

# 1 Asymmetric Information and Adverse Selection

It is amazing that the chapter on Bayesian games spends so little time on the most important class of problems in this field. These are problems of asymmetric information, and the constant problem of adverse selection that goes along with it. These problems generally have several standard benchmarks:

1. One party has a "type" and the other does not.
2. The utility of both parties is dependent on the type.

Let me give you a couple of examples. Say that you are running an insurance company and one of your employees tells you that to insure the average car in Turkey costs 10,000 TL a year. You respond that this is great, why not offer a "no fault" insurance contract for 10,000 TL a year? Well why shouldn't you? That is how much the average car costs to insure. But your employee says that this policy would be a money loser from day one, and could bankrupt the company. Why? Consider little old me, who rarely drives 10,000 Kilometers a year. Would I buy such a policy? No. Am I included in the cost of insuring an average car? Yes. The only people who will buy such a policy are the people who expect to pay at least 10,000 TL a year. The average cost of these people must—by definition—be more than 10,000 TL, so your policy is doomed to lose money.

Another example. Say that you get a loan from the bank for your new business. In order to borrow 10,000 TL you have to offer to pay back 20,000 TL. You complain to the bank manager that they are making extraordinary profits on your loan. The bank manager says, "No, actually about half the projects we fund fail, so we make no profits." But, you point out, that means that only people who expect to make more than 100% profit on the loan can pay back a loan, so many worthwhile projects are forced into bankruptcy. The manager responds that this is too bad, and very disappointing, but it is the only way they can work.

In both of these cases the problem is one of *adverse selection*. The only people who will accept a contract are those who expect to make a profit off of it. This means that only the marginal person expects to break even on the contract. Thus for the person who offers the contract the average person who accepts the contract is worse than the amount the contract costs. In order to overcome this either the offerer has to have monopoly power (in the case of the banks); the person who is buying the contract has to be willing to pay more than the expected value of the contract (why insurance markets work); or the person who is selling the contract has to value the good much more than the person who is buying the contract. This last case we will work on in depth next, where we discuss the market for lemons.

## 1.1 Adverse Selection and the "Market for Lemons"

Adverse selection can cause markets to collapse. Generally they always cause markets to work less than efficiently. As an example of this consider the market for used cars. It is a well known fact that used cars sell for much less than new cars. If you drive a car off of the lot of the dealer and then immediately try to sell it you probably will sell the car for between 10% to 20% less than you bought it for. Why is that?

Perhaps the question you really should be asking is why would someone sell a car that they just bought? The most common reason would be that there is some problem with the car. This problem might be that they... do not like the color? It is too big for the city? No, usually it is some mechanical problem that the person is just too lazy to fix themselves. In the US (where the theory was written) we call such a car a "lemon," but do not ask me why. I do not know. Now when someone has a lemon they might be honest enough to tell you, but often they will not be. This means that the quality you should expect from this car will be less than the quality you should expect from a new car, and thus the price will be lower.

How significant can this impact be? Well lets develop a little model to explain this. Let us say that buyers have a value for the car of  $v_i w$ , where  $w$  is the quality level of the car. I would like to use  $q$  for quality, but that is the quantity in a market, so I use  $w$  for *worth*. Now the buyer will not know that so they have to find their expected utility. Say, for example that the quality level can be one of three levels: high or  $w_h$ , middle or  $w_m$ , and low or  $w_l$  and that the probabilities of these three outcomes are  $\rho_h$ ,  $\rho_m$  and  $\rho_l$  respectively. Then their expected utility will be:

$$E(v_i w) = v_i E(w) = v_i (\rho_h w_h + \rho_m w_m + \rho_l w_l)$$

The key thing to notice is that  $E(w) < w_h$ , so the quality level the buyer expects is lower than the highest quality cars on the market. How could this be a problem? Well let us say that value of a car to the seller is just its quality level, and that the seller knows the quality level. Then the seller with a high quality car will only sell their car if  $v_i E(w) \geq w_h$ .

Let us say that this is not true, or that  $v_i E(w) < w_h$ . The consumer must know this in equilibrium, so their expected utility is now lower:

$$E(v_i w) = v_i E(w) = v_i \left( \frac{\rho_m}{\rho_m + \rho_l} w_m + \frac{\rho_l}{\rho_m + \rho_l} w_l \right)$$

(Notice I have re-weighted the probabilities so they still sum to one.) And at this point again we know that  $E(w) < w_m$  (since this is now the highest quality) and we have to check if  $v_i E(w) \geq w_m$ . If it is false then the middle quality vehicle owners will not enter the market, and  $E(w) = w_l$ .

