Imperfect Information and the Impact on the Economy

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In the 1970's economists realized that imperfect or asymmetric information can have a large impact on the economy. This was first recognized in two great papers, which resulted in a Nobel prize for the authors in 2001. The first, Akerlof: "The Market for Lemons,"[1] brought us the insight that the best quality goods will often not be traded—or that there is **adverse selection** for low quality goods swamping the market. As he pointed out, this can result in market collapse. The second, Spence: "Job Market Signaling,"[2] pointed out that one way we overcome the problem of adverse selection is to take economically wasteful actions simply to inform others about our quality—or **signalling**.[3]

Definition 1 ***Adverse selection*** *is when market conditions select for lower quality goods being traded. This means that the people you most want to trade with choose not to enter the market.*

Definition 2 *A* ***signal*** *is an action where the direct (marginal) cost is higher than the direct (marginal) benefit, but the action is taken to either reveal or hide information about some unobservable characteristic.*

I should mention that this handout will not address *moral hazard*—which is the way people can be less careful about taking risks because they are insured or something similar. This is not because it is unimportant, but because it would be another level of complexity that I just choose not to address. It is very important, but not as important as these two basic insights.

1 Asymmetric Information and the "Market for Lemons"

Let us consider a simple thought experiment. Say that you buy a new car, drive it back to your home or apartment (say, less than 50 kilometers), and try to sell it. What do you think is going to happen? For this discussion assume that all warranties are transferable. I honestly don't have any idea, their isn't a book on this, but I expect you will have to accept between a 10% and 20% discount. Why? Well I think the better question is why are you selling it? I mean, what, you decided you don't like the color on your drive home? The

[1] Akerlof, George A. (1970) "The Market for "Lemons: Quality Uncertainty and the Market Mechanism." The Quarterly Journal of Economics, Vol. 84, No. 3, pp. 488-500

[2] Spence, Michael. (1973) "Job Market Signaling." The Quarterly Journal of Economics, vol. 87, no. 3, pp. 355–374.

[3] Personally I love both these articles, they are both available on JSTOR and I recommend you read them. However some students have reported they're hard to read.

buyer has to suspect their is a problem, and then they might go through the following thought process.

> "Ok, there's probably a problem with the car, so I think he should mark down the price 1%. Wait a minute, he must recognize I will want at least 1% off, so the problem must be pretty severe, so maybe I better get 5% off. Wait a minute, if he's willing to take 5% off, well... should I offer that?"

When is this reasoning going to stop? I really can't say, no matter how much of discount I demand I have to be suspicious if they will take it. I do know one person who bought a car like this, but that person had a mechanic state that the car was in great shape. Even with that reassurance, I don't think I would have bought it. This is a "Market for Lemons," for some obscure reason we refer to a brand new good that is defective in a significant manner as a "lemon" in the United States. Obviously it would be hard to find a bottom in this market that basically the market does not exist—otherwise you'd be able to find a book stating how much such cars are sold for. Notice that this could also explain why (often) warranties are not transferable—the fact that the good is being resold is a sign that it is defective.[4]

But this, you must recognize, is a problem. What if you really did just decide that you didn't like the color? What if it just doesn't fit right into your assigned parking space? What if...there are so many reasons that it would be great if you could simply resell a new car if you didn't like it after a short period of time. Unfortunately it seems this market has collapsed due to incomplete information. If they could *verify* that was your true reason, sure they'd buy it, but they can't verify it.

You have to recognize that this is always a problem in the used car market, except that with a car that is a year or two old there are other reasons that people might want to sell their car. Some people just really value having a new car, sometimes people move and can't take their car with them. However it's always a problem. There are going to be some people who will wait just long enough to sell their car, or might have been in a traffic accident, all these people can't be trusted and obviously aren't just going to tell you that they're trying to rip you off. At this point it's best to just outline a formal model and start thinking about it.

1.0.1 Model

The seller's good will be of an uncontrollable *worth*, denoted w. Nature will determine the worth, it will be from a finite set, $w \in \{w_1, w_2, w_3....\}$ with $w_i > w_{i+1}$. If there are only three classes (our primary case) then $w \in \{w_H, w_M, w_L\}$ with $w_H > w_M > w_L > 0$. The seller will know the worth of the good they are selling, but the buyer will only know the possibilities. The buyer believes that

[4] In the United States, "full" warranties are transferable... which might explain why most warranties are limited.

the probability the worth is of type w_x is ρ_x, $\Pr(w = w_x) = \rho_x > 0$. In this model it is as if "Nature" chooses a worth using those probabilities, and then the seller looks at the worth and decides whether or not to sell it.[5]

We will, simply because we have other things to talk about, assume that both buyers and sellers are risk neutral. We can state the seller's payoff as:

$$\pi(p, w) = \begin{cases} p - w & \text{If the good is sold} \\ 0 & \text{else} \end{cases}$$

where w is the worth of their car and p is the price it is sold at. The buyer's payoffs will be:

$$u(p, w) = \begin{cases} vw - p & \text{If the good is bought} \\ 0 & \text{else} \end{cases}$$

where (of course) w is not known until the good is purchased, and $v > 1$ is a constant expressing how much more the buyer wants the good than the seller.

Now we must specify a model for how price is determined. It is best if the price is as high as possible, thus we will use an **ultimatum module**. We say the seller will make a take it or leave it offer to the buyer, and the buyer can either accept or reject the offer. If the buyer rejects the offer, they have no other options for buying a good of this type. We will further assume that if the buyer is indifferent then they will accept the offer. Given these insights if a subset of goods X is offered in an equilibrium, the price will be $E[vw|X]$.

This module is not intended to be realistic, rather it is simply intended to make analysis of how the price is set simple. We have other things to think about and this is just convenient. (In his original article Akerlof assumed a continuum of types, so this problem didn't occur—see below.)

