A General Overview of General Equilibrium

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Gosh, general equilibrium. Let's take a moment to really think about what we are trying to do. We are trying to develop a simple analytic approach to understanding how prices are determined so that millions of people and millions of goods will all be in balance. Makes you scared, no? It should, it definitely scares me.

It would seem obvious that even the simplest theoretic attempt to understand this would take many steps. There are a lot of trees in the forest we need to look at. And like anytime you go into a forest and start studying trees you can loose sight of the general shape of the forest while doing it. This handout is intended to keep your eyes on the prize when we go wandering in the forest. It will give you a mildly technical overview so that you understand where we are going, and can grasp the steps along the way.

We will start by looking at the fundamental, basic economy—the exchange economy. In this model we don't care about where the goods we consume come from, we just say "there's a pile of food, let's eat." So we're basically looking at only the optimal way to eat the food. After we are done with that we're going to show that producing those goods (given a fixed amount of inputs) is almost the exact same problem. We're then going to tie these two models together into one large economy where we decide how much of which goods to produce.

Before we even get started on any of these problems however we need to take a step back. We need to understand the fundamental theorems that we will use to guide our steps when solving all of these problems. Essentially these theorems show a near equivalence between finding Pareto Efficient allocations and competitive equilibria. We will use these theorems because the Pareto Efficiency problem is so much simpler—we don't care about prices. The equivalence says that we can then go from these problems to help us find the competitive equilibrium.

Throughout this analysis the two goods consumers will consume will be food (F) and clothing (C). When these goods are produced they will be produced using the two standard inputs of labor (L) and capital (K). The price of a unit of food and clothing will be p_f and p_c respectively, the price of a unit of labor will be w, and the price of a unit of capital will be r.

1   The First and Second Welfare Theorems

We start out this discussion by reviewing the first and second welfare theorems. The reason we do this is because these things are very different in the continuous case than in the discrete case, and it is useful to understand what they mean in both cases.

We will start out by defining Pareto Efficiency in the continuum economy. By continuum I am referring to the goods, which throughout this lecture will be Food (F) and Clothing (C) but I am sure you will be able to see how they might be more general. We will first look at the simple case where the amount of food and clothing is fixed (the exchange economy.)

So, first of all, what is the mathematical problem we solve for Pareto Efficiency? Well, one way to write down the Pareto Efficiency problem is to maximize the happiness of one person holding the others constant. For simplicity we will again study the two person case,

and I hope you can see how to generalize it. So this problem is:

$$\max_{F_1,C_1} u_1(F_1, C_1)$$

such that:

$$
\begin{aligned}
F_1 + F_2 &\leq F^0 \\
C_1 + C_2 &\leq C^0 \\
u_2(F_2, C_2) &\geq \bar{u}_2
\end{aligned}
$$

The last condition really summarizes what Pareto Efficiency means. Given we don't hurt person 2 (we keep his utility at $\bar{u}_2$) we want to know how happy we can make person 1. We can use the Lagrangian method to write down this objective function:

$$\max_{F_1,C_1} \min_{\lambda_f,\lambda_c,\lambda_2} u_1(F_1, C_1) + \lambda_2(u_2(F_2, C_2) - \bar{u}_2) - \lambda_f(F_1 + F_2 - F^0) - \lambda_c(C_1 + C_2 - C^0) \tag{1}$$

and we can rewrite this as:

$$\max_{F_1,C_1} u_1(F_1, C_1) + \lambda_2 u_2(F_2, C_2) - \lambda_f(F_1 + F_2 - F^0) - \lambda_c(C_1 + C_2 - C^0) - \lambda_2 \bar{u}_2$$

We can multiply everything by the constant $\frac{1}{1+\lambda_2}$ and get:

$$\max_{F_1,C_1} \frac{1}{1+\lambda_2} u_1(F_1, C_1) + \frac{\lambda_2}{1+\lambda_2} u_2(F_2, C_2) - \frac{\lambda_f}{1+\lambda_2}(F_1 + F_2 - F^0) - \frac{\lambda_c}{1+\lambda_2}(C_1 + C_2 - C^0) - \frac{\lambda_2}{1+\lambda_2}\bar{u}_2$$

And rewrite things as $\alpha = \frac{1}{1+\lambda_2}$, $\mu_f = \frac{\lambda_f}{1+\lambda_2}$, $\mu_c = \frac{\lambda_c}{1+\lambda_2}$ and get:

$$\max_{F_1,C_1} \alpha u_1(F_1, C_1) + (1-\alpha)u_2(F_2, C_2) - \mu_f(F_1 + F_2 - F^0) - \mu_c(C_1 + C_2 - C^0) - (1-\alpha)\bar{u}_2$$

Now something that may not be too obvious, but should be intuitive, is that as $\bar{u}_2$ goes from zero to it's maximum possible $(u_2(F^0, C^0))$ α goes from one to zero. This tells us that we can stop optimizing over α and define Pareto Efficient allocations as allocations that solve:

$$\max_{F_1,C_1} \max_{F_2,C_2} \alpha u_1(F_1, C_1) + (1-\alpha)u_2(F_2, C_2) - \mu_f(F_1 + F_2 - F^0) - \mu_c(C_1 + C_2 - C^0) \tag{2}$$

for all $\alpha \in [0, 1]$. Now let's solve this program. The first order conditions are:

$$
\begin{aligned}
\alpha MU_1^F - \mu_f &= 0 \\
\alpha MU_1^C - \mu_c &= 0 \\
(1-\alpha) MU_2^F - \mu_f &= 0 \\
(1-\alpha) MU_1^C - \mu_c &= 0 \\
-(F_1 + F_2 - F^0) &= 0 \tag{3} \\
-(C_1 + C_2 - C^0) &= 0 \ . \tag{4}
\end{aligned}
$$

The first coefficient we want to get rid of is α:

$$
\begin{aligned}
\alpha MU_1^F &= \mu_f \\
\alpha MU_1^C &= \mu_c
\end{aligned}
$$

$$\frac{MU_1^F}{MU_1^C} = \frac{\mu_f}{\mu_c} \tag{5}$$

Likewise we can repeat these steps and get:

$$\frac{MU_2^F}{MU_2^C} = \frac{\mu_f}{\mu_c} \tag{6}$$

At this point we should notice the essential equivalence played in this program by the weights (μ_f and μ_c) and the prices in a competitive (or *Walrasian*) equilibrium. If we write $\mu_f = p_f$ and $\mu_c = p_c$ these conditions would be equivalent to what we find in a competitive equilibrium. Of course we also need market clearing, but that is given by the last two first order conditions (3 and 4).

