_h - cN_h Q_h$$
$$\max_P (N_h + N_l)\left(B_l(Q_l) - \left(\frac{N_l}{N_h + N_l}c + \frac{N_h}{N_h + N_l}P\right)Q_l\right) + (P - c) N_h Q_h$$

Now we can make the presentation of this function neater if we let $\lambda_l = \frac{N_l}{N_h+N_l}$, then this function is:

$$\max_P (N_h + N_l)\left[(B_l(Q_l) - (\lambda_l c + (1 - \lambda_l P)) Q_l) + (1 - \lambda_l)(P - c) Q_h\right]$$

Now the second problem is just classic monopoly profit maximization, but the problem here is that we only have one P to choose so we have to weigh this incentive against the incentive to maximize the "pseudo-welfare" function

$$B_l(Q_l) - (\lambda_l c + (1 - \lambda_l) P) Q_l$$

If this was all we were trying to maximize then the optimal P would be c and this would end up with the same solution as the true welfare function. But in this case since we are going to try to raise the price (and increase our profits from the high types) we will end up with $P > c$ and we will get less than the optimal welfare.

Now solving this problem is very annoying, we have to substitute out for Q_h and Q_l and then optimize over P, but I will do it. In order to make the solution sensible we have to assume:

Assumption Assume the slopes of the two demand curves are the same, $b_h = b_l = b$. Let me restate the previous notation that $\lambda_l = \frac{N_l}{N_h+N_l}$.

$$\max_P (N_h + N_l)\left[\begin{array}{c}\left(\frac{a_l}{b}(a_l - bP) - \frac{1}{2b}(a_l - bP)^2 - (\lambda_l c + (1 - \lambda_l) P)(a_l - bP)\right) \\ + (P - c)(1 - \lambda_l)(a_h - bP)\end{array}\right]$$
$$\max_P (N_h + N_l)\left[\begin{array}{c}\frac{1}{2b}(a_l - Pb + (P - c) 2b\lambda_l)(a_l - bP) \\ + (P - c)(1 - \lambda_l)(a_h - bP)\end{array}\right]$$

:

and the first order condition is:

$$\frac{1}{2b}(-b)(a_l - bP) + \frac{1}{2b}(2b\lambda_l)(a_l - bP) + \frac{1}{2b}(a_l - Pb + (P - c) 2b\lambda_l)(-b)$$
$$+ (1 - \lambda_l)(a_h - bP) + (P - c)(1 - \lambda_l)(-b) = 0$$

Thus the *second degree price discrimination* unit price is:

$$\begin{aligned}(1-\lambda_l)\left[(a_h-a_l)+bc\right]+\lambda_l bc-P_s b &= 0\\ (1-\lambda_l)\left[\frac{a_h}{b}-\frac{a_l}{b}+c\right]+\lambda_l c &= P_s\end{aligned}$$

Now you can see that as $\lambda_l \to 1$ then the price converges to marginal cost, or in other words efficiency becomes the main concern as this problem converges to first degree price discrimination. As $\lambda_l \to 0$ we get a rather strange expression that I can not readily interpret.

Just to go through the motions,

$$\begin{aligned}Q_h^s &= a_h-b\left((1-\lambda_l)\left[\frac{a_h}{b}-\frac{a_l}{b}+c\right]+\lambda_l c\right)\\ &= a_h-b\left((1-\lambda_l)\left(\frac{a_h}{b}-\frac{a_l}{b}+c\right)+\lambda_l c\right)\\ &= \lambda_l a_h+(1-\lambda_l)\,a_l-bc\\ Q_l^s &= a_l-b\left((1-\lambda_l)\left[\frac{a_h}{b}-\frac{a_l}{b}+c\right]+\lambda_l c\right)\\ &= (2-\lambda_l)\,a_l-a_h\,(1-\lambda_l)-bc\end{aligned}$$

Notice that in common with the standard quantities one always subtracts bc, but the first terms do not appeal logically, other than to notice that as $\lambda_l \to 0$ Q_h^s becomes the efficient quantity, and as $\lambda_l \to 1$ Q_l^s becomes the efficient quantity. And the profits are:

$$\Pi_2=\frac{N_h+N_l}{2b}\left[(a_l-bc)^2+(1-\lambda_l)^2(a_h-a_l)^2\right]$$

When we do such a problem we must not forget that we can not rule out the fact that the Monopolist might just not sell to the low type. In which case we can get their profits from the first degree profit maximization above:

$$\Pi_f=\frac{N_h}{2b}(a_h-bc)^2$$

there is no intuitive explanation for when the Monopolist will not sell to the low demand type.