1.1 Preliminary Analysis—adverse selection in action.

In our very first step we see the bite of adverse selection. Say that the market price is p, which sellers will sell their good? Obviously those with $w \leq p$. But this means that the best goods are not sold! This means that for any price p, the price will be $E[vw|w \leq p]$. Now we get to a depressing fact, unless all the goods are the same $E[w|w \leq p] < p$! In other words, the average is less than the maximum.[6]

Conclusion 3 *If v is too close to one, this iteration will result in only the worst goods being sold, a phenomenons known as* ***market collapse.***

In our analysis, with only a finite number of types all of which have strictly positive worth, when this occurs only the worst quality will be sold. The market will limp along, but with a relatively small number of goods being sold. If either

[5] Please note that in this analysis we will always look for *pure strategy* equilibria. In other words all sellers with a given worth will either sell the good or not.

[6] p is our maximum for worth, $E[w|w \leq p]$ is teh average worth less than p.

it is possible to buy something with a negative worth, or there is a continuum of types, then when this problem occurs the market will literally stop functioning.[7]

Notice what a critical departure this is from full information. With full information as long as $v \geq 1$ trade will occur. Indeed in this simple model an economists concludes that for any $v > 1$ it needs to occur, that if it does not then there is social loss. We now conclude that there are many markets that might not open. This is known as the *missing markets problem.* Based on this analysis we know that there will be many markets which *should* open but do not because of asymmetric information.

How much of a problem is this? We know that asymmetric information makes many markets function imperfectly, but each missing market probably has little worth. I can assert this because I know that otherwise signalling mechanisms would develop to overcome this problem—we will discuss that below. However the total welfare impact of all the missing markets? I have no idea, which is why it keeps some economists up at night.

1.2 How to Find Equilibria in a simple three type model.

The problem that I casually talked about before, iteration of expectations, can be seen in our basic model for the price. For a given p, can we be sure that $E\left[vw|w \leq p\right] \geq p$? No, like I mentioned above (and will fully generate in a continuous model below) it might be that the expected value is strictly lower than the price. This means we have to revise our estimate of the price down, which means some high quality goods which were being sold now aren't be, which would lead us to lower our estimate again, and so on. There is an easier way to find equilibrium when the number of types are small. In reality I will always ask you three type questions.

This algorithm is for my general model of types, $w \in \{w_1, w_2, w_3, ...\}$ with $w_x > w_{x+1}$. Notice that for a given x, $w_{x-1} > w_x$.

Algorithm 4 *The three step algorithm is to:*

1. *For each type (w_x) find $E\left[vw|w \leq w_x\right]$.*

2. *Check that those who are supposed to sell will ($w_x \leq E\left[vw|w \leq w_x\right]$).*

3. *If $x > 1$, check that those who are supposed to not sell will not. ($w_{x-1} > E\left[vw|w \leq w_x\right]$).*

If the two tests are passed for a given x, you have a potential equilibrium. Now let me show the exact formulas and tests in the three type model.

[7] Trust me, with used cars it is definitely possible to buy a car with a negative worth. Could I tell such a car when buying it? Maybe, but I buy new cars.

1.2.1 The Expected Values

The three cases are either all types are sold, high worth goods are not sold, and only low worth goods are sold. (Remember, $w_L > 0$).

1. All types:

$$E[vw|w \leq w_H] = E[vw] = v(\rho_H w_H + \rho_M w_M + \rho_L w_L),$$

   notice here since all goods are sold the expectation is actually unconditional.

2. High worth not sold:

$$E[vw|w \leq w_M] = v\left(\frac{\rho_M}{\rho_M + \rho_L} w_M + \frac{\rho_L}{\rho_M + \rho_L} w_L\right) .$$

   Here we have to use Bayes rule to find the conditional probabilities:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)},$$

   but our application is very simple. In our analysis $B = \{\text{High worth goods not sold}\}$ so $\Pr(B) = \rho_M + \rho_L$, and $A = \{\text{Medium worth good}\}$ is a subset of B, so $\Pr(A \cap B) = \Pr(A) = \rho_M$.

3. Only low worth goods sold:

$$E[vw|w \leq w_L] = v\left(\frac{\rho_L}{\rho_L} w_L\right) = vw_L .$$

1.2.2 Checking whether these are equilibria:

In only one case are both tests strictly necessary. In the best one, there are no other types so you don't have to check they won't sell. In the worst one since $v > 1$ we know the low worth goods will be sold—the question is whether the higher worth goods will be sold.

1. All types sold is an equilibrium if: $E[vw|w \leq w_H] \geq w_H$
2. High worth not sold is an equilibrium if: $w_H \geq E[vw|w \leq w_M] \geq w_M$
3. Only low worth sold is an equilibrium if: $w_M \geq E[vw|w \leq w_L]$.

1.3 The Expectations Trap and Some Examples.

So... can you give me any reason that you think that only one of the three cases above will be an equilibrium? There isn't one, so don't bother. I could let you choose the values of ρ_L, ρ_M, and ρ_H and I could still deliver a model where any one, any two, or all three could be equilibria. This brings us to another problem in markets with imperfect information, and this one is frankly terrifying.

Definition 5 ***Expectations Trap**: In a model of asymmetric information society can be stuck in a bad equilibrium when there are better equilibria out there.*

Because the price is so low, sellers of high worth goods do not enter the market.

Because sellers of high worth goods do not enter the market, buyers can not offer a higher price.

This is a completely self enforcing phenomenon. Let me give some examples just to illustrate.

1.3.1 An example where all three possibilities are equilibria.

Let $v = 3$, $w_L = 2$, $w_M = 6$, and $w_H = 16$, assume $\rho_L = \rho_M = \rho_H = \frac{1}{3}$.

$$\begin{aligned}
E[vw|w \leq w_H] &= E[vw] = 3\left(\frac{1}{3}(16) + \frac{1}{3}(8) + \frac{1}{3}(2)\right) = 26 > w_H = 16 \\
E[vw|w \leq w_M] &= 3\left(\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}}8 + \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}}2\right) = 15 \\
w_H &= 16 > 15 > w_M = 8 \\
E[vw|w \leq w_L] &= 3 * 2 = 6 < w_M = 8 \ .
\end{aligned}$$

So all three cases satisfy the conditions for equilibrium. Now imagine the world in which the market price is six. No sensible seller would sell a medium or high worth good, and thus the consumers can not offer more, but then no seller would sell a medium or high worth good. Both behaviors make perfect sense, its just a tragedy that if both sides could simultaneously be forced to change their strategy then the world would be a better place, where the market price was 26 and all worth goods would be sold.

