Voting,
Various Methodologies and their Properties and Strategy

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Consider an abstract economy, characterized by a set of allocations and a set of people. How are we going to decide what to do? Obviously the competitive economy is one way, but this can only deal with a very specific set of problems. In general we need a different technique. One that is very popular is voting, in this handout we will consider various different types of voting. We will consider both *truthful voting*—voting in a way that most closely represents your preferences—and *strategic voting*—choosing a best response to what the other people are doing. In many ways truthful voting is always better for society than strategic voting. Strategic voting can result in an outcome that is not Pareto Efficient, while truthful voting will never do this. But the problem is that voters don't really care about society, what they care about is their own interests, and any time the two are different honest voting is not what a sensible person will do.

So in terms of *outcomes* strategic voting will be worst, but in terms of *your interests* it will generally be better. The basic lessons of the section on strategic voting will be is some basic guidelines of what you should do to make sure that your best interests are represented.

1 Truthful Voting

Truthful voting will almost always result in something that is at least Pareto Efficient. Unfortunately the result will depend on the institutional rules. We'll first consider the least arguable rule, majority voting. If there are two Pareto Efficient options almost everyone agrees that the one that gets more votes (at least $\frac{n+1}{2}$ where n is the number of voters and is odd) should win. Of course most of the time there will be more than two options, so we'll try to extend this rule by considering only two options at a time. Sometimes this will produce a clear winner.

However Two by Two comparisons are very costly. If we have k outcomes then there will be $\binom{k}{2} = \frac{1}{2}k(k-1)$ comparisons to be made. Imagine, for example, doing this in Turkey where there are 4 parties in the parliament (as of 11/11/2013) but 40 parties recognized by the government. How would you feel about having to make $\frac{1}{2}(40)(40-1) = 780$ comparisons on a ballot?

The most popular voting rule, worldwide, is the Plurality winner. The party that gets the most vote wins. In many countries like Turkey it is actually *proportional*, in other there are m seats in a district and they are divided up among the parties that get the highest number of votes. This does result in dramatically different outcomes, but it is essentially the same voting rule. Variations of this are the runoff rule—have a second election with only the top two candidates, and the sequential run off—drop the candidate that places last in the election. These variations don't really produce that many changes from the basic model, but we'll discuss them where appropriate.

A weird alternative is the Borda Count rule. In this type of elections you would have a number of "points" that you could assign to all political parties and you can break them up any way you want. A more restrictive manner is to require people to list all political parties from best to worst. This method is not often used for significant decisions, usually it is used where people are not too concerned about the outcome or they think people won't be able to behave strategically. An example of an election that uses this type of rule is the Eurovision song contest, each country votes for their favorites and then the one that gets the most votes gets the most points from that country, and so on. The final winner is decided by summing up the votes from the countries in the contest. Another example is the Heisman Trophy for the best college American football player, the voters there get to give points to their top choices, and then whomever gets the most points wins the trophy. This type of scoring rule is also used to decide World Cups. In downhill skiing, for example, whomever places first in a race gets x points, second place gets y points, and so on. Then the winner for the season is the one who gets the most points in all races.

1.1 Majority Voting, Two by Two Comparisons and the Condorcet Winner

The simplest voting rule is Majority Voting, if there are two options almost all voting rules boil down to this very simple rule. Majority voting is if there are n people (assume n is odd) then if $\frac{n+1}{2}$ favor an option it is chosen.

If there are more than two options then the outcome can depend on the methodology, and there are various methodologies that can be used. In every one of these methodologies we want to look at the *beats* relationship. Let the set of outcomes be Ω.

Definition 1 *We say that X* ***beats*** *Y if a majority of voters prefers X to Y, and we denote it $X \rhd Y$.*

This is a ridiculously easy thing to calculate, and if something beats every other alternative then it's obviously something we should care about.

So in this method we compare two options at a time. You then take the winner by majority vote and compare it to a third option and so on until you run out of options. Lets look at an example and see how this would work.

1	2	3
A	C	B
B	B	C
C	A	A

(1)

Now if people voted over A versus B, which one would win? Two people (2 and 3) prefer B to A, so B will win—or $B \triangleright A$. Now if we compare B to C who will win? 1 and 3 prefer B to C so $B \triangleright C$ and B wins every contest.