4.1 Third Degree or Identification Based Price Discrimination.

We will work with the same demand curves as before, but we will loosen the assumptions a little bit.

$$\begin{aligned}Q_h &= a_h-b_h P_h\\ Q_h &= a_l-b_l P_l\\ \frac{a_h}{b_h} &\geq \frac{a_l}{b_l}\end{aligned}$$

here the main distinction between the two demand curves is who will pay more for the first unit, this is all that is important.

4.1.1 Price and Quantity if the Monopolist can not Price Discriminate.

In this type of question the first thing we want to know is what the Monopolist would charge if they could only charge one price to both demanders. The critical problem in this analysis is that the demand curve is kinked.

$$Q = Q_h + Q_l = \begin{cases} a_h - b_h P & P \geq \frac{a_l}{b_l} \\ (a_h + a_l) - (b_h + b_l) P & P \leq \frac{a_l}{b_l} \end{cases}$$

I recommend you always start by assuming both types demand, and then seeing whether this is true or not.

Notation 1 *Let* $A = (a_h + a_l)$ *and* $B = (b_h + b_l)$

The problem you are solving we have solved above,

$$\max_Q \left(\frac{A}{B} - \frac{1}{B} Q \right) Q - cQ$$

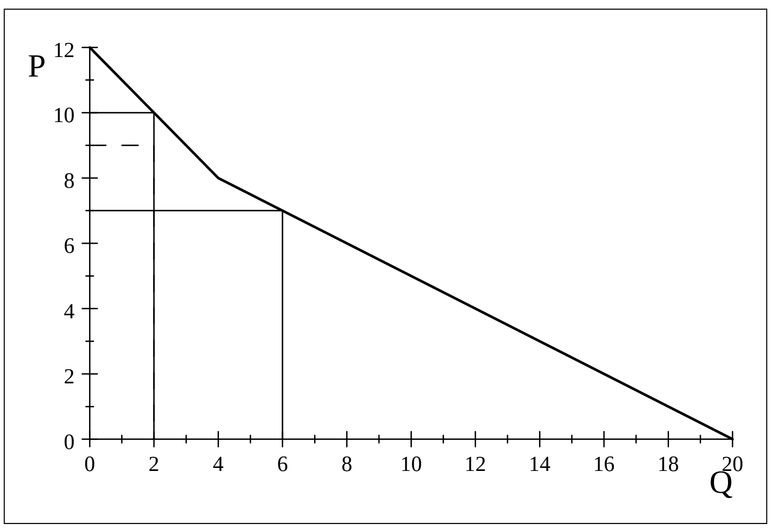
the marginal revenue is:

$$MR = \frac{A}{B} - \frac{2}{B} Q$$

and the optimal quantity is found from:

$$\begin{aligned} MR &= MC \\ \frac{A}{B} - \frac{2}{B} Q &= c \\ Q &= \frac{1}{2} (A - Bc) \\ P &= \frac{1}{2} \frac{A}{B} + \frac{1}{2} c \end{aligned}$$

Now, look at this solution. This solution may not make sense, to understand why let's graph the aggregate demand curve when $b_h = b_l = 1$, $a_h = 12$, $a_l = 8$.



If $Q = 6$ (for example) then we have no problem, when we find out the market clearing price at $Q = 6$ it is 7, both parties are willing to buy at that price so the Monopolist will sell to everyone. If, on the other hand, $Q = 2$ then we can see that we are to the left of the kink, and actually the Monopolist will need to set the price of 10 to clear the market. However if we are doing our analysis using the aggregate demand curve we will think that the market clearing price is 9, not 10.

How do we figure out that the Monopolist does not want to sell to the low demand types without drawing the graph? Well it's quite simple, we will find that $P > \frac{a_l}{b_l}$, in which case we should realize the low demand types will not want to buy anything. For example here $\frac{a_l}{b_l} = 8 < 9$ which is the price we thought we found.

So if that is true then we do the standard profit maximization assuming only the high demand types will buy.

$$\max_{Q} \left(\frac{a_h}{b_h} - \frac{1}{b_h} Q \right) Q - cQ$$

and the solution will be just like we found above, with a_h for A and b_h for B.

4.1.2 Price and Quantity if the Monopolist can Price Discriminate.

This problem is exactly like we have done above:

$$\max_{Q_l, Q_h} \left(\frac{a_h}{b_h} - \frac{1}{b_h} Q_l \right) Q_l + \left(\frac{a_h}{b_h} - \frac{1}{b_h} Q_h \right) Q_h - c \left(Q_h + Q_l \right)$$
$$\max_{Q_l, Q_h} \left(\frac{a_h}{b_h} - \frac{1}{b_h} Q_l \right) Q_l - cQ_l + \left(\frac{a_h}{b_h} - \frac{1}{b_h} Q_h \right) Q_h - cQ_h$$
$$\left[\max_{Q_l} \left(\frac{a_l}{b_l} - \frac{1}{b_l} Q_l \right) Q_l - cQ_l \right] + \left[\max_{Q_h} \left(\frac{a_h}{b_h} - \frac{1}{b_h} Q_h \right) Q_h - cQ_h \right]$$

Where we can go from the second to the third line because the expressions in the brackets are independent of the other variable. The solution is above, all you have to do is change the labels on the slopes and intercepts.