1.3.2 An example where Medium worth goods will always be sold.

Let $v = 2$, $w_L = 4$, $w_M = 6$, and $w_H = 12$, assume $\rho_L = \rho_M = \frac{1}{4}$, $\rho_H = \frac{1}{2}$.

$$\begin{aligned}
E[vw|w \leq w_H] &= E[vw] = 2\left(\frac{1}{2}(12) + \frac{1}{4}(6) + \frac{1}{4}(2)\right) = 16 > w_H = 12 \\
E[vw|w \leq w_M] &= 2\left(\frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{4}}6 + \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{4}}4\right) = 10 \\
w_H &= 12 > 10 > w_M = 6 \\
E[vw|w \leq w_L] &= 2 * 4 = 8 > w_M = 6 \ .
\end{aligned}$$

In this example medium types will always be sold. Imagine now that consumers think that every good they buy is of low worth. Medium worth sellers will think, hey, I will be better off if I sell my good, so they will start selling their goods. Sooner or later the customers have to recognize this, and all sellers will be able to start demanding 10. However at this point we are at an equilibrium, the high worth sellers might *wish* they could make enough money to enter the market, but unless there is some magical intervention they will not be selling.

Looking at this model, you probably think that any enlightened government could convince people that they should be more trusting. I am quite confident that if the government told you that used cars were worth, say, another 10,000 TL you would gladly just offer more for used cars.[8]

But, you have to recognize that the government doesn't really know the distribution of types, whether there is another equilibrium where people are more trusting, or not. There are probably programs that might work *if* there is a better equilibrium out there. Should the government spend billions to try and get people to be more optimistic even though it might fail?[9]

1.4 A Continuous Model—the original Akerlof Analysis

A model with continuos supply and demand curves is difficult to derive, but once it is derived it will help your thought process to see the way it behaves. We will assume that we have a continuum of buyers and sellers, which both have uniform distributions. Critically, unlike before we assume that the values of customers follows a distribution as well. This allows us to derive a continuous demand curve, with those who value the good the most being the first to buy.

Now some of you who aren't comfortable with math might get a little intimidated by this assumption, but it really isn't that difficult. All we are doing is just making our demand and supply curves smooth, if I didn't do this then the curves would be step functions, jumping down when one person is no longer willing to buy in the market. We can also normalize quantity to be between zero and 100 without any loss. If this is true and the values of consumers are distributed uniformly over $[\underline{v}, \bar{v}]$ our inverse demand curve is:

$$P = \left(\bar{v} - \frac{\bar{v} - \underline{v}}{100} Q\right) E[w]$$

where $E[w]$ is the expected quality level of the car. The quality levels of the car will be distributed uniformly on $[0, \bar{w}]$ so our inverse supply curve will be:

$$P = \frac{\bar{w}}{100} Q$$

[8] That is what we call sarcasm.

[9] I just want to mention how practically impossible this is. The sensible approach would be to collect data and use sophisticated econometric techniques to figure out if there is another equilibrium out there. However this new equilibrium will be far from the current one, which gives rise to a very serious problem.

The iron rule is don't sell your car if it's worth is above the market price—so by definition you won't have any data on the high worth cars. I mean, an occasional one might be sold due to desperate situations (moving far away, etceteras) but that won't tell you the true distribution without some heroic assumptions.

If the government is going to base their policy on these assumptions... and they're basically untestable...

Well, don't vote for them.

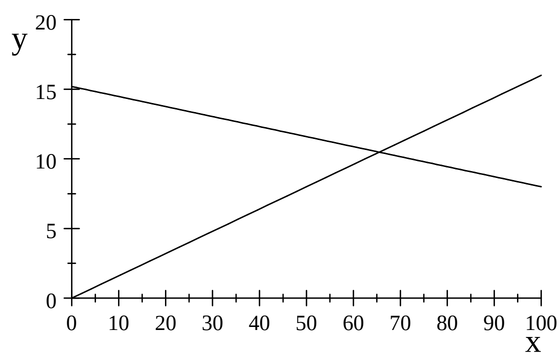
and the marginal car will have a quality level of P. Now, one final technical detail, what is $E[w|w \leq P]$? I.e. given that every car on the market has a quality level below P what is the expected quality of a car?

$$E[w|w \leq P] = \frac{1}{P}\int_0^P wdw$$

$\frac{1}{P}$ is an adjustment of the probabilities so that the probabilities sum to 1. Now let's solve this:

$$\frac{1}{P}\int_0^P wdw = \frac{1}{P}\left[\frac{w^2}{2}\right]_0^P = \frac{P^2}{2P} = \frac{P}{2}$$

Before I find the equilibrium let's graph the demand and supply curves. To do this we need to make some assumption about the quality levels of cars on the market, as a first pass let's assume that all cars are offered for sale. We also need some parameters, so let's assume that $\bar{w} = 16$, $\bar{v} = 1.9$ and $\underline{v} = 1$. In this case we can see that the supply curve is $P = \frac{16}{100}Q$ and the demand curve is $P = (1.9 - .00\,9Q)\,E[w]$, or our first demand curve is :$P = (1.9 - .00\,9Q)\,8$.