Now we want to get rid of μ_f/μ_c from these conditions, resulting in the final form:

$$\frac{MU_1^F}{MU_1^C} = \frac{MU_2^F}{MU_2^C} \tag{7}$$

So Pareto Efficiency can be characterized as equations 7, 3, and 4.

Now we need to compare these conditions with a competitive equilibrium. Or more formally a Walrasian equilibrium. In this economy income is given by an initial endowment of food and clothing, $\left(F_1^0, C_1^0\right)$ and $\left(F_2^0, C_2^0\right)$. To find a person's income we then multiply these by the market prices. $I_1 = p_f F_1^0 + p_c C_1^0$, $I_2 = p_f F_2^0 + p_c C_2^0$.

Definition 1 *A Walrasian equilibrium is a (p_f, p_c) and an allocation $(F_1^*, C_1^*, F_2^*, C_2^*)$ such that:*

1. *Consumers maximize their utility given (p_f, p_c) and their income $I_i = p_f F_i^0 + p_c C_i^0$.*

2. *Market clearing occurs in both markets. (Demand is equal to (fixed) Supply).*

We all know that this can be characterized by:

$$\frac{MU_1^F}{MU_1^C} = \frac{p_f}{p_c}, \frac{MU_2^F}{MU_2^C} = \frac{p_f}{p_c} \tag{8}$$

$$p_f F_1^* + p_c C_1^* = p_f F_1^0 + p_c C_1^0, \; p_f F_2^* + p_c C_2^* = p_f F_2^0 + p_c C_2^0 \tag{9}$$

$$F_1^* + F_2^* = F_1^0 + F_2^0 = F^0 \tag{10}$$

$$C_1^* + C_2^* = C_1^0 + C_2^0 = C^0 \tag{11}$$

And now we can restate the first and second welfare theorems in this environment.

Theorem 2 (First Welfare Theorem) *Every Walrasian equilibrium is Pareto Efficient.*
Proof. *Clearly the equations in line 8 can be set equal so that we have equation 7, furthermore equations 10 and 11 are equivalent to 3 and 4. Finally we know that by the definition of Walrasian equilibrium that each customer is maximizing his utility. Thus the solution is Pareto Efficient.* ∎

This Theorem is always accompanied by a second Theorem. In general this Theorem is not too useful, but in this topic it will be very useful. An allocation $(F_1^e, C_1^e, F_2^e, C_2^e)$ is *strictly interior* if $F_1^e > 0, C_1^e > 0, F_2^e > 0, C_2^e > 0$.

Theorem 3 (Second Welfare Theorem) *If preferences are all convex, then every strictly interior Pareto Efficient allocation is a Walrasian equilibrium.*

Proof. *We use the fact that it is strictly interior to know that 7 actually holds, on the boundary it may not. We use the fact that preferences are convex to make sure that maximizing the sum of utilities also implies maximizing each person's utility. Given these insights then if we let μ_f in equations 5 and 6 be p_f and μ_c be p_c then this is clearly a competitive equilibrium except that we need to specify the income of the two people. The easiest way to do this is if $(F_1^e, C_1^e, F_2^e, C_2^e)$ is the Pareto Efficient allocation, then let $I_1 = p_f F_1^e + p_c C_1^e$, $I_2 = p_f F_2^e + p_c C_2^e$. Clearly the consumers will choose to consume their endowments.* ■

Now often times in class I present "baby" versions of theorems. Easier versions that are simpler to prove. Well this is not one of those cases. All of the assumptions in the Second Welfare Theorem are completely standard. It's harder to go from Pareto Efficient to a competitive equilibrium because you need to make sure that the global maximization problem will have the same solutions as the local maximization problems. And, unfortunately, we need all the assumptions made here to do that. But don't worry about it, these assumptions are all fairly reasonable and will always be met in this class.

Now why is this not too useful in general? Well that's simply because it's really only a tool for revolutionaries. As a social designer it's nice to know that you could radically change endowments and get a new Pareto Efficient outcome, but basically you're talking about taking from the rich and giving to the poor. You're talking revolution.

Now why is it useful here? Because solving the Pareto Efficiency problem only requires 7, 3, and 4. Basically we only have to determine $(F_1^e, C_1^e, F_2^e, C_2^e)$. In competitive equilibrium we also need to determine (p_f, p_c). So we have fewer parameters to work with. It is also of independent interest, it is nice to know what the set of possible competitive equilibria are and this is a convenient way to do that.

2 The Exchange Economy

2.1 The Edgeworth Box

Now let's start thinking about this from the viewpoint of person 1 in an exchange economy. She will have an initial endowment of (F_1^0, C_1^0), thus their income given prices is $I_1 = p_f F_1^0 + p_c C_1^0$. Obviously their income will be different for different prices, but it will always pass through (F_1^0, C_1^0), for example if $(F_1^0, C_1^0) = (2, 5)$ and $(p_f, p_c) \in \{(1, 2), (2, 1)\}$ then

their budget set would look like one of the two triangles below.
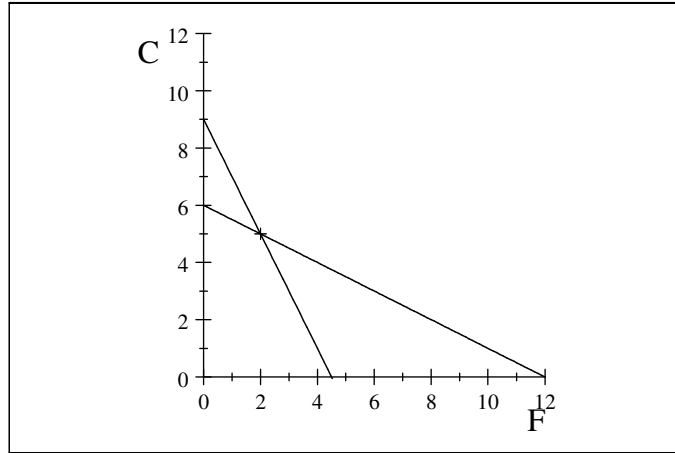
The small cross is her initial endowment.