Notice that if we have such a winner the order we look at the options really doesn't matter. No, B won individually against both A and C so no matter whether A or C wins that election the winner will loose to B. in it then B will win every further contest, and thus B will always win in this format, or B is a **Condorcet Winner**.

Definition 2 *A* Condorcet Winner *is an option that will get a majority of the votes in every pair-wise competition. Formally X is a Condorcet Winner if $X \triangleright Y$ for every $Y \in \Omega \backslash X$ (which is all of the other options.)*

Whenever we have a Condorcet Winner the order in which we choose options does not matter. At some point that option will come up and it will win every subsequent contest, so it will always be chosen. In a very real sense it is clearly the option that should be chosen. However we don't always have a Condorcet Winner. The following example is usually called the *Arrow's Paradox* for the man who first discussed it.

1	2	3
A	C	B
B	A	C
C	B	A

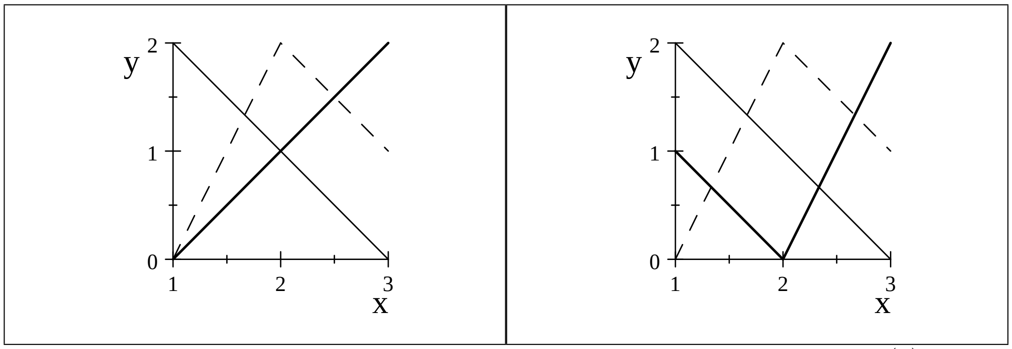
(2)

In this model $A \triangleright B$, $B \triangleright C$, $C \triangleright A$. So if we use a majority voting rule then order matters a lot, any option can be chosen with the appropriate order. If we start with A versus B, then A will win and then it will loose to C. If we start with A versus C then C will win and B will beat C in the head to head contest. If we start with B versus C then B will win and A will beat B in the final round. This is very disturbing, how can we be sure this won't happen?

1.1.1 Single Peaked Preferences

Let us graph the preferences of the voters in both cases. The way we will graph it is that we will list the options from A to C along the bottom of the graph, and give each option a value associated with its distance from the bottom. For example for person 1 in either economy above A would be worth 2, B would be worth 1 and C would be worth 0. The graph of both example 1 and example

2 are below.



(3)

In both of these graphs on the horizontal axis 1 is A, 2 is B and 3 is C (I have limited graphing capabilities.) Person 1 is the light solid line, person 3 is the dashed solid line, and person 2 is the heavy solid line.

The graph on the left represents example 1, notice how each person has a single peak to their preferences? By this I mean that we can represent their preferences as if they have one best option, and the rank of other options gets lower the further you get from that peak. Person 1's peak is at A, and C is worse than B because it is further from A. Likewise person 2's peak is at C in example 1, and A is worse than B because it is further from C.

On the other hand in the graph on the right (example 2) one person does not have single peaked preferences, person 2 now has double peaked preferences. Option B is the worst and options A and C are both better, so her preferences have a double peak. Even more significantly this happens no matter how we order the options. If we make option B 1 then person 1 will have double peaked preferences. If we make option C 1 then person 2 will have double peaked preferences.

If each voter has single peaked preferences then we know there will be a Condorcet winner and we know something even further. We call a voter the **median voter** if half the people have a most preferred option that is above this person and half the people have a most preferred option that is below it (notice the median voter is in both groups.) In most cases there is a natural ordering over the options and the median is obvious, like consider the choice over size of government.

Voter	Donald	Louie	Huey	Daisy	Dewey
Optimal Size of Government	5	600	150	100	195

(4)

Here with a little quick calculation one can see that the median is Huey at 150. Daisy and Donald want a smaller government, and Dewey and Louie want a larger government.