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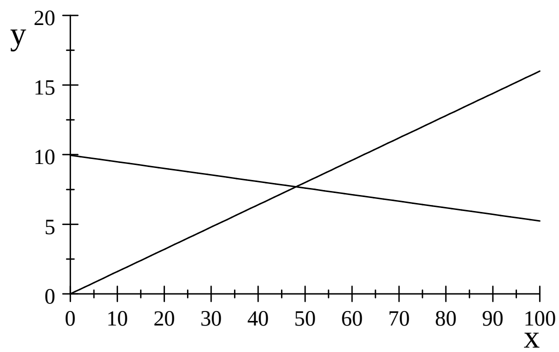
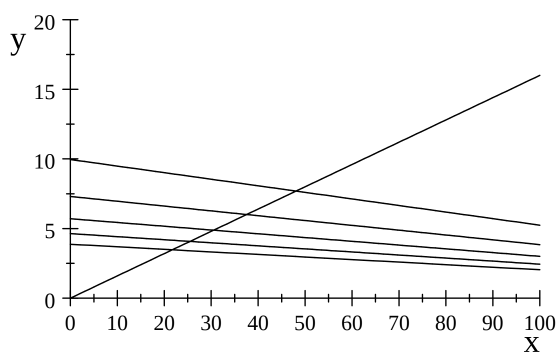
Now notice that our original guess (that all cars would be sold) is wrong. So what is the value of the marginal car now? Well we can see that

$$\begin{aligned}\frac{16}{100}Q &= (1.9 - .00\,9Q)\,8 \\ Q &= 65.517 \\ P &= \frac{16}{100}Q = \frac{16}{100}(65.517) = 10.483\end{aligned}$$

So this gives us a new demand curve, $P = (1.9 - .009Q)\,\frac{10.483}{2}$



which is lower than before. Uh ohh. You see what is happening... Let's go through several further steps in one graph. Each lower demand curve is the result of figuring out the actual quality of cars that will be sold using the previous demand curve.



and you can see that with each iteration of the process the demand curve shifts further and further down. What will be the final result? Well to do that we have to find out what the equilibrium will be. This is actually easier than doing what we did above.

The trick to finding the equilibrium is realizing that the marginal consumer has to be willing to buy the average car, and the marginal supplier has to be willing to sell it. So this means

$$\begin{aligned}(1.9 - .009Q)\,\frac{P}{2} &= P \\ 1.9 - 2 &= .009Q \\ Q &= -11.111\end{aligned}$$

and unfortunately quantity can not be negative. What does this mean? No one will buy a car, and the market will collapse. In this model in general what we need is:

$$\begin{aligned}
P &= \left(\bar{v} - \frac{\bar{v} - \underline{v}}{100} Q\right) \frac{P}{2} \\
Q &= 100 \frac{\bar{v} - 2}{\bar{v} - \underline{v}} \\
P &= \frac{\bar{v}}{100} Q = \frac{\bar{v}}{100}\left(100 \frac{\bar{v} - 2}{\bar{v} - \underline{v}}\right) = \bar{v} \frac{\bar{v} - 2}{\bar{v} - \underline{v}}
\end{aligned}$$

Notice that one thing we can be sure of is that $\bar{v} > 2$ is necessary. We can see this by looking at one person's incentives:

$$v_i E\left[w | w \leq P\right] \geq P$$

and this means if we have a uniform distribution that:

$$\begin{aligned}
v_i \frac{P}{2} &\geq P \\
v_i &\geq 2\ .
\end{aligned}$$

Hah, so now you know why I made the consumer with the highest utility only value the good at $1.9 * E\left[w\right]$. The disturbing fact is that *in this market trade is only possible if the consumer values the car twice as much as the seller.*

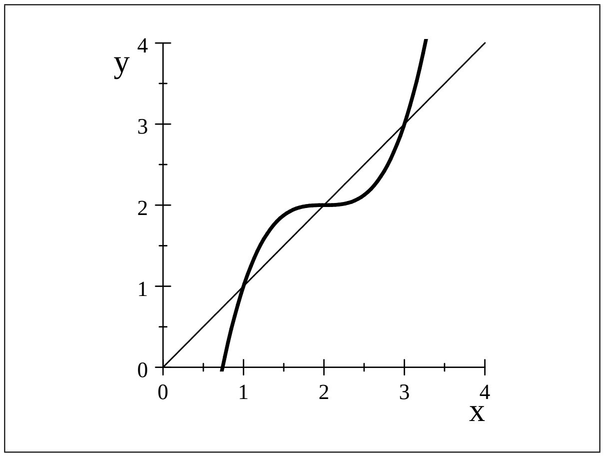
Does that bother you? It bothers me. Of course with other distributions of quality is different then this result wouldn't be true. What is always true is that there is some $v_i^* > 1$ that is necessary for trade. Why do we care? Because whenever $v_i > 1$ it is Pareto improving to trade, so there will always be some people with values $v_i^* > v_i > 1$ who should be trading and won't be.

1.4.1 Graduate analysis of the Continuum model—Mas-Colell, Whinston, and Green

Just to take this to an unnecessarily high level, higher than you will need for this class, I want to repeat the analysis in a graduate textbook. The key thing is that when worth has a continuum of types we no longer need to worry about how the price is determined. Since for $w > w^*$ we must have $w > p$ and for $w < w^*$ we must have $w < p$ we know that $w^* = p = vE\left[w | w \leq p\right] = vE\left[w | w \leq w^*\right]$ (for simplicity all customers are identical again.)

Thus we have two functions, w^* and $vE\left[w | w \leq w^*\right]$. We know that $vE\left[w | w \leq w^*\right]$ is an increasing function (for each w^* it gives us a precise value). Since it is a function we can use Brouwer's Fixed Point theorem to know that either there is a w^* such that $vE\left[w | w \leq w^*\right] = w^*$ or if $\bar{w}$ is the maximum worth good, $vE\left[w | w \leq \bar{w}\right] > \bar{w}$ and so there is an equilibrium where all goods are sold.

However, like above there could be many possible crossings of $vE\left[w | w \leq w^*\right]$ and w^*. For example $vE\left[w | w \leq w^*\right]$ could look like the dark line in the following graph:



Here there are three crossings of the dark line ($vE[w|w \leq w^*]$) and the thin line (w^*), thus three equilibria. In this case only one of these equilibria is *stable*, for small perturbations in expectation we will always converge to the center crossing. In a general model there could be many equilibria, and many stable equilibria.

2 Signalling

So what should you do in this situation? Well one option that some people follow is they try to find a trustworthy mechanic to certify that their car has no mechanical problems. This is a solution, but it is economically wasteful. After all they would not seek the certification if they wouldn't pass the inspection, so the mere fact they want to be certified indicates they do not need it. This is a *signal*, an action which is economically wasteful but overcomes some asymmetric information problem.