4.1.3 When Will the Monopolist Price Discriminate? When will Price Discrimination be Pareto Improving? Increasing Welfare?

The answer to when the Monopolist will price discriminate is so easy that it is almost laughably trivial. This is because it is a simple implication of rationality. **If a Monopolist can use n variables to maximize their profits they will only use n-1 if two of the variables end up being the same when they maximize over all n.** Or in other words, if the two prices they charge when they can price discriminate end up being the same:

$$P_h^t = \frac{1}{2}\frac{a_h}{b_h} + \frac{1}{2}c = \frac{1}{2}\frac{a_l}{b_l} + \frac{1}{2}c = P_l^t$$
$$\frac{a_h}{b_h} = \frac{a_l}{b_l}$$

Notice how incredibly trivial this is to check. Are the intersections of the demand curves with the vertical axis the same? If so then the Monopolist will not price discriminate.

So when will it be Pareto Improving? Again this isn't too hard to figure out intuitively. First of all notice that if the Monopolist charges one price to both types and sells to both types then obviously it must be higher than the price when he sells to the low types and lower than the price when he sells to the high types. To be precise in the case we are analyzing in this handout:

$$\begin{aligned}
P &= \frac{1}{2}\frac{a_h + a_l}{b_h + b_l} + \frac{1}{2}c \\
&= \frac{b_h}{b_h + b_l}\frac{1}{2}\frac{a_h}{b_h} + \frac{b_l}{b_h + b_l}\frac{1}{2}\frac{a_l}{b_l} + \frac{1}{2}c \\
&= \frac{b_h}{b_h + b_l}\frac{1}{2}\frac{a_h}{b_h} + \frac{b_l}{b_h + b_l}\frac{1}{2}\frac{a_l}{b_l} + \frac{b_h}{b_h + b_l}\frac{1}{2}c + \frac{b_l}{b_h + b_l}\frac{1}{2}c \\
&= \frac{b_h}{b_h + b_l}\left(\frac{1}{2}\frac{a_h}{b_h} + \frac{1}{2}c\right) + \frac{b_l}{b_h + b_l}\left(\frac{1}{2}\frac{a_l}{b_l} + \frac{1}{2}c\right) \\
&= \frac{b_h}{b_h + b_l}P_h^t + \frac{b_l}{b_h + b_l}P_l^t
\end{aligned}$$

and obviously if a group is getting a lower price they must like the pricing scheme where this happens better. Thus high types like it when the Monopolist charges one price, low types like it when the Monopolist charges two.

So the only case in which it might be Pareto Improving is clearly when the Monopolist only sells to the high types if they can only set one price. In this case the price charged to the high types is exactly the same, and the only difference is now the Monopolist sells to the low types. Thus clearly the low types will like it better, the Monopolist will like it better, and the high types will be indifferent, so it is Pareto Improving.

Answering when it will increase welfare is much more complicated. Now we have to actually figure out the profits and Consumer Surplus and sum them up. Assume that the Monopolist sells to both types if they can only charge one price, above we argued logically that it was Pareto Improving if they did not sell to the low types, so our welfare will increase if that is the case (all parties surplus is increasing or constant.) So we can find that:

$$\begin{aligned}
CS_h &= \frac{1}{2b_h}Q_h^2 \\
CS_l &= \frac{1}{2b_l}Q_l^2 \\
\Pi &= \begin{cases} \left(\frac{A}{B}-\frac{1}{B}\left(Q_h+Q_l\right)\right)\left(Q_h+Q_l\right)-c\left(Q_h+Q_l\right) & \text{if } P_h=P_l=P \\ \left(\frac{a_l}{b_l}-\frac{1}{b_l}Q_l\right)Q_l+\left(\frac{a_h}{b_h}-\frac{1}{b_h}Q_h\right)Q_h-c\left(Q_h+Q_l\right) & \text{else} \end{cases}
\end{aligned}$$

$$W\left(P_h=P_l=P\right)=\frac{1}{2b_h}\left(Q_h\right)^2+\frac{1}{2b_l}\left(Q_l\right)^2+\left(\frac{a_h+a_l}{b_h+b_l}-\frac{1}{b_h+b_l}\left(Q_h+Q_l\right)\right)\left(Q_h+Q_l\right)-c\left(Q_h+Q_l\right)$$