Now let's assume that $(F^0, C^0) = (6, 8)$, then as a social planner we care about the fact that this person can not consume outside this box, thus it is sensible to include this box in our analysis.
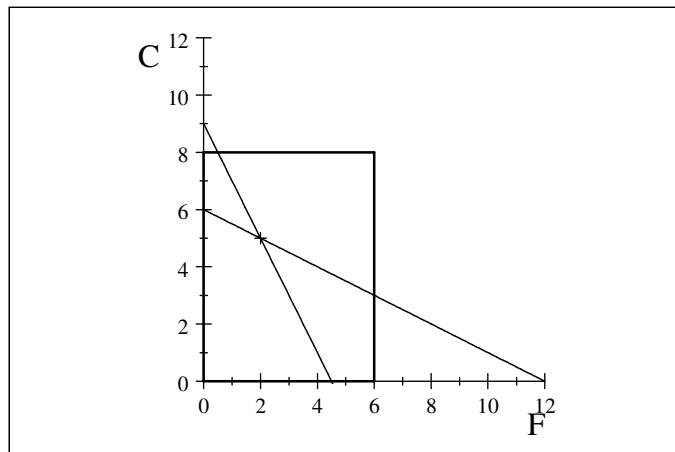
Notice that if we get the prices wrong the consumer might want to consume outside of that box. This is our problem, not hers. She doesn't care if supply equals demand, we are the only ones who care about that. However let's remove all points outside of that box just to

be careful.
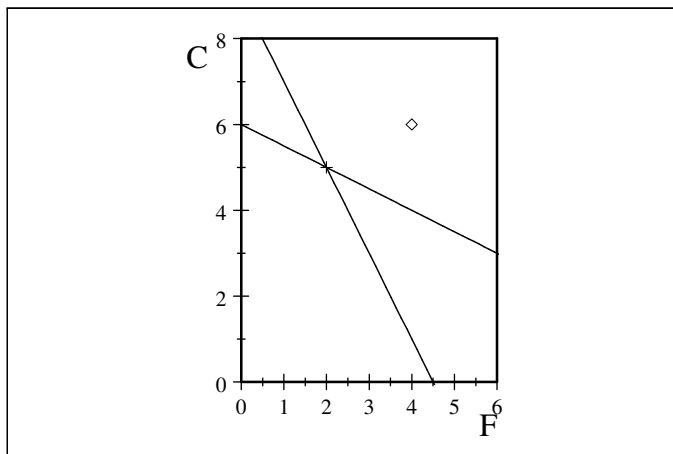
And consider an arbitrary point like the diamond. Notice something pretty cool about that point in this graph. Obviously that point represents an amount of F and C for person 1 to consume, these are $(F_1, C_1) = (4, 6)$, but it also represents an amount for person 2 to consume, $F_2 = F^0 - F_1 = 2$, $C_2 = C^0 - C_1 = 2$. Thus we can think of person 1's utility increasing to the upper right (with the origin being her origin) and person 2's utility increasing to the lower left—and the point (F^0, C^0) will be his origin.

This model is called the *Edgeworth Box* and is a marvelous tool for representing the two good/two person exchange economy graphically.[1]

2.2   The Pareto Efficient Allocations

Since we are first going to study the Pareto Efficiency problem we don't actually care about initial endowments and incomes, so lets drop the budget sets from our graph and instead draw some indifference curves. Assume $U_1(F_1, C_1) = F_1 C_1^2$ and that $U_2(F_2, C_2) = F_2^2 C_2$. Now in order to draw the indifference curves for person 2 in this graph it is convenient to write his utility in terms of (F_1, C_1), $U_2(F^0 - F_1, C^0 - C_1) = (F^0 - F_1)^2 (C^0 - C_1)$, thus the formula for an indifference curve for person 1 is:
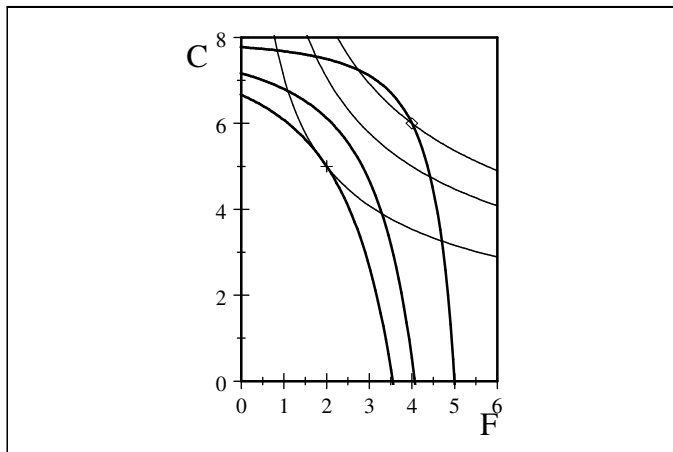
$$C_1 = \left(\frac{U_1}{F_1}\right)^{\frac{1}{2}},$$

and for person 2 is:

$$C_1 = C^0 - \frac{U_2}{(F^0 - F_1)^2} \ .$$

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[1]Francis Ysidro Edgeworth first presented this model in: "Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences," 1881.

I will now add these to our Edgeworth box for various values of U_1 and U_2.

The darker lines are the indifference curves for person 2, the lighter lines are the indifference curves for person 1. Now, remember that the slope of an indifference curve is exactly MU_f/MU_c, or the marginal rate of substitution. Thus Pareto Efficiency can be characterized as the points where the indifference curves are tangent inside of the Edgeworth Box. This is called the *Contract Curve*.

Definition 4 *The* Contract Curve *is the set of Pareto Efficient allocations in an Exchange Economy.*

The reason for this terminology is because it seems obvious that if people trade then they will end up somewhere on the contract curve. Both parties will clearly agree to any Pareto Improving trade, thus in the end the final allocation will be Pareto Efficient. Walrasian equilibrium is just one way to select one of these Pareto Efficient points.

In general this can be written as $C_1(F_1)$ for $F_1 \in [0, F^0]$ and we will always assume that $C_1(F_1)$ is a strictly increasing function. It will be strictly increasing if both people have monotonic utility functions (more is better) and it will be a function if at least one person has strictly convex preferences. Notice we could also write this as $C_2(F_2)$ for $F_2 \in [0, F^0]$, its just a matter of perspective. We use $C_1(F_1)$ because we always put person 1's origin in the lower left and F on the horizontal axis.