Let me introduce you to another model that captures the impact of signalling in it's most dramatic light. How much do you think your education will help you in your future job? For example, are you sure you're going to use the foreign language that you have (or will) learn? Are you sure that you will even use English? So what is the point of you learning all of these languages when you aren't going to use them? And what about your history classes? In the US this problem is even worse because people do not choose their major before they go to college. I'm thankful for that—otherwise I would be a biologist right now—but it means that you take a lot of general classes before you decide on your major. How much do you think I use my class on the sociological impact of journalism? Sigh. So why do we take these classes? Why are you in college? Well, that's a good question.

Spence realized that part of the answer is signalling. It's not that what you learn at Bilkent will be that useful, but the fact that you got into Bilkent and graduated with a good GPA that is useful information. I.e. it's not what you learn but the fact that you CAN learn. That is a skill that many employers value, and with good reason. That is probably the skill that you will retain from all of this education until you die. What we teach you is not as important as the fact that you learn whatever we teach.

Signalling is important, because it can reduce the problem of adverse selection, but it also can create its own problems because people can waste resources signalling when there really isn't any social benefit. First, a proper definition.

Definition 6 *A* signal *is an action where the direct (marginal) benefit is below the direct (marginal) cost, but this action is taken to either reveal or hide information about the person signalling.*

To give a simple example, traditionally a man is supposed to open doors for women. If a girl's boyfriend does this he is signalling to the girl that he values her.[10] The man is not doing this because he enjoys opening doors, rather he is signalling that he values women—and this woman in particular. When you think about it, many actions we take have a signalling quality. Indeed after thinking about it for long enough it's hard to find actions which have no signalling component.

We will look at two models of signalling, the first is simple signalling—certification, and then we will look at more complicated one but one which is very worthwhile. A model where education is useless, but workers still get an education to signal.

2.1 Certification

Let's be clear about one thing, certification will only work *if* there is a trustworthy company providing these services. This company has to value it's reputation more than any bribes that might be offered to get a good rating. If you watch Pawn Stars or similar shows for long enough you will see plenty of examples of "certificates" that are barely worth the paper they are written on. Both the buyer and the seller needs to know and trust the company. This is why certification does not work in the market for used cars—you can always find an untrustworthy mechanic, and frankly a car is too complicated to evaluate in a few hours. Around the turn of the century a used car dealer in the United States was attempting to make a name for themselves by certifying their used cars, but they don't seem to have succeeded. It's hard to establish this sort of reputation, and often just not worth it.

Generally you see certification in collectibles markets—comic books, trading cards, stamps, currency, etcetera. These are goods that can have an infinite shelf life if handled carefully and each item is—usually—not worth that much. A standardized rating has a high value in such a market, and the size of bribe people might offer is hardly worth it. At the same time it's relatively easy for an expert to rate such a good. Comic Etc uses a Overstreet standard ONE (Overstreet Numerical Equivalency) grading system, which has 14 grades. The

[10] Of course, some girls might inerpret this as a signal he thinks she is weak, such is life.

value of the comic obviously depends both on rarity and the certified rating. Of course we're looking for a simpler model.

2.1.1 Model

We will be following the model in 1.0.1 except now sellers can have their good certified for a cost of $\chi > 0$. This certification reveals the true worth of the good, w_x.

2.1.2 The decision to certify

Recognize that since the seller gets to make a take it or leave it offer to the buyer, they will demand vw_x. Thus summarizing their decision is quite simple.

If they are not selling their good In this case we know that the market price—p, is too low, so $p \leq w_x$. Thus the only question is whether the price they can get with certification is worth it or not, in other words is $vw_x - \chi - w_x \geq 0$, or is $vw_x - \chi \geq w_x$. Notice that this type of certification is for the good of the economy, these goods should be sold and now will be.

If they are currently selling their good In this case we know that $p - w_x \geq 0$, so the relevant decision they need to make is whether $vw_x - \chi - w_x \geq p - w_x$ or not. In short, if $vw_x - \chi \geq p$ they should certify, if it is not they should not.

Notice first of all that this is economically wasteful signalling, they were already selling their good and now they're wasting money to get a higher price. From a social standpoint it's hard to describe this as anything but wasteful.

Also notice that this will make things worse those people who can't afford to get certified. Basically the condition for certification—$vw_x - \chi \geq p$ is increasing in w_x, so the first to drop out will be the highest worth sellers. This will suppress the price, which might then encourage the next highest group... and of course at each iteration the profits of those who do not certify is decreasing. This is simply bad for the market as a whole.

2.2 Spence Job Market Signalling—Education as a Signal

So, Spence hypothesizes that education may serve no purpose other than to signal that you are bright and hard working. If he is correct, wouldn't it follow that people could get education in a model where education literally has no value? I.e. nothing you have learned helps you out at all, but you still learn it because obviously it shows that you are smart and hard working?

Because it is the simplest model, this is the best one for you to learn. Notice I am not saying that education is worthless. I personally feel that the lessons you can learn this semester could help you for the rest of your life. Not the precise models, but the general insights.[11] So let's first precisely lay out the model.

2.2.1 Model

There are two types of workers, H or high quality and L or low quality. The probability a given worker is high quality is λ ($\Pr(H) = \lambda$). High quality workers have a higher productivity in the workplace (denote the productivity of type $x \in \{H, L\}$ by π_x, then we are assuming $\pi_H > \pi_L > 0$) and a lower marginal cost of getting education (this marginal cost is c_x, and $c_L > c_H > 0$). Workers choose an education level $e \geq 0$, and the utilities of the workers is:

$$u(w, e, x) = w - c_x e$$

Let $\beta(e) = \Pr(H|e)$—or the probability that a firm believes the worker is of the high type given the amount of education they received. Then the firm's profits are:

$$\pi(w, e) = \beta(e)\pi_H + (1 - \beta(e))\pi_L - w(e).$$

We assume that their is one worker, and that there are multiple firms competing over price for this worker's services. To be clear, we are using a **Bertrand module** to remove any concerns about how the wage will be set. If firms compete over price and the worker sees firms as identical, a standard result will show that $\pi(w, e) = 0$ in equilibrium. The only critical assumption here is that worker's have some market power.