Now compare this to the Pareto efficient outcome. As long as  $v_i > 1$  (which we will always assume) every type of car should be sold, if only the low quality cars are coming into the market then there could be a lot of buyers who would

like to buy. This is the basic problem in this market, and explains the price difference between new cars and “slightly used.” There is a lemons problem in this market, and it can cause a collapse.

I want to now present a continuous model, where we have a continuous demand and supply curve. In order to make the demand curve continuous we will assume that there is a continuum of buyers, each of whom buys an *extremely small* amount. The suppliers can be discrete, since the supply curve will be the expected supply curve in our analysis. For technical convenience the sellers will have a uniform distribution. The distribution of buyers will satisfy the following reduced form convenience argument. If  $Q$  units are sold in this market then the value of the marginal consumer will be:

$$v_m = a - bQ$$

This means that our inverse demand curve will be:

$$P = (a - bQ) E[w]$$

where  $E[w]$  is the expected quality level of the car. The quality levels of the car will be distributed uniformly on  $[0, \bar{w}]$  given  $P$  each person will supply  $F(P)$  in expectation. Thus our supply curve for an individual will be  $q = F(P)$ , and if we assume there is only one individual our inverse supply curve is:

$$P = \bar{w}Q$$

and the marginal car will have a quality level of  $P$ . Now, one final technical detail, what is  $E[w|w \leq P]$ ? I.e. given that every car sold in the market has a quality level below  $P$  what is the expected quality of a car?

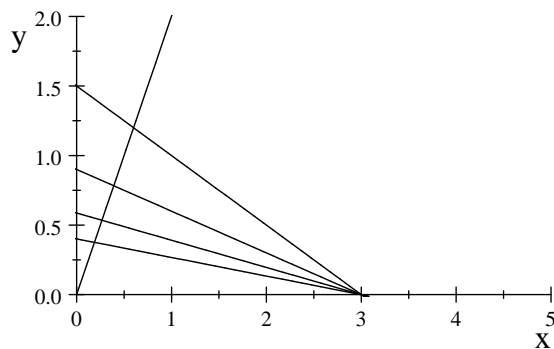
$$E[w|w \leq P] = \frac{1}{F(P)} \int_0^P wf(w) dw = \frac{\bar{w}}{P} \int_0^P \frac{1}{\bar{w}} w dw = \frac{1}{P} \int_0^P w dw = \frac{1}{P} \left[ \frac{w^2}{2} \right]_0^P = \frac{P^2}{2P} = \frac{P}{2}$$

Now the traditional way to solve such a model is to find the intersection of supply and demand. To graph such a case let us assume that  $a = \frac{3}{2}$  and  $b = \frac{1}{2}$ . Now, what should we assume about  $E[w]$ ? Let us solve this model iteratively, by first assuming that it is the unconstrained expectation,  $\frac{\bar{w}}{2} = 1$ , and then

iterate.

$$\begin{aligned}
 P_1 &= \frac{3}{2} - \frac{1}{2}Q \\
 P_1 &= 2Q \\
 \Rightarrow Q &= \frac{3}{5}, P_1 = \frac{6}{5} \\
 P_2 &= \left(\frac{3}{2} - \frac{1}{2}Q\right) \frac{3}{5} \\
 \Rightarrow Q &= \frac{9}{23}, P_2 = \frac{18}{23} \\
 P_3 &= \left(\frac{3}{2} - \frac{1}{2}Q\right) \frac{9}{23} \Rightarrow Q = \frac{27}{101} \\
 P_4 &= \left(\frac{3}{2} - \frac{1}{2}Q\right) \frac{27}{101}
 \end{aligned}$$

Graphically the situation looks like the following:



Each time we find a new price the demand curve shifts down. Now clearly this is a bad process for finding an equilibrium. How can we do it more neatly? The two equations we have to satisfy are:

$$\begin{aligned}
 P &= (a - bQ) E[w|w \leq P] \\
 P &= \bar{w}Q
 \end{aligned}$$

Now in general the first one could be a complicated formula. For example if the worth is distributed uniformly on  $[\underline{w}, \bar{w}]$  then the formula is  $\frac{1}{2}P + \frac{1}{2}\underline{w}$ . However in this case it is simply  $\frac{P}{2}$  so the solution is:

$$\begin{aligned}
 P &= (a - bQ) \frac{P}{2} \\
 Q &= \max \left\{ \frac{a - 2}{b}, 0 \right\} \\
 P &= \bar{w}Q = \max \left\{ \frac{\bar{w}}{b} (a - 2), 0 \right\}
 \end{aligned}$$

Notice that only cars will only be sold if  $a \geq 2$  and that all cars will only be sold if  $a \geq b + 2$ . In the example I gave (of course) the market will collapse, so no cars will be sold in equilibrium.