2.2.1   The Two Possible Contract Curves.

In this class we will only analyze three utility functions. The Cobb-Douglass: $U(F, C) = F^\alpha C^\beta$, the Leontief or Perfect Compliments: $U(F, C) = \min\{\alpha F, \beta C\}$ and the Linear or Perfect Substitutes: $U(F, C) = \alpha F + \beta C$. I personally guarantee that the contract curve will always pass through the points $(0, 0)$ and (F^0, C^0) so this leaves us with two possible shapes for the contract curve. It will either be linear—like the case on the left below, or it

will be a smooth curve—like the two cases shown on the right below.
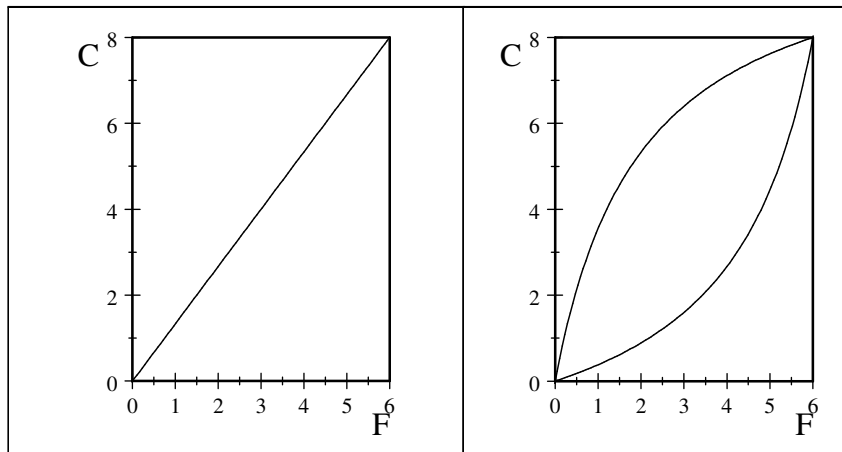
It will always be linear if one of the two consumers has either Leontief or Linear preferences, or the two consumers have the same marginal rate of substitution. It will be curved if both people have Cobb-Douglass utility functions with different marginal rates of substitution.

## 2.3 Going from the Contract Curve to the Competitive Equilibrium

In our exchange economy we start with an initial endowment for both people, $\left(F_1^0, C_1^0\right)$ and $\left(F_2^0, C_2^0\right)$, and we want to use the Contract Curve we just found to find the Competitive equilibrium. Now we know that every point on the Contract Curve is a Competitive equilibrium for some initial endowments, so what we need to know is given our initial endowment which point on the Contract Curve will we end up at?

Notice that each point on the Contract Curve has implicit prices associated with it, these are the μ_f and μ_c from the Pareto Efficiency problem. So, if we are to end up at point $C_1(F_1)$ on the contract curve we must have the prices be $\mu_f(F_1)$ and $\mu_c(F_1)$. So one way to solve this problem is to invert it. First for each Pareto Efficient allocation find the $p_f(F_1) = \mu_f(F_1)$ and $p_c(F_1) = \mu_c(F_1)$, then find the F_1 so that $p_f(F_1)F_1 + p_c(F_1)C_1(F_1) = p_f(F_1)F_1^0 + p_c(F_1)C_1^0$.

Now how do we do this graphically? For a given point on the contract curve the ratio of the prices is given by the slope of the contract curve. Thus the budget constraint for that point is given by a line that is perpendicular to the slope of the contract curve. In other words you first find the line that is tangent to the contract curve and then find the line that is perpendicular to that one. If that perpendicular line passes through your initial endowment, you are done. If not you have to do it again.

Let me do this graphically if the contract curve is not straight.
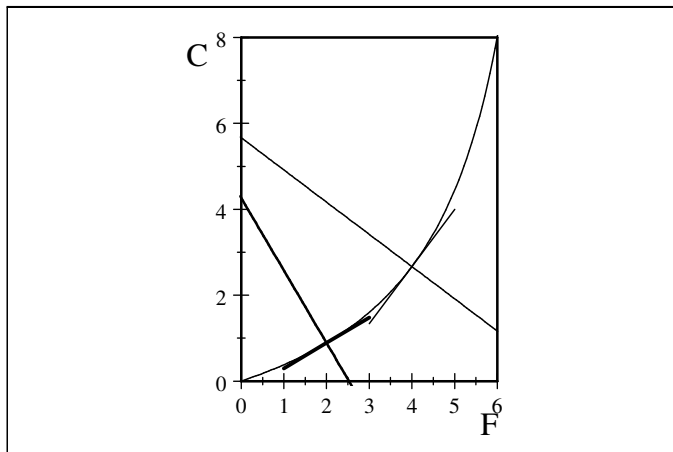
So this is the analysis done when $F_1 = 2$ (the lower cross) and when $F_1 = 4$ (the upper cross). Obviously if our initial endowment lies on one of these two lines we are done, otherwise we might have to do it again.

This is much easier if the Contract curve is linear, because then the slope of the contract curve is constant. In fact the form of the contract curve must be $C_1 = \frac{C^0}{F^0} F_1$. Isn't that nice?[2] Thus $\frac{\partial C_1}{\partial F_1} = \frac{C^0}{F^0} = \frac{p_c}{p_f}$. It is important to note that this is p_c/p_f because this is the line that is perpendicular to the budget line. For the budget line $\frac{\partial C_1}{\partial F_1} = -\frac{p_f}{p_c}$, and the slope of lines that are perpendicular is the negative reciprocal of it, or p_c/p_f. Notice that you can normalize prices any way that you want, but a particularly convenient normalization is $p_c = C^0$ and $p_f = F^0$. Given this the critical F_1 must satisfy:

$$
\begin{aligned}
p_f(F_1) F_1 + p_c(F_1) C_1(F_1) &= p_f(F_1) F_1^0 + p_c(F_1) C_1^0 \\
F^0 F_1 + C^0 C_1(F_1) &= F^0 F_1^0 + C^0 C_1^0 \\
F^0 F_1 + C^0 \left(\frac{C^0}{F^0} F_1 \right) &= F^0 F_1^0 + C^0 C_1^0 \\
F^0 F^0 F_1 + C^0 C^0 F_1 &= F^0 F^0 F_1^0 + F^0 C^0 C_1^0 \\
F_1 &= \frac{\left(F^0\right)^2 F_1^0 + F^0 C^0 C_1^0}{\left(F^0\right)^2 + \left(C^0\right)^2} \\
&= \frac{\left(F^0\right)^2}{\left(F^0\right)^2 + \left(C^0\right)^2} F_1^0 + \frac{F^0 C^0}{\left(F^0\right)^2 + \left(C^0\right)^2} C_1^0
\end{aligned}
$$

And gosh wasn't that simple. As you can see below, if the contract curve is not linear then this methodology is not useful at all—the formulas are just too complex.