Throughout we will be looking for *pure strategy* equilibria, which here means that education is a function of type. In short, all workers of a given type will choose one level of education for sure.

2.2.2 Preliminary analysis

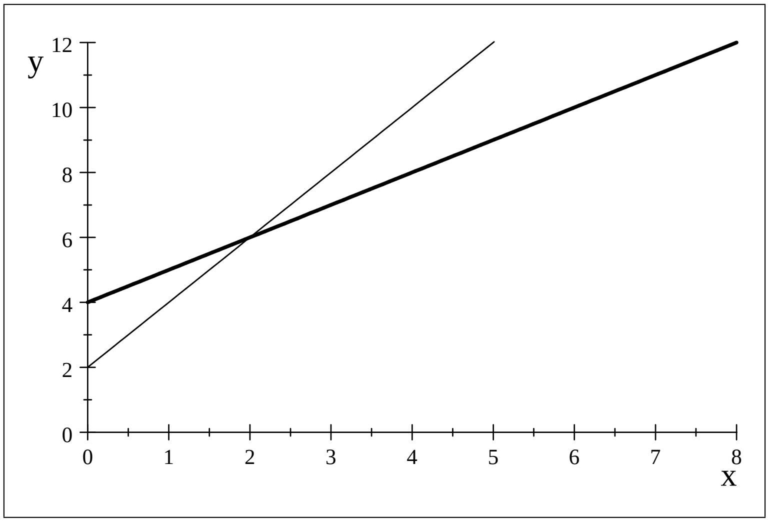
Workers will choose at most two wage/education pairs. The critical aspect of the model is that the more desirable workers have the lower cost of signalling, in this case, getting an education. Clearly in this case the assumption is justified. For both tasks you need intelligence, hard work, and an ability to learn new things. Our assumption that the marginal cost of education makes the model easy to solve because indifference curves will be linear. To be precise the formula for an indifference curve is:

$$\begin{aligned} u(w, e, x) &= \bar{u} = w - c_x e \\ w &= c_x e + \bar{u} \\ \frac{\partial w}{\partial e} &= c_x \ . \end{aligned}$$

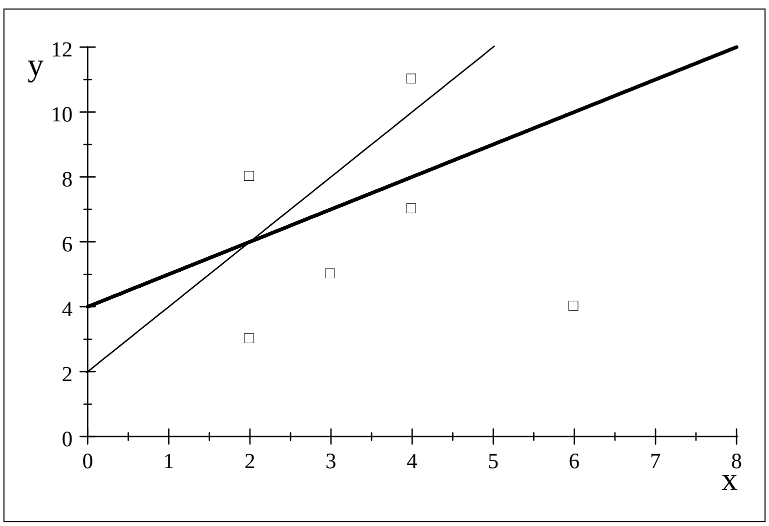
[11] Of course from now on you are free to say. "Dr. Hasker says that education is useless." Except, of course, you've locked yourself in a paradox. Because by attributing this to me you are proving some of your education was worthwhile.

Notice that in this model a high wage is good, and a high education level is bad[12]. Also, the high type has a flatter indifference curve.

Just to make things simple, let's graph a couple of indifference curves. For this graph $\bar{u}_H = 4$, $c_H = 1$ (the dark line) $\bar{u}_L = 2$, $c_L = 2$ (the light line):



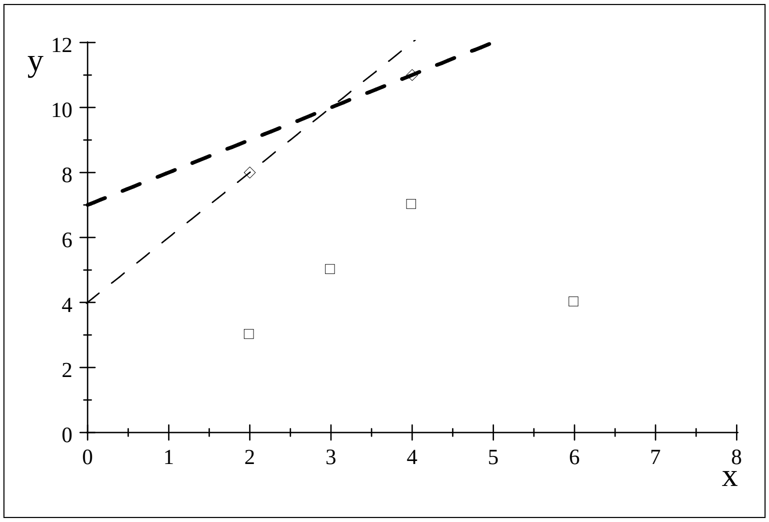
Loosely speaking we can think of any equilibrium as a bunch of pairs of wages and educations :$\{w_i, e_i\}_{i=1}^n$, or we can just put a bunch of points onto our graph.



Now let's imagine these two types of workers choosing among these bundles. It should be fairly clear that the optimum for the low types is $(w, e) = (8, 2)$ and

[12] See, I told you it was unrealistic.

for the high type is $(w, e) = (11, 4)$.