Theorem 3 (Median Voter Theorem) *If everyone has single peaked preferences then the most preferred outcome of the Median voter is also the Condorcet Winner.*

Proof. *Consider a contest where the Median voter's preferred option (m^*) is pitted against any other option (o). Without loss of generality assume that o is higher than m^*. By single peaked preferences this means that all people who's most preferred option is below m^* prefer m^* to o, and m^* gets a majority of the votes.* ■

An important thing to recognize is that while the median voter's option will be Pareto efficient (since it is her most preferred option) it may not be close to the average voter's preferences at all. If welfare is defined as maximizing the sum of people's preferences then the average of the voters most preferred option may be welfare maximizing. In this example the average size of government the voters would select is 210.

One can look at this result in two different ways, first of all the average person could be fairly upset with this policy, but on the other hand it may not be a good idea to let people with extreme desires (like Louie above) control the electoral process. The median is very insensitive to outliers like Louie, the average will always be affected by them. Say, for example Louie decided that the optimal size for a government was 1500, do you think the society should choose 390—two times as high as the next highest person? Or what if Dewey convinced Louie that he was right and the optimal size was 195, should the amount of government collapse to 129?

Notice also that while single peaked preferences guarantee that there is always a Condorcet winner they are not necessary. Indeed we can easily generate such an example by just adding a couple of options to example 1.

1	2	3
A	C	B
B	B	C
C	A	A
E	E	D
D	D	E

(5)

Now no one has single peaked preferences, but B easily wins against E and D so it is still the Condorcet Winner. You can generate a thousand similar examples, many much more subtle than this, but this example is enough for now.

I should point out that this result can also be changed by different levels of voter participation. If Louie and Dewey gave up on the political system and stayed home then the median would be Daisy, and government would shrink by 33%. If Donald and Daisy gave up then the median would increase 30%. In a small community this is rarely a big problem because each voter knows how important their vote is, but in general it can be. It is quite common for people on both ends of the political spectrum to give up on the electoral process. One of the reasons for the dramatic shift to the right in the US in the mid 1990's to the mid 2000's was because a significant number of evangelical Christians (generally a very conservative group)[1] decided to get involved in politics. A

[1] I should mention that my Dad considers himself an evangelical Christian and hasn't voted for a Republican since the 1960's.

major reason for Obama's strong overall showing in the 2008 election was that his voters were extremely enthusiastic about voting for him and showed up in droves (large numbers). The overall turnout was not that high but the turnout in areas strongly supporting Barack Obama set records.

1.2 Plurality Voting or First Past the Post.

One of the biggest problems about finding the Condorcet winner is that it can take so long. If there are many candidates then it requires the voters to vote many different times or make many a lot of comparisons. Not exactly feasible in most societies, it may become more feasible with the internet but it is still rarely used. The simplest alternative is to simply have everyone vote once and whichever alternative gets the most votes wins. But this is often selects the "wrong" option, for example consider the following example.

1	2	3	4	5
A	A	C	D	E
B	B	B	B	B
C	C	A	A	A
D	D	D	C	C
E	E	E	E	D

(6)

Here B is the Condorcet winner, even more it is second best for every single person. It is an obvious compromise candidate, but it will never be selected. However if everyone votes for their most preferred outcome A will win. Hmmmm, it is interesting to note that in most countries that have first past the post (like Great Britain and the United States) the result of this is essentially a two party system. The reason for this will become apparent when we look at strategic voting.

Notice that in this example B will not win in any of the variations on the plurality rule I've mentioned. It won't appear in the runoff (which A would win every time), and it will be the first to go in sequential runoffs (which again A would win every time.)

A minor variation on this rule used in many countries (like Turkey) is to have multiple winners in each election, and usually to assign power (seats) to each winner based on their percentage of the vote. The only difference between this model and strict first past the post is that in general there are more than two political parties.

1.3 The Borda Count Rule

Another voting system that is sometimes used is the Borda Count Rule. For example in the Eurovision contest you vote for your favorite country by phone, and then Turkey votes for it's top ten countries by giving 12 points to its top choice, 10 to its second highest, and then 8 to 1 for ranks 3 to 10. This is essentially a Borda Count Rule, for each option you assign that option a number of points based on its distance from the worst choice and then the winner is the option with the most points.

