

Handout on: Free Entry in an Oligopolistic Industry

Dr. Kevin Hasker

April 26, 2016

1 Introduction

This handout will deal with the messy topic of free entry and the difference between maximizing social welfare and allowing firms to enter freely.

First of all, some assumptions to allow us to handle general models. The main variable of interest will be n , the number of firms, and we want to simplify everything until we can focus on this variable alone. Thus:

Assumption All firms have the same cost and revenue structure, there is fixed cost of entry into the market of $F > 0$.

Assumption There will be a unique equilibrium, which will be symmetric.

Assumption The total quantity supplied, $Q(n)$ will be weakly increasing in n ($\frac{dQ}{dn} \geq 0$).

This assumption (and downward sloping demand) means that the price each firm receives will be decreasing in n , and thus the *operating profits* ($\pi(n)$) will be decreasing in n , or $\frac{\partial \pi}{\partial n} \leq 0$. We can also write consumer surplus as $CS(Q(n)) = CS(n)$ and know that $\frac{dCS}{dn} \geq 0$.

2 Free Entry versus Welfare Maximizing, the General Case

Entry will occur as long as the operating profits ($\pi(n)$) are higher than the fixed cost (F) thus the free entry equilibrium is a n^{fe} such that:

$$\pi(n^{fe}) \geq F > \pi(n^{fe} + 1) \quad (1)$$

We will usually ignore the integer problem (n has to be a natural number) and just characterize it as:

$$\pi(n^{fe}) = F \quad (2)$$

Now let us look at the welfare maximization problem, it will be to maximize:

$$W(n) = CS(n) + n(\pi(n) - F)$$

and the derivative of this function is:

$$\frac{dW}{dn} = CS'(n) + n\pi'(n) + (\pi(n) - F)$$

to compare this to free entry let's consider it at the free entry equilibrium. At this point $\pi(n^{fe}) - F = 0$ and so it simplifies to:¹

$$\frac{dW}{dn^{fe}} = CS'(n^{fe}) + n\pi'(n^{fe}) . \quad (3)$$

If it so happens that $CS'(n^{fe}) = -n\pi'(n^{fe})$ then free entry will be social welfare maximizing, but as you can imagine this would be a coincidence. There are two effects that are competing with each other:

1. Business Stealing: Profits of existing firms fall with entry, $n\pi'(n^{fe}) < 0$.
2. Appropriation Effect: The social planner cares about Consumer Surplus, which is increasing in n , $CS'(n^{fe}) > 0$.

If these two effects balance out then free entry is welfare maximizing, but this is not generally true.²

2.1 Classic Cournot—Constant Marginal Costs and Linear Demand

As a paradigmatic example consider the symmetric Cournot equilibrium. Inverse demand is $P = a - bQ$ and costs are $c_i(q) = cq$.

2.1.1 Equilibrium and Firm's Profits.

With n firms, firm one's profit maximization problem is:

$$\max_{q_1} \pi_1(q_1, Q_{-1}) = (a - b(q_1 + Q_{-1}))q_1 - cq_1 , \quad (4)$$

where $Q_{-1} = q_2 + q_3 + q_4 + \dots + q_n = \sum_{j=2}^n q_j$, or the output of all other firms. Given this convention there isn't much difference between solving this problem and the two firm problem, at least at first:

$$\frac{\partial \pi_1}{\partial q_1} = (a - b(q_1 + Q_{-1})) - bq_1 - c = 0 . \quad (5)$$

However now we have a problem, we have n variables to solve for. How do we get around this? Well we use the fact that we are looking for a symmetric equilibrium where $q_i = q$ for all firms. This means that $q_1 = q$ and $Q_{-1} =$

¹We ignore the integer problem here and throughout, or assume equation 2 is true for a natural number.

²It is in perfect competition only because average cost and marginal cost are equal to the price. In this model of oligopoly this will not necessarily be the case.

$(n - 1)q$, and the first order condition is easy to solve.³

$$(a - b(q + (n - 1)q)) - bq - c = 0 \quad (6)$$

$$a - c = b(q + (n - 1)q) + bq \quad (7)$$

$$a - c = (b(1 + (n - 1)) + b)q \quad (8)$$

$$a - c = ((1 + (n - 1)) + 1)bq \quad (9)$$

$$a - c = (n + 1)bq \quad (10)$$

$$q(n) = \frac{1}{n + 1} \frac{a - c}{b} \quad (11)$$

$$Q(n) = nq(n) = \frac{n}{n + 1} \frac{a - c}{b} \quad (12)$$

$$P(n) = a - bQ(n) \quad (13)$$

$$= a - b \left(\frac{n}{n + 1} \frac{a - c}{b} \right) \quad (14)$$

$$= a - a \frac{n}{n + 1} + c \frac{n}{n + 1} \quad (15)$$

$$= \frac{1}{n + 1} a + \frac{n}{n + 1} c \quad (16)$$

Now notice that because we have constant marginal cost:

$$\pi(q, P) = (P - c)q \quad (17)$$

And we can easily see that

$$\begin{aligned} P(n) - c &= \frac{1}{n + 1} a + \frac{n}{n + 1} c - c \\ &= \frac{1}{n + 1} (a - c) \end{aligned} \quad (18)$$

thus

$$\pi(n) = \frac{1}{n + 1} (a - c) \frac{1}{n + 1} \frac{a - c}{b} = \frac{1}{(n + 1)^2} \frac{(a - c)^2}{b} \quad (19)$$

We're going to be working with this a lot, so it's useful to note that the efficient quantity is:

$$\begin{aligned} Q_e &= \frac{a - c}{b} \\ bQ_e &= (a - c) \end{aligned} \quad (20)$$

so:

$$\pi(n) = \frac{1}{(n + 1)^2} \frac{(bQ_e)^2}{b} = \frac{1}{(n + 1)^2} bQ_e^2. \quad (21)$$

³Please notice that we can only start analyzing equilibrium **after** taking the first derivative. If we do it before it's the same as saying "if firm one increases his output all other firms will react by increasing their output one to one." How are they going to do that? Everyone chooses their output at the same time; it's a simultaneous game. If you do you'll find that they'll always produce the joint profit maximizing output—or the industry profits will always be the monopoly profits.

2.1.2 Consumer Surplus

With linear demand we can write Consumer Surplus as:

$$CS(Q) = \frac{1}{2} (P(0) - P(Q))Q \quad (22)$$

and this is a very simple model to work with:

$$CS(n) = \frac{1}{2} \left(a - \left[\frac{1}{n+1}a + \frac{n}{n+1}c \right] \right) \left[\frac{n}{n+1} \frac{a-c}{b} \right] \quad (23)$$

where the first term in brackets is the price as a function of n , and the second is quantity. Or:

$$CS(n) = \frac{1}{2} \left(\frac{n}{n+1}a - \frac{n}{n+1}c \right) \left(\frac{n}{n+1} \frac{a-c}{b} \right) \quad (24)$$

$$= \frac{1}{2} \left(\frac{n}{n+1} \right)^2 \frac{(a-c)^2}{b} \quad (25)$$

$$= \frac{1}{2} \left(\frac{n}{n+1} \right)^2 bQ_e^2 \quad (26)$$

Where the third line is uses the substitution of $bQ_e = a - c$.

2.1.3 Welfare at Free Entry

Now at the free entry equilibrium we want to analyze:

$$\frac{dW}{dn^{fe}} = CS'(n^{fe}) + n\pi'(n^{fe}) \quad (27)$$

$$CS'(n) = \frac{1}{2} bQ_e^2 2 \left(\frac{n}{n+1} \right) \left(\frac{1}{n+1} - \frac{n}{(n+1)^2} \right) \quad (28)$$

$$= \frac{1}{2} bQ_e^2 2 \left(\frac{n}{n+1} \right) \left(\frac{n+1}{(n+1)^2} - \frac{n}{(n+1)^2} \right) \quad (29)$$

$$= \frac{1}{2} bQ_e^2 2 \left(\frac{n}{n+1} \right) \frac{1}{(n+1)^2} \quad (30)$$

$$= bQ_e^2 \frac{n}{(n+1)^3}, \quad (31)$$

and for the other side of the market:

$$\pi'(n) = bQ_e^2 \left(-2 \frac{1}{(n+1)^3} \right) \quad (32)$$

$$= -2bQ_e^2 \frac{1}{(n+1)^3} \quad (33)$$

$$n\pi'(n) = -2bQ_e^2 \frac{n}{(n+1)^3}. \quad (34)$$

Thus:

$$\frac{dW}{dnf_e} = bQ_e^2 \frac{n}{(n+1)^3} - 2bQ_e^2 \frac{n}{(n+1)^3} \quad (35)$$

$$= bQ_e^2 \frac{n}{(n+1)^3} (1-2) \quad (36)$$

$$= -bQ_e^2 \frac{n}{(n+1)^3} < 0. \quad (37)$$

Wow, I'd actually like to put an exclamation mark at the end of that. There is *too much* entry in a Cournot free entry equilibrium, or the business stealing affect dominates. Rather surprising, huh?

