Nazerattin Hoca and the Ants

Once there was not but once there was, a time when the Emperor Tamerlane wanted to build a watchtower on the highest place in all his lands.

He called his greatest scout and said to him, "Scout, I want to find the highest spot in all my lands so I can build a watchtower from which you can see forever. Find this land for me and I will reward you richly, fail and I will have you killed."

So the scout set off, and he searched and searched. For days, for weeks, for months he searched and finally, after a year and a day, he returned. "I have found it!" he cried. "I have checked everywhere within a hundred leagues and I have found the highest spot!"

"Take me to it," said Tamerlane, and the scout took Tamerlane and all the court to his spot. All were filled with amazement, to think how hard the scout had searched and look how high this spot was! But Tamerlane—with his eagle eyes—stared into the distance and saw some land that was higher. "You have failed!," he cried, and the scout was killed.

Next he called his most promising, most diligent corporal and sent him off. Again he searched for a year and a day, checking thousands and thousands of places. Again he failed. Again he died.

And it was the same with the captain, and the colonel. Each time he chose his best, each time after a year and a day they returned, and each time they failed.

Finally he came to his General. "My General, you are the cleverest and most diligent in all of my army. Find me the highest point of land so I can build a watchtower from which you can see forever."

The General was terrified, for days he wandered this way, and that way, looking for the highest spot. He would go a hundred meters, look around. Go another direction for a ways, look around, and on and on. But every time he checked he wasn't at the highest spot, and he became very sad. Finally one morning he was riding by Nazerattin Hoca's house while Nazerattin Hoca was having breakfast in the garden.

Nazerattin Hoca said to the General, "General, I have seen you wandering around for days. First you go this way, then you go that. Please tell me what you are doing." The General sighed and said, "I am looking for the highest point in all the lands so Tamerlane can build a watchtower from which you can see forever." And Nazerattin Hoca put down his glass of tea, looked at the General and said, "Is that all? Why don't you ask the ants?"

The General snickered, then he giggled, then he laughed, great gouts of laughter bursting out of his belly, he laughed better than he had in forty years—and he was only thirty five. "Nazerattin Hoca, the ants are too small, they could never see if they are on the highest point of the land." "That is true, but they can find it," said Nazerattin Hoca, "and they can do it before the sun sets today."

"Well then," giggled the General, "let us see this marvel." Since he expected

to be at this task for the rest of his life he had a day to waste. Nazerattin Hoca went over to an ant hill in his garden, leaned down, and squeaked at the ants. The ants squeaked, jumped up and down, and off they went. The ants in front, Nazerattin Hoca behind, and the General following him snickering with every step.

First they went a little this way, then that, then straight on and then they stopped and started milling around in a great big circle. The General looked around and said, "Nazerattin Hoca, the ants have failed. I agree that if we go forward or backwards we will go down, but the land rises to the right. This is a ridge, not the highest point in all the lands." Nazerattin Hoca smiled at him, and waited. Soon the ants took off again, up the slope to the right. The next time they stopped they circled again for a while, then they started jumping up and down and squeaking excitedly. "This is the place," said Nazerattin Hoca. The General laughed, and said "No it is not, why I can see higher land..." and he looked to his right, to his left, behind. Then he got out his telescope and looked to his right, to his left, and behind. He scanned the horizon as far as one could see, and scanned it again. There was no place higher than the place the ants had led him to.

The General was stunned. "But Nazerattin Hoca, how did the ants know they were on the highest spot." "Ahh," said Nazerattin Hoca, "that was simplicity itself. If you are on the highest point of the land then the ground is flat, or the slope of the ground is zero $(\frac{df(x)}{dx}=0)$." The General thought about this for a while, and was truly amazed, but then he had another thought. "But how did they know not to stop on the ridge? The ground was flat back there." And Nazerattin Hoca said, "You remember how they were milling around in a circle? They were checking the second important fact about the top of a hill. If you are at the top of a hill then in all directions the slope of the land must be decreasing $(\frac{d}{dx}\frac{df(x)}{dx} \leq 0 \text{ or } \frac{d^2f(x)}{dx^2} \leq 0)$." But there was one final thing that was bothering the General, "How did

But there was one final thing that was bothering the General, "How did they do it so quickly, they did in a morning what four good men in four years and four days could not." "Ahh, said Nazerattin Hoca, "this is the heart of their wisdom. If you follow this one piece of advice then all of the rest will follow. All they did was go forward whenever the ground was rising—the slope of the land was positive—and back if the slope was negative $(\frac{df(x)}{dx}>0\Rightarrow x\uparrow, \frac{df(x)}{dx}<0\Rightarrow x\downarrow)$."
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The General could not thank Nazerattin Hoca enough, and immediately went back to Tamerlane and told him what he had found. And he even—being honest to the core—told Tamerlane how. He said to Tamerlane, "You should name the watchtower 'Ant Tower' for without the ants there would be no tower." However Tamerlane, in his infinite wisdom, named it Lion Tower, after all the ants might have been ant lions.