Now let me briefly go over what you have to do in the more complicated case, where the contract curve is not linear. Without loss of generality[3] we can assume that $U_1(F_1, C_1) =$

[2] Let me remind you that in answering the question you will have to derive this.
[3] Since utility is ordinal.

$F_1 C_1^\beta$ and $U_2 (F_2, C_2) = F_2 C_2^\gamma$, where $\beta \neq \gamma$. Then one can show that the contract curve is:

$$\frac{\beta F_1 C^0}{\gamma F^0 + (\beta - \gamma) F_1} = C_1$$

and the slope of it is:

$$\frac{\gamma F^0 \beta C^0}{(\gamma F^0 + (\beta - \gamma) F_1)^2} = \frac{\partial C_1}{\partial F_1} = \frac{p_c}{p_f}$$

This is a mess, but let's just continue the argument with $p_f = 1$. Then the budget constraint is:

$$F_1 + \frac{\gamma F^0 \beta C^0}{(\gamma F^0 + (\beta - \gamma) F_1)^2} \left(\frac{\beta F_1 C^0}{\gamma F^0 + (\beta - \gamma) F_1} \right) = F_1^0 + \frac{\gamma F^0 \beta C^0}{(\gamma F^0 + (\beta - \gamma) F_1)^2} C_1^0$$

$$\frac{\gamma F^0 \beta C^0}{(\gamma F^0 + (\beta - \gamma) F_1)^2} \left(C_1^0 - \frac{\beta F_1 C^0}{\gamma F^0 + (\beta - \gamma) F_1} \right) = F_1 - F_1^0$$

And this is a cubic in F_1, so it is not useful at all. Indeed I have found that there is no simple way to go from the contract curve to the competitive equilibrium if the contract curve is not linear.

Notice that direct calculation of the competitive equilibrium in this economy is quite simple. One can show that:

$$F_1 = \frac{1}{1+\alpha} \frac{p_f F_1^0 + p_c C_1^0}{p_f}$$

$$F_2 = \frac{1}{1+\beta} \frac{p_f F_2^0 + p_c C_2^0}{p_f}$$

so the equilibrium can be found by

$$\frac{1}{1+\alpha} \frac{p_f F_1^0 + p_c C_1^0}{p_f} + \frac{1}{1+\beta} \frac{p_f F_2^0 + p_c C_2^0}{p_f} = F^0$$

This function is linear in price and simple to solve. It's a pity that the contract curve does not help in this case, but it happens.

2.4 Walras's Law

In the last section I made constant use of a fundamental fact, that we only could solve for relative price, and at the last I actually only solved for demand for one good. I didn't worry about demand for the other good. Why could I do this? Well it's based on a basic law in general equilibrium, which is called Walras's Law.

Definition 5 (Walras's Law) *In a competitive equilibrium of an n good economy:*

1. *You can only solve for $n - 1$ prices.*

2. *You only need to check market clearing in $n - 1$ markets.*

The basic property that gives us these rules is budget balancing. Because of budget balancing only relative price matters. Because of budget balancing if $n - 1$ markets clear the last one has to clear by default. Notice how much work this saves you, especially since in our analysis usually $n = 2$. Thus you only have one price to find and one market to clear. The proof is very simple, especially for the first property. I'm only going to prove it in the two good case for simplicity, but it should be easy to see how it extends. We will explicitly assume throughout this proof that all prices are strictly positive, there are no free goods.
Proof. By budget balancing we know that:

$$p_f F_i^* + p_c C_i^* = p_f F_i^0 + p_c C_i^0$$

since $p_f > 0$ we can divide through by it and get:

$$F_i^* + \frac{p_c}{p_f} C_i^* = F_i^0 + \frac{p_c}{p_f} C_i^0$$

now normalize the other prices, $\tilde{p}_c = \frac{p_c}{p_f}$ and you only have to solve for $n - 1$ prices.

For the second property again we use that

$$F_i^* + \tilde{p}_c C_i^* = F_i^0 + \tilde{p}_c C_i^0$$

for all consumers $i \in \{1, 2, 3, ..., I\}$, but then if we sum the budget constraints:

$$\Sigma_{i=1}^I F_i^* + \tilde{p}_c \left(\Sigma_{i=1}^I C_i^* \right) = \Sigma_{i=1}^I F_i^0 + \tilde{p}_c \left(\Sigma_{i=1}^I C_i^0 \right)$$

By definition $\Sigma_{i=1}^I F_i^0 = F^0$ or supply, and $\Sigma_{i=1}^I C_i^0 = C^0$. (Just in case you are intimidated by this, let me point out that $\Sigma_{i=1}^I F_i^* = F_1^* + F_2^* + F_3^* + ... + F_I^*$—it's just saying we're adding up everyone's demand.)

$$\Sigma_{i=1}^I F_i^* + \tilde{p}_c \left(\Sigma_{i=1}^I C_i^* \right) = F^0 + \tilde{p}_c C^0$$

Now by market clearing in clothing we have that $\Sigma_{i=1}^I C_i^* = C^0$, making this substitution we have:

$$\Sigma_{i=1}^I F_i^* + \tilde{p}_c C^0 = F^0 + \tilde{p}_c C^0$$

and obviously $\tilde{p}_c C^0$ cancels. But this means

$$\Sigma_{i=1}^I F_i^* = F^0$$

and we are done. Notice that since $\tilde{p}_c > 0$ we could have just as easily checked market clearing in food. ∎

This result is important in many ways. Perhaps the most important of the two results (analytically) is that again we see that only relative price matters. In its full glory this is the theorem that explains why *neutral* inflation (that affects all prices simultaneously) has no impact on welfare. If all prices rise that causes no impact on relative prices, and thus society doesn't care. Of course, unfortunately, all prices do not adjust simultaneously. For example Bilkent (and most employers) gives us a raise every six months, not every day. One factor in this raise is inflation adjustment, but most salaries do not react immediately to every raise in other prices. More generally part of what impacts welfare in Turkey is the exchange rate, and as you are probably aware it doesn't seem to be related to Turkish inflation at all. When I came to Turkey in 2001 the exchange rate was about 1.65 (new)

TL to the dollar. In 2008 that was still it's approximate level (actually it was lower). This is despite the fact that inflation in Turkey was higher than inflation in the United States throughout that period. Then the financial crisis hit, and the exchange rate immediately went up even though Turkey was one of the few countries that was not directly impacted by the financial crisis. Currently (22 February 2012) it is at 1.79, and in the last year it has fluctuated between 1.5 and 1.9 TL/USD. It is transparent that the exchange rate is not at all based on the relative inflation rate in Turkey and the US, thus it is clear that inflation is not neutral.