In example 6 the way points would be assigned is:

	1	2	3	4	5	Total
A	4	4	2	2	2	14
B	3	3	3	3	3	15
C	2	2	4	1	1	10
D	1	1	1	4	0	7
E	0	0	0	0	4	4

(7)

and in this case this would result in the Condorcet Winner being selected. However notice that if instead of assigning 4 points for the top option we assigned 5 then A would win, and this a minor variation on the Eurovision points assignment. The importance of the exact number of points assigned to a particular rank makes this a popular system only if parties can negotiate on this fact. World Cup sports competitions often use this sort of ranking system and—for example—in the 2008-2009 season Biathlon changed the amount of points they gave to the winner. If parties can not agree on the points system then this system will quickly fall into disfavor. Notice that in the standard Borda count we assigned points to every option but in the Eurovision contest (and World Cups) points are only assigned to the top ten options. There are also other variations like "vote for any two," etceteras.

It is important to point out that the Borda Count rule does not always select the Condorcet Winner, consider the following example.

1	2	3	4	5		1	2	3	4	5	Total
A	A	A	D	E	A	4	4	4	0	2	14
B	B	B	B	B	B	3	3	3	3	3	15
C	C	C	C	A	C	2	2	2	2	1	9
D	D	D	E	C	D	1	1	1	4	0	7
E	E	E	A	D	E	0	0	0	1	4	5

(8)

Now A is both the plurality winner and the Condorcet winner, however the Borda count rule still selects the outcome B. Notice that B is still Pareto efficient and it is still the second best option for everyone, so perhaps it is better—or perhaps not.

The more important issue is that the way we assign points to each rank can affect the outcome. What can we conclude for an arbitrary rating system?

Definition 4 *We say A* ***rank dominates*** *B if for every rank x the number of voters who think A is rank x or higher is more than B with A being strictly higher for at least one rank. (Bottom is rank zero.)*

Then if B is rank dominated A will always beat it. This is similar to *first order stochastic dominance* for the geeks in the audience. It is clear that something is rank dominated should not win—for example if one racer always comes in front of another then clearly they are better—but things which are rank dominated will be fairly rare. Notice that if A Pareto dominates B then it also rank dominates B, so the outcome must be Pareto efficient.

1.4 The General Impossibility of Social Choice

A rather controversial result in the field of voting models is that the only methodology that always works is Dictatorship—let one person always choose the outcome. You might think this should be addressed after we talk about strategic behavior, but the same result holds in both cases and it's easier to understand without the strategic baggage.

Theorem 5 (Arrow's Impossibility) *If a social choice (voting) rule is:*

1. *Always able to produce a decision and able to rank all possible outcomes in a transitive manner.*[2]
2. *Always selects a Pareto Efficient outcome.*
3. *The order between two options only depends on the two options (Independent of Irrelevant Alternatives) OR if people change their mind about an option in a way that increases that option's rank then the social ordering also increases that option's rank. (Maskin Monotonicity).*

Then it is dictatorial.

How do you feel about this? Well, it may be part of the reason it seems that not every country can be a democracy, but I rather doubt that. To me it merely says that there are some problems where society just can't choose. Tough luck, but then again if I make no assumptions about preferences I often either can't find an equilibrium or the equilibrium makes no sense. A lot of the problem here is that we want it to "always produce a decision." Yea, well, it don't always work like that, right?

Generally the failure of this theorem merely means that the agenda or History matters. If there is no Condorcet winner then just select an agenda and you'll get a winner—it just won't be independent of the agenda. It is unfortunate that this is true but I can show you many examples of this. Take the QWERTY keyboard for example (named after the six letters in the upper left hand corner). This keyboard was designed to make sure that typists *didn't type too fast*. When they first invented the typewriter they were afraid that typists would just type so fast that the machine would break, so with flawless logic they decided to design the keyboard to make this very hard. Why didn't they just trust the typists to use common sense? Well, that's engineers for you. But the result is that the most common keyboard in the world is designed to make sure that you can not type very fast. There are better keyboards, and typists

[2] Transitive means that if A is better than B and B is better than C then A is better than C.

who are experienced using them are significantly faster than the best QWERTY typists, but we still use the worse keyboard.[3] Makes you laugh, ehh?

To be specific, consider the following voting rule.