But this brings us to a completely obvious point. *Workers will choose at most one wage/education level per type, and we might need a third wage/education pair to explain their choices.*

Firms will set wages to expected productivities, and education will either be informative or not. The point of the Bertrand module of wage competition is simplicity. We will cover this model when we analyze oligopoly, but here and elsewhere we will assume this simply because it makes things simple. In equilibrium you can show that the expected profits must be zero:

$$\pi(w, e) = 0 = \beta(e)\,\pi_H + (1 - \beta(e))\,\pi_L - w(e)$$

or

$$w(e) = \beta(e)\,\pi_H + (1 - \beta(e))\,\pi_L$$

now what possible values can $\beta(e)$ attain in a pure strategy equilibrium? It should be clear that either we could have the education level chosen by the high types—e_H—be the same as the education level chosen by the low types (e_L) or not. If $e_H \neq e_L$ then $\beta(e_H) = 1$, $w(e_H) = \pi_H$, and $\beta(e_L) = 1$, $w(e_L) = \pi_L$. If $e_H = e_L = \bar{e}$ then education is uninformative and obviously $\beta(\bar{e}) = \lambda$—or the firm learned nothing from the worker's education and $w(\bar{e}) = \lambda\pi_H + (1 - \lambda)\,\pi_L$. Thus we can completely describe the firm's by $\beta(e)$ and we can see that at most we will need beliefs like:

$$\beta(e) = \begin{cases} 1 & \text{if } e_1 \leq e \\ \lambda & \text{if } e_2 \leq e < e_1 \\ 0 & \text{if } e < e_2 \end{cases}.$$

I should mention we will assume all firms have the same beliefs, and that they are all of this form. There are many other models for beliefs that would do the job, here we are just focusing on simple beliefs—these different models will not add or subtract any equilibrium outcomes we will find.

Describing worker behavior So, we have now learned that an equilibrium can be described as a (w_H, e_H), a (w_L, e_L) and an outside option of dropping out and taking whatever they will give you: $(\underline{w}, 0)$. Like usual, it is best to describe the optimal behavior of the informed agents first—in this case the worker.

First, we want the high types to choose (w_H, e_H) over (w_L, e_L) or:

$$IC_H : w_H - c_H e_H \geq w_L - c_H e_L$$

and we want the low types to choose (w_L, e_L) over (w_H, e_H):

$$IC_L : w_L - c_L e_L \geq w_H - c_L e_H \ .$$

Notice that we know the type of a worker merely by the marginal cost of education, c_H in the first equation and c_L in the second. These are testing the incentive compatibility of the contracts offered, and thus are called *incentive compatibility constraints*, often denoted IC_H for the first and IC_L for the second.

We also want both parties to participate, or prefer getting the specified level of education to dropping out. These are referred to as the *individual rationality* constraints and are:

$$\begin{aligned} IR_H &: \quad w_H - c_H e_H \geq \underline{w} \\ IR_L &: \quad w_L - c_L e_L \geq \underline{w} \ . \end{aligned}$$

One of these constraints is not binding, can you tell which one it is? It's fairly simple to show, and the intermediate step actually has an enlightening implication.

Lemma 7 *In any equilibrium,* $u(w_H, e_H, H) \geq u(w_L, e_L, L)$ *and thus* IR_H *is never a binding constraint.*

Proof. *We know that* $w_H - c_H e_H \geq w_L - c_H e_L$ *since* $c_H < c_L$ *we know that* $w_L - c_H e_L \geq w_L - c_L e_L$. *Combining these two constraints gives us* $u(w_H, e_H, H) \geq u(w_L, e_L, L)$, *and also verifies that as long as* IR_L *is satisfied so is* IR_H. ■

2.2.3 The Equilibria

Since we are looking for pure strategy equilibria it should be clear that they can be of two classes:

1. Everyone can do the same thing (*pooling* equilibria)

2. Each type does something different (*separating equilibria.*)

Yes, I know, it seems silly to point it out but these two classes are very different in a lot of ways so it is well worth our time to discuss each separately.

Pooling Equilibria These equilibria are much easier to describe because $e_H = e_L = \bar{e}$ so we don't need to consider incentive compatibility constraints. Instead all we need to look at is the individual rationality constraint for the low types. To be precise $w(\bar{e}) = \lambda\pi_H + (1-\lambda)\pi_L$, $\underline{w} = \pi_L$, and so the constraint we need to satisfy is:

$$\begin{aligned}
\lambda\pi_H + (1-\lambda)\pi_L - c_L\bar{e} &\geq \pi_L \\
\lambda\pi_H + (1-\lambda)\pi_L - \pi_L &\geq c_L\bar{e} \\
\lambda(\pi_H - \pi_L) &\geq c_L\bar{e} \\
\lambda\frac{(\pi_H - \pi_L)}{c_L} &\geq \bar{e}
\end{aligned}$$

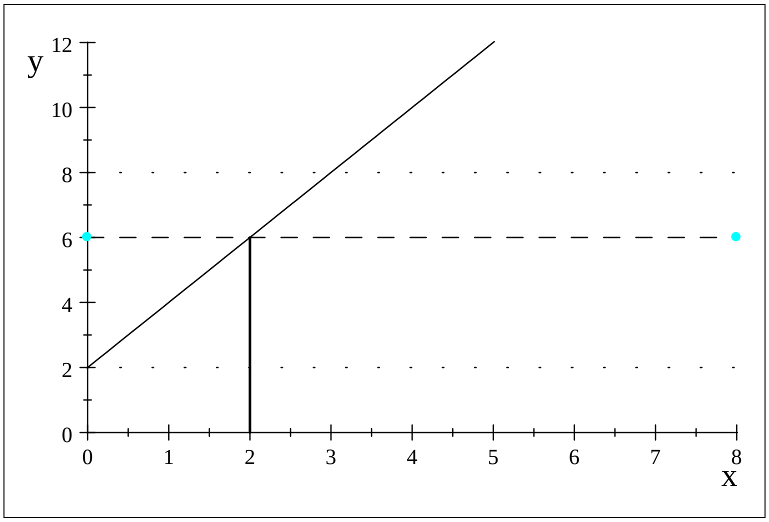
and we notice there are a whole bunch of these equilibria. The Pareto efficient one—$\bar{e} = 0$—is possible but there are also ones where everyone gets an education even though it has no value. Why is this? Because by going to school they are informing their future employers of something—that they are not losers. Yes, yes, some of them are lying, but so what? They are actively trying to hide their low productivity from the employers. They are signalling to hide an underlying characteristic.