From our point of view it just makes life a heck of a lot easier. Really, it does. Now I know that I only have to solve for one price and check one market clearing condition. Unfortunately I must mention a problem this causes. I like answers to questions to be integers, and you can trust me that consumption levels will be integers. Unfortunately though you may decide to normalize by p_c and I might decide to normalize by p_f, so there is no way the ratio of prices will always be an integer. Generally I am satisfied to have it a simple fraction, like 2/3, 3/2 or 3/7.

3   The Shell Game—the Edgeworth Box Production Economy.

Now it's time to let you guys in on a little joke all Economists play on their students.[4] In the first semester we spend approximately a third of the class studying consumer theory. Then we spend the next third of the class studying producer theory. I call this the shell game because while the ball is under a different cup its the same game.[5] Almost exactly, consumption, production, who cares? Well there is a critical difference in that problem, in consumer theory there is a binding income constraint, and understanding the impact of this constraint is important. But now we're primarily looking at Pareto Efficiency. No income? No difference!

To explain this a little more formally let me first ask you if it would make any difference in analysis to use (L, K) instead of (F, C) in equation 1. Obviously not, it's the same mathematical problem. Second, is there any reason we can't write $U_1(L_1, K_1) = f_F(L_f, K_f)$ and $U_2(L_2, K_2) = f_C(L_c, K_c)$? Again obviously not. So we can rewrite equation 1 as:

$$\max_{L_f, K_f} \min_{\lambda_l, \lambda_k, \lambda_c} f_F(L_f, K_f) + \lambda_c \left(f_c(L_c, K_c) - \bar{C} \right) - \lambda_l \left(L_f + L_c - L^0 \right) - \lambda_k \left(K_f + K_c - K^0 \right)$$
(12)

Now there's absolutely no harm in multiplying through by μ_f, and writing $\mu_c = \mu_f \lambda_c$, $\mu_l = \mu_f \lambda_l$, $\mu_k = \mu_f \lambda_k$ resulting in the problem:

$$\max_{L_f, K_f} \min_{\mu_l, \mu_k, \mu_c} \mu_f f_F(L_f, K_f) + \mu_c \left(f_c(L_c, K_c) - \bar{C} \right) - \mu_l \left(L_f + L_c - L^0 \right) - \mu_k \left(K_f + K_c - K^0 \right)$$
(13)

Since this is the same problem we can use all the technology we just developed to solve it. However let me rewrite the equation a little bit more.

$$\max_{L_f, K_f} \min_{\mu_l, \mu_k, \mu_c} \left(\mu_f f_F(L_f, K_f) - \mu_l L_f - \mu_k K_f \right) + \left(\mu_c f_c(L_c, K_c) - \mu_l L_c - \mu_k K_c \right) - \mu_c \bar{C} + \mu_l L^0 + \mu_k K^0$$

[4] Don't tell anyone I told you this. Please. Not just my job but my entire academic career could be on the line!!!!

[5] http://en.wikipedia.org/wiki/Shell_game

and point out that like before we can drop the $\mu_c \bar{C}$ just by considering the problem for all μ_f/μ_c, resulting in the problem:

$$\max_{L_f,K_f} \max_{L_c,K_c} \min_{\mu_l,\mu_k} \mu_f f_F (L_f, K_f) - \mu_l L_f - \mu_k K_f + \mu_c f_c (L_c, K_c) - \mu_l L_c - \mu_k K_c + \mu_l L^0 + \mu_k K^0$$

(14)

Does this problem look at all familiar to you? Well let's write profit maximization:

$$\max_{L_f,K_f} p_f f_F (L_f, K_f) - wL_f - rK_f + \max_{L_c,K_c} p_f f_F (L_f, K_f) - wL_f - rK_f$$

Now we have to be clear that in a general equilibrium we also have to have market clearing:

$$\begin{aligned} L_f + L_c &= L^0 \\ K_f + K_c &= K^0 \end{aligned}$$

But if we add in these constraints to our profit maximization, make sure the production functions are well behaved (basically they need to be concave—have decreasing or constant returns to scale and be convex with regards to (L,K)—just like in Utility) then the two problems are identical. This tells us that we can reinterpret the constraints in the Pareto Efficiency problem as our market prices, and the two problems are exactly the same. (Given (μ_f, μ_c) or (p_f, p_c), these values must be determined outside the problem we are considering). That is yet another short and sweet proof of the first and second welfare theorems in the production economy.

3.1 The Production Possibilities Frontier

Whoops, I lied. (This is part 2 of the Shell game, convincing you the problems are exactly the same and then pointing out they are not. The ball was under the third cup all along!) In production theory we do want to go one step further, we want to construct a *production possibilities frontier*. This is simply the boundary of the *production possibilities set*, which is the equivalent of the budget set for the macro economy.

Definition 6 *The* production possibilities set *is the set of outputs ((F,C)) that can be produced given resources ((L^0, K^0)).*

Definition 7 *The* production possibilities frontier (PPF) *is the boundary of the production possibilities set.*

Like I said this is really the macro equivalent of the budget set for a closed economy. Given that trade is not possible people have to consume within their resources, and this means they have to consume something in the production possibilities set. Obviously they won't want to consume something on the strict interior (where in effect some resources are wasted) so they'll consume on the production possibilities frontier just like they consume on the budget constraint in consumer theory.