Definition 6 *Incumbency voting: choose an order over the allocations. Make the first allocation the* ***incumbent****, then go through the order considering each option at a time. If the new option wins a majority of votes it is the new incumbent, if it does not then the old incumbent is still the incumbent. Continue until you are at the last item in the list.*

This rule is not a social choice rule because of the (arbitrary) order over the allocations. However this is exactly how democracy works—though the order is not exactly arbitrary, and often times the vote is over three or more candidates. This is precisely why I say that history matters, if we include a history, an order over the outcomes, then we will get a choice. It will always be Pareto efficient if we use pairwise comparisons, and in the top cycle (see below) but other than that we can't predict exactly what will happen.

2 Strategic Voting

The fundamental problem with strategic voting is that expectations matter. Like in models of adverse selection, if you don't believe that good things can happen they won't happen. We will place some restrictions on player's strategies, but unfortunately this will leave a lot of cases where bad things still happen. Of course being Economists our favorite definition of bad is Pareto Dominated, so let me give you two examples where B is bad and is selected by two of our voting rules.

Now strategic voting won't matter in the pair-wise majority model if there is a Condorcet winner, but consider the following Economy:

1	2	3
A	D	C
B	A	D
C	B	A
D	C	B

(9)

which we will refer to as the *Condorcet Plus Paradox*. It's the Condorcet Paradox but now we've added in something that is Pareto Dominated, B. In this model what matters is the order in which the pairwise comparisons are made, and for the correct order we can end up with B winning.

For the Borda Count rule consider the following Economy, and assume that

[3] SOME would mumble about Windows here, but not me, oh no, I would never criticize Windows.

the points people vote with are on the right.

1	2	3	4	5		1	2	3	4	5	Total
A	A	A	E	D	A	1	1	1	2	2	7
B	B	B	C	C	B	4	4	4	1	1	14
C	C	C	A	A	C	0	0	0	3	3	6
D	D	D	B	B	D	3	3	3	0	4	13
E	E	E	D	E	E	2	2	2	4	0	10

(10)

Notice that voters 4 and 5 are being honest, voters 1-3 are very concerned that C will win, and if they increase their votes for C then it might. Now you might point out to them, very reasonably, that if they all increased their votes for A then A could win, but they might respond that they don't think A will win. Notice that A is the Condorcet Winner, but the voters aren't involved in an election where this matters. Again, we have a Nash equilibrium were something that is Pareto Inefficient is elected.

With Plurality voting getting B selected in either election is depressingly easy. All we have to do is have the voters come into the election believing that the only viable choices are B or C. In both cases it is an equilibrium to elect B for any reasonable restrictions on players behaviors.

So what is the problem here? The problem in Plurality voting and the Borda Count rule is that *expectations matter.* If I expect something bad to occur then something bad can occur. In fact in all of the models we can easily make any $X \in \Omega$ a Nash equilibrium, all we have to do is have all voters come into the election believing everyone else is going to vote for X. This means that no voter will be *pivotal.*

Definition 7 *A voter is* ***pivotal*** *if when he or she changes his or her vote the outcome of the election changes.*

As you should realize in most elections your vote doesn't matter. For example I am a resident of Indiana, which is a heavily Republican state. I vote Democratic almost all of the time, and I know that almost always my vote is wasted. The Republican always wins, and my vote didn't matter at all. You can imagine how excited I was in 2008 when Obama (a Democrat) actually won Indiana! Finally! I get to vote for the winner! Of course, I still probably could have not voted and he still would have won. Ahh well, at least I felt good.

Lemma 8 *If the number of voters is three or more than any outcome $X \in \Omega$ can be a Nash equilibrium.*

Proof. *We assume that when people think their vote won't matter, they vote for the option that they expect to win. This is a utility maximizing strategy, if perhaps something of a weird one.*

Now assume that every voter believes that every other voter will vote for X (or rank it as their best option in the Borda Count rule). Then no voter is pivotal because X will win no matter how he votes. Thus he might as well vote for X, and the other people's beliefs are right. Thus X wins. ■

Silly, right? And completely counter-intuitive. If you find the proof hard look at the definition of pivotal again. If you aren't pivotal you might as well do anything. Anything *could* be to vote for X, so go ahead! Do it! Let's point out how ridiculous this is. We can have X be the *worst* outcome for every single person and yet it still could be a Nash equilibrium.