The important fact to realize is that *trade can only take place if the uninformed party values the object traded relatively more than the informed*. Now in general this might seem hard to understand. Consider the insurance market, there the buyer is risk adverse while the insurer (a large firm) is risk neutral. This difference in attitudes to risk guarantees that the uninformed values the insurance contract more than the informed. At prices where the informed is just willing to trade the uninformed is eager to trade. Consider the loans market. Here the bank expects to make supernormal profits on the good loans, so they value their good loans much more than the borrower, who often will be exactly indifferent between taking the loan and not.

## 1.2 Adverse Selection and the Market for Loans.

In this case the payoff for the banks occurs after the projects return is realized. Thus the bank makes its profit off of the marginal type who will accept the loan, and loses if they offer the loan to someone who is of a worse type. Unfortunately they can not tell the types apart. Assume that each loan is for the same fixed amount, and that lenders require a dividend of  $D > 1$  if the project is successful. For simplicity assume there are two types of borrowers, those who get a good return,  $R$  and those who get a return of zero. The proportion of good borrowers is  $\gamma > 0$ . Then as long as  $R \geq D$  both types will borrow money, and the lender can offer loans if

$$\begin{aligned} -1 + \gamma D &\geq 0 \\ D &\geq \frac{1}{\gamma} \end{aligned}$$

Thus such contracts exist only if  $R\gamma \geq 1$ . In other words the lender must be making a high enough profit off of the good type of borrower to subsidize the bad type of borrower. Notice that there can be competition in this market, but it will never be able to drive dividends below  $\frac{1}{\gamma}$ .

## 1.3 Adverse Selection and Insurance Contracts.

Now to give an example based on risk aversion. First of all we need to write down risk averse preferences and learn something about them. Generally in this sort of problem we are just analyzing utility based on a wealth level. Let  $w$  be the (uncertain) amount of wealth someone will have, then an elegant representation of preferences over  $w$  is

$$u(w) = \begin{cases} \frac{w^\alpha}{\alpha} & \alpha \neq 0 \\ \ln(w) & \alpha = 0 \end{cases}$$

if  $\alpha \leq 1$  then this person is risk averse. Notice explicitly that  $\alpha < 0$  is allowed, this might seem strange because it means that every utility level is negative,

but what is important is that the marginal utility of wealth is positive:

$$\frac{du}{dw} = \alpha \frac{w^{\alpha-1}}{\alpha} = w^{\alpha-1} > 0$$

Notice as well that as long as  $\alpha \leq 1$  these preferences are concave:

$$\frac{d^2u}{dw^2} = (\alpha - 1) w^{\alpha-2} \leq 0$$

this gives us a lovely result called ‘‘Jensens inequality.’’

$$\begin{aligned} \int_x^y u(w) dw &\leq u\left(\int_x^y w dw\right) \\ E[u(w)] &\leq u(E[w]) \end{aligned}$$

which means in simple language that this person prefers the average outcome of any lottery to the lottery. That is one way of saying that this person is risk averse. Now on with the example.

Assume that there are two possible states of the world, in one you face a loss of  $L$  in the other you do not. Let  $\lambda$  be the probability of a loss of quantity  $L > 0$  and  $w$  your current wealth level ( $w \geq L$ ), then your expected utility without insurance is:

$$U = (1 - \lambda) \frac{w^\alpha}{\alpha} + \lambda \frac{(w - L)^\alpha}{\alpha}$$