Why do we want to take this step? Well simply speaking to determine the equilibrium output prices, (p_f, p_c). As one might expect they will be determined by the slope of the production possibilities frontier. Taking this step analytically is quite simple. Remember that we can write the contract curve in the production economy as either $L_f (K_f)$ or $L_c (K_c)$, thus given that production is on the contract curve (or it is efficient) we can write the production functions as $F = f_F (L_f (K_f), K_f)$ and $C = f_C (L_c (K_c), K_c)$. Now we use a theoretically simple trick. Notice that:

1. The production function is strictly increasing in both L and K.

2. Therefore $L_f(K_f)$ is strictly increasing in K_f and $L_c(K_c)$ is strictly increasing in K_c.

This tells us that the function $f_F(L_f(K_f), K_f)$ and the function $f_C(L_c(K_c), K_c)$ are both strictly increasing in capital. Notice that in these functions there is no labor. The amount of labor is determined by the amount of capital. So we can write an implicit function of food and clothing in terms of nothing but the amount of capital used, $F = g_f^{-1}(K_f)$ and $C = g_c^{-1}(K_c)$. We know that both these functions are strictly increasing in capital. But this means they have an inverse, which we can write as $g_f(F) = K_f$ and $g_c(C) = K_c$. Now we substitute these functions into our resource constraint, $K_f + K_c = K^0$, and we have a production possibilities frontier:

$$g_f(F) + g_c(C) = K^0 \tag{15}$$

Wasn't that easy? Yea, well, in practice it's actually almost impossible if the contract curve is not linear. If the contract curve is linear then $C_j = \frac{C^0}{K^0} K_j$ (for $j \in \{c, f\}$) and then if the production function is constant elasticities of substitution with respect to labor and capital (for example Linear, Leontief, or Cobb-Douglass) it's easy to find the function g_j^{-1}. In general it is basically intractable. (The easiest example is two Cobb-Douglass production functions that do not have the same marginal rate of technical substitution. It can't be done analytically, take my word for it.) One final point, while in utility theory it doesn't matter in production we will usually want the production function to have decreasing returns to scale. Thus we may add a parameter to make this true. For example we could have $f(L, K) = u(L, K)^\rho$, where $\rho < 1$ or we could have $f(L, K) = \ln u(L, K)$. In both cases $u(L, K)$ will be either Leontief, Linear, or Cobb-Douglass.

3.1.1 The General Shape of the Production Possibilities Frontier

To graph the production possibilities frontier we want to solve for it as a function of C in terms of F, this is quite simple in abstract, $C = g_c^{-1}(K^0 - g_f(F))$ and as long as both production functions are decreasing or constant returns to scale it can have only two basic shapes:
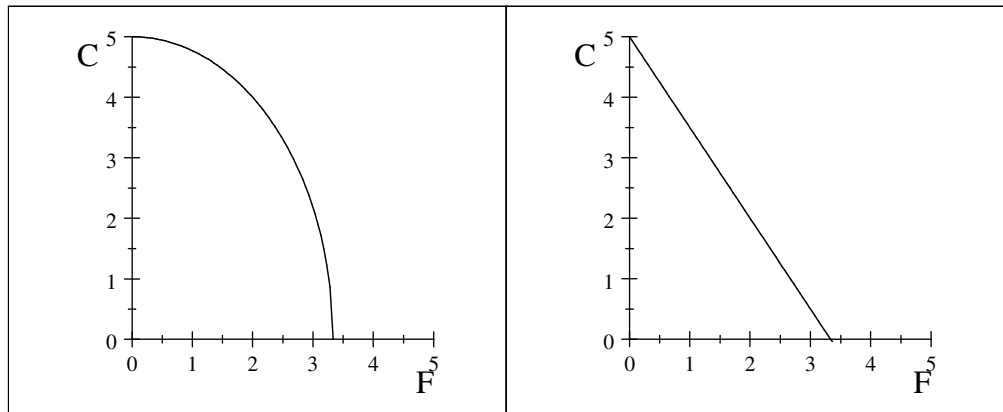
The one on the right is very rare. It requires that both production functions have constant returns to scale (CRS) and that the marginal rate of technical substitution (MRTS) for the two firms is the same. More generally the one on the left is normal.

It should be fairly obvious that the function will be decreasing, in order to produce more F you have to devote more resources to its production, which means that you have to produce less C.

The fact that it is concave is a little more subtle. Remember that a concave (and decreasing) function has a decreasing slope—it is getting steeper and steeper as we produce more and more F. In other words as you try and produce more and more F you have to give up more and more C. I think the easiest way to explain it is concave is to explain why it's not if one of the production functions has strictly increasing returns to scale. If you have increasing returns to scale then we all know that the most efficient way to produce output for that good is to devote all our resources to it. This is essentially what the definition means, $f(2K, 2L) > 2f(K, L)$. Thus every time we reduce the amount of C we produce by 1 we may get more than 1 unit of F in return. Obviously this will mean that the slope of the production possibilities frontier is increasing (but always negative—it's converging to zero) as we devote more and more resources to F, this would produce a decreasing and convex production possibilities frontier.

It is more subtle why if the two firms do not have the same MRTS but do have constant returns to scale it will be decreasing, and frankly at this point you should just take my word on it. The essential insight is that when you move along the production possibilities frontier to increase the output of one good you have to use a worse balance of inputs to produce the other. Not happy with that explanation? Well, find me a better one.

And why is it linear when we have CRS and the same MRTS? Because the optimal amount to give up from one firm is also the optimal amount to give to the other, and because we have CRS the transferred inputs will be changed to outputs in a one to one manner.

The important fact is that under decreasing or constant returns to scale the Production Possibilities set is always a subset of any Budget set that contains it. I.e. society would be better off if they could just find the "optimal budget set" and ignore the fact that they can't produce what they want.