So what do we do now? Well, let's rule out silly behavior. Unfortunately this gets us very little more. I'll talk about two different ways that one could want to do this. First a completely obvious way to do this would be to have everyone vote *as if* they were pivotal even when they are not.

Definition 9 *A voter is using* weakly undominated strategies *(*WUD*) if he or she always votes as if he or she was pivotal. Or in other words to maximize the potential impact of their vote.*

This is a widely accepted criterion, and in pairwise elections will at least always give us one winner. But the key problem is *pair.* If there are three or more options then I have to choose "the lessor of two evils"[4], and that means that I can be pivotal and vote for something that is Pareto Dominated. Look at both of the examples above, this is exactly what is happening. Everyone who is voting is choosing the better of the two options he or she thinks is viable, but that results in him or her voting for a Pareto Dominated outcome.

A more restrictive requirement, but perhaps even more obvious, is that we require people to be honest when their vote won't matter.

Definition 10 *A voter is using* default honest strategies (DH) *if he or she always votes to represent his or her true preferences when indifferent.*

This won't solve any problems, but perhaps you'll like thinking of this as the fallback plan. A subtle but important point is that in a proportional representation plurality rule contest using weakly undominated strategies often means that you will not use default honest strategies—your vote might matter in a way that doesn't seem obvious at first.

2.1 Pairwise voting and Agenda Control

If we use the pairwise method then at least we have exactly pinned down the way someone will vote in a pairwise election. However, unfortunately, if we don't have a Condorcet Winner then this won't pin down what can happen. Instead we then have to think about what the Agenda is going to be, or in other words what order the pairs are going to be considered. First of all, what is an Agenda?

Definition 11 *An* Agenda *is some order of presentation of the options to be voted on*

Then the standard model of voting over an Agenda is:

[4] Trust me, I do this a lot in elections.

Definition 12 *In the standard model voters vote to accept the first item or reject it and move to the second one. This continues as long as there is more than one option left in the agenda. Thus if the voters vote to reject everything except the last item then the last item is chosen by default.*

So what can we find that can survive for *some* agenda. Well it's actually quite simple to figure out by construction. The first item in the agenda should be the thing that you want to be selected. The next one should be something that it beats. The third item should be something that the second option beats, and so on. If you can construct such an agenda that covers all of the options then it can be selected. To be precise this definition is:

Definition 13 *X is in the* top cycle *of Ω if there is an agenda over Ω where X is the first item in the agenda, and it beats the second item, and for every n the n'th item beats the $(n+1)$'th item.*

But we want some simpler way to figure this out, and something that can be done using only the *beats* relationship. A more constructive way to look at the top cycle is:

Definition 14 *The* top cycle *of Ω is the largest cycle of the $\rhd$ relationship that is not dominated with regards to the $\rhd$ relationship. I.e. nothing in the cycle is beaten by anything outside of it, and everything in the cycle beats at least one other option in the cycle.*

Let me give some examples to make this clear. For example in the Economy 9 we can see that $A \rhd B \rhd C \rhd D \rhd A$. Now you might point out that another such cycle is $A \rhd C \rhd D \rhd A$, but this is not the *largest*. As another example consider:

1	2	3
E	E	C
A	D	D
B	A	A
C	B	B
D	C	E

(11)

In this economy we have a Condorcet winner, E. Thus the cycle $A \rhd B \rhd C \rhd D \rhd A$ is dominated by E and it's not the top cycle. So, you ask, can I construct an agenda where B wins in Economy 9? Yes, it's easy. This agenda is (B, C, D, A). We show this by backward induction. In stage 3 if they reject D they will select A, and since everyone is using a weakly undominated strategy we choose to accept D. Thus in stage 2 if we reject C we accept D, and so we accept C. Thus in stage 1 if we reject B we accept C, thus we accept B.

Now I didn't explain why the person who sets the agenda chose this peculiar order, perhaps she likes B. I don't know (that would make sure that B was not Pareto dominated), the point is that for the economy that we are analyzing we could have things that are not Pareto Efficient be selected.

2.2 Strategic Plurality Voting

I'm not going to start this section with complex analysis, instead I'm going to look at the impact of a very simple insight.

Insight, Plurality **Choose the lesser of two evils**, or **Don't waste your vote**—weakly undominated strategies always tells you to vote for one of the top two candidates.