2.1.4 Full Welfare Maximization

Just for the fun of it let's find the social welfare optimum. Given the simplicity of this model it isn't that hard:

$$\begin{aligned} CS(n) + n\pi(n) - nF &= \frac{1}{2} \left(\frac{n}{n+1} \right)^2 bQ_e^2 + n \frac{1}{(n+1)^2} bQ_e^2 - nF \quad (38) \\ &= \frac{1}{2} \left(\frac{n}{n+1} \right)^2 bQ_e^2 + \frac{n}{(n+1)^2} bQ_e^2 - nF \\ &= \frac{n(n+2)}{(n+1)^2} \frac{1}{2} bQ_e^2 - nF \end{aligned}$$

$$\frac{dW}{dn} = \frac{1}{2} bQ_e^2 \left[\frac{(n+2)}{(n+1)^2} + \frac{n}{(n+1)^2} - 2 \frac{n(n+2)}{(n+1)^3} \right] - F \quad (39)$$

$$= \frac{1}{2} bQ_e^2 \left[\frac{2n+2}{(n+1)^2} - 2 \frac{n(n+2)}{(n+1)^3} \right] - F \quad (40)$$

$$= \frac{1}{2} bQ_e^2 \left[\frac{2(n+1)}{(n+1)^2} - 2 \frac{n(n+2)}{(n+1)^3} \right] - F \quad (41)$$

$$= \frac{1}{2} bQ_e^2 \left[\frac{2}{(n+1)} - 2 \frac{n(n+2)}{(n+1)^3} \right] - F \quad (42)$$

$$= \frac{1}{2} bQ_e^2 \left[\frac{2(n+1)^2}{(n+1)^3} - \frac{2n(n+2)}{(n+1)^3} \right] - F \quad (43)$$

$$= \frac{1}{2} bQ_e^2 \left[\frac{[2n^2 + 4n + 2] - [2n^2 + 4n]}{(n+1)^3} \right] - F \quad (44)$$

$$= \frac{1}{2} bQ_e^2 \left[\frac{2}{(n+1)^3} \right] - F \quad (45)$$

$$= \frac{1}{(n+1)^3} bQ_e^2 - F \quad (46)$$

So the optimum is characterized as:

$$\pi(n) = \frac{1}{(n+1)^2} b Q_e^2 = (n+1) F \quad (47)$$

In other words in the socially optimal equilibrium each firm earns:

$$\pi(n) - F = (n+1) F - F = nF \quad (48)$$

where as in the free entry equilibrium each firm earns nothing. Notice how extremely large this difference can be, a government very well might want to severely restrict entry in such an industry.

3 Conclusion

The intuition for this result is fairly clear, ex post. Yea, I know, everything should be clear ex post—but many things are harder to understand. As the number of firms gets large the impact of the marginal firm on the market price is small. The price ($P(n) = \frac{1}{n+1}a + \frac{n}{n+1}c$) is almost completely determined by marginal cost (c) fairly quickly. So how is the marginal firm making profit? By stealing it from the other firms. The hard bit is wondering what one would have to do overturn this basic logic.

4 Appendix: Cournot with Quadratic Costs

Just to be fair I will consider another simple cost specification, $c(q) = \frac{1}{2}cq^2$ and we will simply solve the model without explaining or simplifying the steps.

$$\left. \frac{\partial \pi_1}{\partial q_1} \right|_{q_1=q_j=q} = (a - b(q + (n-1)q)) - bq - cq = 0$$

$$\begin{aligned} q &= \frac{a}{b+c+bn} \\ Q &= \frac{na}{b+c+bn} \\ P &= a \frac{b+c}{b+c+bn} \end{aligned}$$

$$\begin{aligned} \pi(n) &= a \frac{b+c}{b+c+bn} \left(\frac{a}{b+c+bn} \right) - c \frac{1}{2} \left(\frac{a}{b+c+bn} \right)^2 \\ &= \frac{1}{2} a^2 \frac{2b+c}{(b+c+bn)^2} \end{aligned}$$

$$\begin{aligned}
CS(n) &= \frac{1}{2} \left(a - a \frac{b+c}{b+c+bn} \right) \frac{na}{b+c+bn} \\
&= \frac{1}{2} a^2 b \frac{n^2}{(b+c+bn)^2}
\end{aligned}$$

$$\begin{aligned}
CS' &= a^2 b n \frac{b+c}{(b+c+bn)^3} \\
\pi'(n) &= -a^2 b \frac{2b+c}{(b+c+bn)^3} \\
n\pi'(n) &= -na^2 b \frac{2b+c}{(b+c+bn)^3}
\end{aligned}$$

Thus at the free market equilibrium (n^{fe}):

$$\begin{aligned}
CS'(n) + n\pi'(n) &= a^2 b n \frac{b+c}{(b+c+bn)^3} - na^2 b \frac{2b+c}{(b+c+bn)^3} \\
&= -a^2 b^2 \frac{n}{(b+c+bn)^3} < 0
\end{aligned}$$

we get the same result.