You probably are thinking that this is ridiculous, you would never do that. Well, when I was growing up in Indiana you only had to go to high school until you were 16, but you couldn't graduate from high school until you were 18. I grew up in a rural county in Indiana and I promise you that many of my classmates never used anything they learned in those last two years. However even though they could be working (you can work at 16) they stayed. Why? Because they didn't want to be one of those losers who couldn't even finish high school. They were pooling with the bright kids who had a good reason to finish those last two years. It might seem weird, but this happens all the time. Notice that one big thing this model explains is the *degree effect.* Why is it that dropping out in the winter of your senior year means that you can't get any good jobs? You have taken all but one year of classes, so why is it such a big deal? Because you are signalling to employers that you are a loser, that's why. It's not the amount you learn in that last semester but rather the signal that you could finish your degree that makes them value drop outs so low.

We can easily graph the space of equilibria by specifying a value for π_H, π_L, and λ. Let $\pi_H = 8$, $\pi_L = 2$, $\lambda = \frac{2}{3}$ then using $c_L = 2$ like before the binding constraint is that:

$$w - 2e \geq 2$$

thus we are interested in the indifference curve where $u(w, e, L) = 2$, and the relevant graph is:



The critical point is where the low types are just ready to drop out of school, or where the indifference curve $u(w,e,L) = 2$ crosses the dashed line. The faint dotted lines are the productivity of the high and low types.

Separating Equilibria In a separating equilibria $w(e_L) = \pi_L$, and like before $\underline{w} = \pi_L$, this suggest we should first look at IR_L, or the low types individual rationality constraints:

$$IR_L : \pi_L - c_L e_L \geq \pi_L \Rightarrow 0 \geq c_L e_L$$

and we arrive at the conclusion that now $e_L = 0$. We still have both of the incentive compatibility constraints to analyze, including the fact that $(w_L, e_L) = (\pi_L, 0)$ these are:

$$\begin{aligned} IC_H &: \pi_H - c_H e_H \geq \pi_L \\ IC_L &: \pi_L \geq \pi_H - c_L e_H \ . \end{aligned}$$

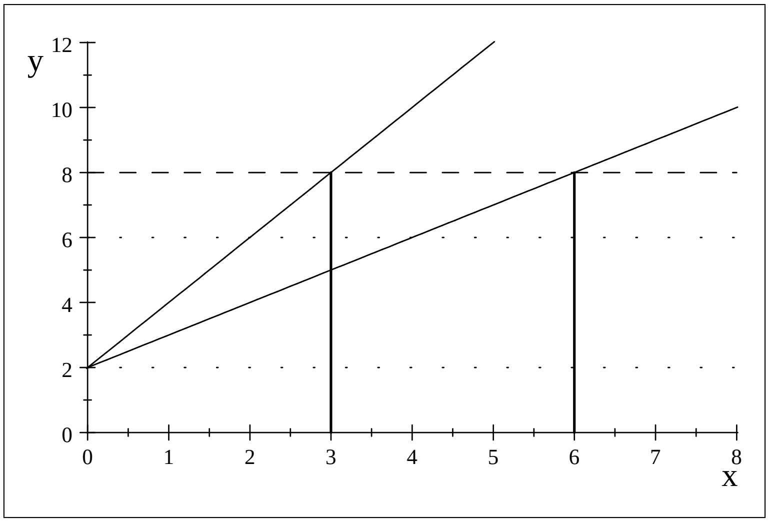
The former will give us an upper bound on the level of education and the latter a lower bound. To be specific solving IC_H:

$$\begin{aligned} \pi_H - \pi_L &\geq c_H e_H \\ \frac{\pi_H - \pi_L}{c_H} &\geq e_H \end{aligned}$$

and solving IC_L we get:

$$\begin{aligned} c_L e_H &\geq \pi_H - \pi_L \\ e_H &\geq \frac{\pi_H - \pi_L}{c_L} \end{aligned}$$

so we conclude that $\frac{\pi_H - \pi_L}{c_H} \geq e_H \geq \frac{\pi_H - \pi_L}{c_L}$. The first inequality makes sure the high types will go to school, the second that low types will not. Again, we can see the set of possible equilibria graphically, now we need the indifference curve where $u(w, e, H) = 2$ as well as $u(w, e, L) = 2$.



and the equilibrium separating level of education can be anywhere between the two lines.

Now again, you might think, how realistic is this? I mean, using university as our example it's always four years, and that's just the right amount of time. However in Great Britain university is three years. Now British kids go to high school for an extra year, but you could go to school in England and get your degree in three. Would that put you at a disadvantage in the workplace? No. The length of college is just a convention, we have settled on four years but in this model it could be anything from three to six. If it was three, then the low productivity types would be just indifferent between going to school and dropping out, if it was six then the high productivity would be just as badly off as the low productivity.

2.2.4 Generalization, productive education, and other closing comments.

How does this model change if their are more types? Now we can have a mixture of pooling and separating, and we get closer to a description of reality. In reality there's obviously pooling both in high school and college, and there are some extreme types who can't even graduate high school. This really does nothing more than add to the realism of the model, but it makes the analysis more complex and there's nothing to learn there. Lemma 7 always holds, which makes analysis much simpler.

What about productive education? In separating equilibria the low types always get the optimal amount of education and the high types get too much in order to separate. In pooling equilibria it always at least the optimal amount for the low types.

In general while this model seems complicated, the basic insight is something you use every day. You want to learn something about attribute x, which is not directly observable (honesty, kindness, intelligence, etceteras). Thus you look at some behavior that is lower cost if this person is strong in this attribute (like education for intelligence) and you decide what the value for x is based on this behavior.