If you vote for a candidate that is sure not to win then there is no chance of your vote mattering. This is equivalent to saying you want the front runner to win. If this is true then sure, go ahead and vote for whomever you like, but if this is not true then you should think twice.

To get a good understanding of the benefit of strategic voting let's look at the March 2019 Mayoral election in Istanbul. This election was rerun, in the first round the CHP candidate one by an incredibly small margin. In the first election (results as of 8 April 2019) the voting was:

	CHP (E. İmamoğlu)	AKP (B. Yıldırım)	Other Party	Not Voting
Number of votes	4,169,987	4,146,042	230,763	973,974
% of voters	48.79%	48.51%	2.70%	Not Applicable
% of registered voters	43.80%	43.55%	2.42%	10.23%

Thus, as of 8 April, CHP (Ekrem İmamoğlu) won by $23,945$ votes. Notice this is about one tenth (10.3%) of the voters who voted for a third party, and only 2.5% of the voters who didn't vote. These results make the basic insight about *not* choosing the lesser of two evils even more apparent. Of that 2.7% that bothered to vote, do you think it possible that 30,000 would actually have rather had the AKP win? I would be surprised if they didn't. However since they voted with their hearts for another candidate they left it up to the 97.3% that voted strategically, and they might has well have stayed home with the 10.23% who didn't vote. And what do you think the odds are that of the people who didn't vote 2.5% of them really wanted the AKP candidate to win? I think that's practically certain. Of course if that group voted, maybe others in that group would have voted, maybe it would have balanced out—we will never know.

And the CHP supporters who voted for another party or didn't vote? If the CHP had managed to rally a third of those who didn't vote to vote for them then there would have been no rerunning of the election. In elections participation matters, and furthermore not following the "lesser of two evils" advice can result in outcomes you wish you could have changed. Of course, in the end, you should do what you want, but you need to be aware of the cost.

The rerunning of the election on 23 June proves that Turkish voters understand these things and vote strategically. Many suspected that the only reason the election was rerun was because the AKP lost. This would provide strong motivation for non-AKP voters to vote for the CHP, and marginal voters for the AKP will either not vote or vote for the CHP. The political parties certainly seemed to feel that the March results should be respected. Before the election every marginal party except the Saadet Party (a religious party) and the Vatan Party (a nationalist party) withdrew their candidates for Mayor. Notice that the voters for these parties would probably back the AKP over the CHP. The end result should be that CHP will win by a dramatic margin, which is exactly what happened.

	CHP (E. İmamoğlu)	AKP (B. Yıldırım)	Other Party	Not Voting
Number of votes	4,741,868	3,935,453	69, 978	612, 682
% of voters	54.21%	44.99%	0.8%	Not Applicable
% of registered voters	49.81%	41.33%	0.7%	6. 44%

Not only did the CHP candidate win but his winning margin is higher than the number that didn't vote. The total participation dramatically increased, but despite this the votes for the AKP decreased.

It was clearly bad for the AKP to rerun the election. If they had accepted the March victory by the CHP then they would have been able to claim that the loss was a mere happenstance. Now the CHP has a clear mandate and the AKP has decisively lost Istanbul.

I should mention that there is a downside to strategic voting. If everyone votes as if they are pivotal, then theoretically bad outcomes might win.

Lemma 15 *With weakly undominated strategies, if $X \rhd Y$ for some $Y \in \Omega$ then X can be a Nash equilibrium.*
Proof. *All we have to do is make sure that coming into the election everyone believes that X and Y are the only viable candidates. If everyone believes this then the majority will vote for X.* ■

2.3 Strategy with the Borda Count

With the Borda Count there are two general strategies that someone wants to follow. Both of them first require that we pin down the two viable options, and then mess with our ranking of these two options.

Insight 1, Borda Count Promoting, If your option doesn't seem viable, you should always give maximum points to the option that you think is best among the viable options.

You should falsely "promote" an option that you think is viable to the best position. Notice that in the example 10 this is exactly what voters $1-3$ are doing. They think B is viable so they give it maximum points.

Insight 2, Borda Count Burying, you should always give the minimum points to the option you think is most viable that you do not want to win.

The bottom line is exactly the same as in plurality voting. "Choose the lesser of two evils" i.e. put all the votes you can behind the option you want to win of the two viable options and put the minimum number of votes behind the other option.