And if this person has a choice between this and full insurance they will choose full insurance (with a premium of  $p$ ) if

$$\begin{aligned} \frac{(w - p)^\alpha}{\alpha} &\geq (1 - \lambda) \frac{w^\alpha}{\alpha} + \lambda \frac{(w - L)^\alpha}{\alpha} \\ w - p &\geq ((1 - \lambda) w^\alpha + \lambda (w - L)^\alpha)^{\frac{1}{\alpha}} \\ w - ((1 - \lambda) w^\alpha + \lambda (w - L)^\alpha)^{\frac{1}{\alpha}} &\geq p \\ w - \left( (1 - \lambda) w^\alpha + \lambda w^\alpha \left(1 - \frac{L}{w}\right)^\alpha \right)^{\frac{1}{\alpha}} &\geq p \\ w \left( 1 - \left( (1 - \lambda) + \lambda \left(1 - \frac{L}{w}\right)^\alpha \right)^{\frac{1}{\alpha}} \right) &\geq p \end{aligned}$$

You can take it for granted that even when  $\alpha < 0$  these inequalities hold. Or you can check it yourself, it does work out.

The firm will insure this person if:

$$\begin{aligned} (1 - \lambda)p + \lambda(p - L) &\geq 0 \\ p &\geq \lambda L \end{aligned}$$

now if  $\alpha < 1$  by Jensens inequality we know that:

$$(1 - \lambda) w^\alpha + \lambda (w - L)^\alpha > ((1 - \lambda) w + \lambda (w - L))^\alpha = (w - \lambda L)^\alpha$$

Thus  $w - ((1 - \lambda) w^\alpha + \lambda(w - L)^\alpha)^{\frac{1}{\alpha}} > w - ((w - \lambda L)^\alpha)^{\frac{1}{\alpha}} = \lambda L$  and the firm can insure this client.

Now let us extend the model by having two types of consumers. Bad consumers have the loss with probability  $\lambda_b$ , good with probability  $\lambda_g < \lambda_b$ . The the probability of bad consumers be  $\beta$ .

What can be the equilibria in this model? There can be one type of equilibrium where only the bad types are insured, for example  $p_b = w - ((1 - \lambda_b) w^\alpha + \lambda_b(w - L)^\alpha)^{\frac{1}{\alpha}}$  will satisfy this condition. There will can be a second equilibrium where both types are insured, in this case we must have:

$$p \leq w \left( 1 - \left( (1 - \lambda_g) + \lambda_g \left( 1 - \frac{L}{w} \right)^\alpha \right)^{\frac{1}{\alpha}} \right)$$

and:

$$\beta((1 - \lambda_b)p + \lambda_b(p - L)) + (1 - \beta)((1 - \lambda_g)p + \lambda_g(p - L)) \geq 0$$

or:

$$p \geq ((1 - \beta)\lambda_g + \beta\lambda_b)L$$

Thus in order for this equilibrium to exist we must have:

$$\left( 1 - ((1 - \beta)\lambda_g + \beta\lambda_b) \frac{L}{w} \right)^\alpha \geq (1 - \lambda_g) + \lambda_g \left( 1 - \frac{L}{w} \right)^\alpha$$

This condition becomes easier to satisfy as the person becomes more risk averse, or  $\alpha$  decreases. To see this first consider the case where  $L = w$ , and notice that problem is the upper bound on  $p$ , and this constraint becomes more relaxed when  $\alpha$  decreases.

$$p \leq w - (1 - \lambda_g)^{\frac{1}{\alpha}} w = w \left( 1 - (1 - \lambda_g)^{\frac{1}{\alpha}} \right)$$

Since  $1 - \lambda_g < 1$  as  $\alpha$  decreases  $(1 - \lambda_g)^{\frac{1}{\alpha}}$  decreases thus  $p$  can increase. In general there are two impacts of decreasing  $\alpha$ .

$$p \leq w \left( 1 - \left( (1 - \lambda_g) + \lambda_g \left( 1 - \frac{L}{w} \right)^\alpha \right)^{\frac{1}{\alpha}} \right)$$

the first order impact is to increase  $\left( (1 - \lambda_g) + \lambda_g \left( 1 - \frac{L}{w} \right)^\alpha \right)^{\frac{1}{\alpha}}$ , and a second order impact is to decrease  $\left( 1 - \frac{L}{w} \right)^\alpha$ , but the first order dominates. Indeed as  $\alpha \rightarrow -\infty$

$$\begin{aligned} \lim_{\alpha \rightarrow -\infty} w \left( 1 - \left( (1 - \lambda_g) + \lambda_g \left( 1 - \frac{L}{w} \right)^\alpha \right)^{\frac{1}{\alpha}} \right) &= w \left( 1 - \left( 1 - \frac{L}{w} \right) \right) \\ &= L \end{aligned}$$

thus if this person is risk averse enough both types can be insured.