And when we find the quantities and substitute:

$$\begin{aligned}
P &= \frac{1}{2}\frac{a_h+a_l}{b_h+b_l}+\frac{1}{2}c \\
Q_h &= a_h-b_h\left(\frac{1}{2}\frac{a_h+a_l}{b_h+b_l}+\frac{1}{2}c\right) \\
Q_l &= a_l-b_l\left(\frac{1}{2}\frac{a_h+a_l}{b_h+b_l}+\frac{1}{2}c\right) \\
Q &= \frac{1}{2}\left(a_h+a_l\right)-\frac{1}{2}\left(b_h+b_l\right)c
\end{aligned}$$

$$\begin{aligned}
W\left(P_h=P_l=P\right) &= \frac{1}{2b_h}\left(a_h-b_h\left(\frac{1}{2}\frac{a_h+a_l}{b_h+b_l}+\frac{1}{2}c\right)\right)^2+\frac{1}{2b_l}\left(a_l-b_l\left(\frac{1}{2}\frac{a_h+a_l}{b_h+b_l}+\frac{1}{2}c\right)\right)^2 \\
&\quad +\frac{1}{4}\frac{\left(\left(a_h+a_l\right)-\left(b_h+b_l\right)c\right)^2}{\left(b_h+b_l\right)} \\
&= \frac{3}{2}\left(b_h+b_l\right)\left(\frac{1}{2}\frac{\left(a_h+a_l\right)}{\left(b_h+b_l\right)}-\frac{1}{2}c\right)^2+\frac{1}{2}\frac{b_hb_l}{b_h+b_l}\left(\frac{a_h}{b_h}-\frac{a_l}{b_l}\right)^2
\end{aligned}$$

Boy, was that a lot of work to get it into a form that actually seems to make some sense. The last term is a weight time the difference between the vertical intercepts squared, and the first term is the price minus the marginal cost, or the per unit profit margin, squared. I would just like to mention that there is no way I could have done that without using Scientific Workplace. The problem when the Monopolist charges two different prices is substantially easier:

$$\begin{aligned}
W\left(P_h\neq P_l\right) &= \frac{1}{2b_h}Q_h^2+\frac{1}{2b_l}Q_l^2+\left(\frac{a_l}{b_l}-\frac{1}{b_l}Q_l\right)Q_l+\left(\frac{a_h}{b_h}-\frac{1}{b_h}Q_h\right)Q_h-c\left(Q_h+Q_l\right) \\
&= \left(\frac{a_l}{b_l}-c-\frac{1}{2b_l}Q_l\right)Q_l+\left(\frac{a_h}{b_h}-c-\frac{1}{2b_h}Q_h\right)Q_h
\end{aligned}$$

$$\begin{aligned}
Q_l &= \frac{1}{2}\left(a_l-b_lc\right) \\
Q_h &= \frac{1}{2}\left(a_h-b_hc\right)
\end{aligned}$$

$$
\begin{aligned}
W(P_h \neq P_l) &= \left(\frac{a_l}{b_l} - c - \frac{1}{2b_l}\frac{1}{2}(a_l - b_l c)\right)\frac{1}{2}(a_l - b_l c) + \left(\frac{a_h}{b_h} - c - \frac{1}{2b_h}\frac{1}{2}(a_h - b_h c)\right)\frac{1}{2}(a_h - b_h c) \\
&= \frac{3}{4b_l}(a_l - cb_l)\frac{1}{2}(a_l - b_l c) + \frac{3}{4b_h}(a_h - cb_h)\frac{1}{2}(a_h - b_h c) \\
&= \frac{3}{8b_l}(a_l - cb_l)^2 + \frac{3}{8b_h}(a_h - cb_h)^2
\end{aligned}
$$

Now, how do we compare them? Well fortunately you have your handy-dandy Scientific Workplace which has Maple imbedded, and so you can just subtract them and ask the program to simplify it:

$$
\frac{3}{2}(b_h + b_l)\left(\frac{1}{2}\frac{(a_h + a_l)}{(b_h + b_l)} - \frac{1}{2}c\right)^2 + \frac{1}{2}\frac{b_h b_l}{b_h + b_l}\left(\frac{a_h}{b_h} - \frac{a_l}{b_l}\right)^2
$$
$$
-\left(\frac{3}{8b_l}(a_l - cb_l)^2 + \frac{3}{8b_h}(a_h - cb_h)^2\right) = \frac{1}{8}\frac{b_h b_l}{b_h + b_l}\left(\frac{a_h}{b_h} - \frac{a_l}{b_l}\right)^2
$$

wasn't that easy? As you can see Welfare is always higher without price discrimination in this case. However, notice that the low demand types are still getting a worse deal—their price is still higher. So is this measure of Welfare important or the Pareto argument? Your choice.