In the example above voters $1-3$ are doing exactly this. They are afraid C might win if they put more votes on it, so they don't. Of course the result of this strategy is that D gets a lot of points, but voter 4 is already giving D maximum points, and voter 5 hates D even worse than B.

But again, the importance of the word *viable* comes into play again. If they all thought A was viable it would win hand's down. It is the Condorcet winner after all, but they don't, so they won't. It's depressing, but it can happen.

However with the Borda Count rule there is another very good reason not to use it in elections that people care a lot about. It can be that there is *no* pure strategy equilibrium. To see this consider the following rather large economy. In this economy there are two types of voters, α (who like A) and β (who like B and think A is the worst outcome). The optimal strategy for type α voters is for all of them to vote A the best, and half of them use the strategy $B(\alpha)$ and the other half use the strategy $C(\alpha)$—or make sure that the other options only get 3 points from them. But this leaves group β with compete control over the election. They bury A (which is actually voting honestly) and whatever they promote to the best becomes the outcome. They have two different strategies they might want to use ($B(\beta)$ or $C(\beta)$), and whichever option they promote wins.

Type	α	β	Votes	α	α	β	
Number	(6)	(5)		(3)	(3)	(5)	(5)
	A	B	A	2	2	0	0
	B	C	B	0	1	2	1
	C	A	C	1	0	1	2
Strategy Name				$B(\alpha)$	$C(\alpha)$	$B(\beta)$	$C(\beta)$

But this means that the α voters have to change their strategy. Say the β voters use the strategy $B(\beta)$, then type α voters will want to bury B. But they must increase their votes for C to do this, and $C(\beta)$ makes C the outcome. But now type α voters will want to bury C, bringing us back to where we started.

There is a cheap fix to solve this problem, but it turns the Borda Count rule into Plurality voting.

Definition 16 *A voter can* express indifference *between outcomes if for each option they have to choose an amount of points in an interval, say* $[0,k]$ *where* k *is the number of options.*

But now, hmmm, I promote the viable option I want to win to k, I bury the viable option I want to lose to 0, and might as well bury all the other options just to be sure... OK, so I am just voting for the option I want to win. I am using a plurality strategy.

In other words either we allow them to use a strategy which is plurality voting *or* we know that there might be a mixed strategy equilibrium. Not good, not good at all. Do you really want to tell voters that they have to strategize right to have a chance of winning? I don't. What we want to be able to tell them is to vote honestly as much as possible.

2.4 Logrolling

One final concept I want to cover is the trading of votes. This is very common in legislatures, and indeed is part of any coalition government. It's kind of a funny term though. Let me first give a few definitions:

log—a usually bulky piece or length of a cut or fallen tree.

rolling—a form of the verb *roll*, which means "to impel forward by causing to turn over on a surface."

logrolling—the exchange of assistance or favors, especially the trading of votes by legislators to secure favorable action on projects of interest to each one.

All these definitions are from Mirriam Webster online. The last one is the one we will talk about here, but how did it come from the former? Well Mirriam Webster is a good dictionary, knowing we would ask they mention that this comes from a tradition of helping your neighbors roll logs into a pile, but the transformation sure is wild.

This idea is somewhat controversial. Intuitively it is the old adage: "I scratch your back and you scratch mine." This issue is somewhat controversial because while sometimes it increases the total welfare (assume we can add up people's utilities, or they can be measured in units of money like the lira) but it can sometimes decrease total welfare. This can be quite simply illustrated with an example (x is greater than zero):

Voter	1	2	3	
Issue A	10	-3	$-x$	
Issue B	-3	8	$-x$	
Issue A & B	7	5	$-2x$	

(12)

Now in this case voters 1 and 2 would both benefit if both issues were approved, so if they can they would like to logroll. Voter 3 would be hurt but as long as he is not hurt too much the total net benefit will be positive.

$$\begin{aligned} U(A) &= 10-3-x=7-x \\ U(B) &= -3+8-x=5-x \\ U(A)+U(B) &= 12-2x \end{aligned} \tag{13}$$

so if $x<6$ it is welfare enhancing, but if $x>6$ it will decrease total welfare to have both laws passed.

I should point out that in most proportional representation countries logrolling is part of the standard practice of how to form governments. When a Coalition government forms they often have to court some small parties. These parties will insist on certain issues being decided their way in return for supporting the main party. This is logrolling.