# 4 Pulling it All Together—The Robinson Crusoe Economy and International Trade

Now we want to pull everything together. We've already shown that we can go from the Pareto Efficiency problem for two firms to a Production Possibilities Frontier. Now we want to make the glorious assumption that we can think of all the people in society as one person, called Robinson Crusoe (http://en.wikipedia.org/wiki/Robinson_Crusoe). Robinson Crusoe was a character in a Daniel Defoe novel published in 1719. He was shipwrecked on an island all alone except for a native (who he named Friday because he met him on a Friday—such respect for the locals). Since we all know natives don't count we can treat him basically as the only person on the island.[6]

This is a paradigmatic name for this type of economy because in this case we clearly will have a production possibilities set and one consumer who's goal is to do the best for himself

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[6] I **am** being sarcastic.

that he can. Now let's graphically represent this problem:
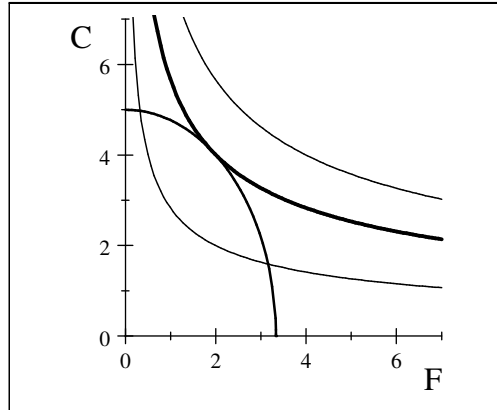
I've drawn several indifference curves in this space, and the optimal indifference curve is the heavy dark one. The consumer obviously wants to consume on the highest indifference curve he can, and in general that will be when the indifference curve is tangent to the production possibility set. We can write this as:

$$MRS = \frac{MU^f}{MU^c} = \frac{g'_f(F)}{g'_c(C)} = MRT \ .$$

MRT is short for the marginal rate of transformation. Now we can prove a very important result, known as the *decentralization theorem.* Basically Robinson Crusoe doesn't need to solve the Pareto Efficiency problem, he can have a split personality. One part of his personality can simply maximize his utility, a second (or possibly second and third) can solve the production problem, and a final part of him must make sure that supply is equal to demand in all markets.

Theorem 8 (Decentralization Theorem) *Robinson Crusoe can either:*

1. *Solve three problems:*

    (a) *Solve the revenue maximization problem of $R = \max_{F,C} p_f F + p_c C$ such that $g_f(F) + g_c(C) \leq K^0$*

    (b) *Solve the utility maximization problem given the income constraint of $I = R$*

    (c) *Choose prices so that all markets clear.*

2. *Solve four problems:*

    (a) *Solve $\pi_f = \max_{L_f, C_f} p_f f_F(L_f, C_f) - wL_f - rK_f$*

    (b) *Solve $\pi_c = \max_{L_c, C_c} p_c f_c(L_c, C_c) - wL_c - rK_c$*

    (c) *Solve the utility maximization problem given the income constraint of $I = \pi_f + \pi_c + wL^0 + rK^0$*

    (d) *Choose prices so that all markets clear.*

The proof of this theorem is basically "this is what the second welfare theorem tells us." So I'm not going to repeat myself. Now why, you may ask, would we ever want to do this? We have one simple maximization problem given a constraint and you want to change it into—maybe—four different problems? You must be crazy. Well, I did say Robinson Crusoe had a split personality, so he's crazy. Why not join the fun?

OK, you're right. Mathematically speaking the standard approach is simpler. But theoretically this equivalence is extremely important. First think about the equivalence between 1.a and 2.a and 2.b. This clearly can be extended to having thousands, millions of firms that are all trying to maximize profits, as long as prices clear markets these two problems will be equivalent. Thus we can have many profit maximizing firms, and we don't need to worry about it. We can always, without loss of generality, analyze them as if they were one super firm, that didn't really care about who produces what it just wants to maximize revenue. At this point let me point out a technical equivalence between the incomes in the two scenarios. Let (F^*, C^*) be the optimal amount of food and clothing to produce, then in scenario 1 Robinson's income is $I = p_f F^* + p_c C^*$. In the second scenario $I = \pi_f + \pi_c + wL^0 + rK^0 = p_f F^* - wL_f^* - rK_f^* + p_c C^* - wL_c^* - rK_c^* + wL^0 + rK^0$. By market clearing we know that $L_f^* + L_c^* = L^0$, $K_f^* + K_c^* = K^0$, making these substitutions we get $I = p_f F^* + p_c C^* - wL^0 - rK^0 + wL^0 + rK^0 = p_f F^* + p_c C^*$. If we couldn't show this equivalence then these wouldn't be the same problem.

Now to go to many consumers we need to specify that each consumer is endowed with a specific initial endowment of labor and capitol *and* a share of the profits from each firm. In abstract we are thinking about them as shareholders, of course a simple approach would be to say that one owns each firm. Then given these endowments and the total amount of food and clothing this is just an exchange economy. Once we've done it twice we can do it a thousand times, a million times. As long as prices clear markets the decentralization theorem tells us that the Robinson Crusoe economy can be thought of as just one general economy. A little bit simpler to analyze, but equivalent.

A final technical note, you may wonder: "But my labor endowment is not fixed, I optimize over it." We won't go there because that would be basically a third "good." (Actually we assume it's a bad, you would rather not work.) We don't want to do a three good optimization problem. Of course you could just drop food and then you would be back to the model we've already analyzed. (We live in the idyllic land where we just walk out our door and pick apples and steaks off trees.) Basically it won't produce any difference.

4.1   International Trade

So how would this change if we analyzed a world where Robinson Crusoe could trade with others? For simplicity we assume in this new world the prices he would trade at, $\left(p_f^w, p_c^w\right)$ are fixed. We will want to compare these prices with the local prices, $\left(p_f^l, p_c^l\right)$. Now first of all since relative price is all that matters if $p_f^w/p_c^w = p_f^l/p_c^l$ then it won't cause any difference. Crusoe will just ignore the rest of the world and continue his idyllic existence. He could trade with them, but why? The prices they charge are equivalent to the prices he would charge. But what if they are not? Then we get a rather stunning result.

Theorem 9 *If $p_f^w/p_c^w \neq p_f^l/p_c^l$ Robinson Crusoe is always strictly better off trading with the rest of the world.*

Wow, and this is actually pretty easy to prove. The proof is similar to the proof that the cost function is always decreasing in input prices. In that proof we always include the intermediate step of "continue producing using the old mix of inputs at the new prices." We do this because this *must* lower the costs of production, then we can optimize. Here we will use the same trick, because then the old consumption point is in the new budget set. Thus Robinson Crusoe must be at least as well off as before because he can always continue to consume what he did before. Then we choose the optimal amount of output and strictly increase his income, so now he must be strictly better off. Formally the proof goes:

Proof. Let (F_0^*, C_0^*) be his original consumption level, and have Robinson Crusoe continue producing (F_0^*, C_0^*) at the new prices. Robinson Crusoe must be at least as well off as before because he can continue to consume (F_0^*, C_0^*) if he likes. Now let the firm maximize it's revenue at the new prices, this will result in a $R_n > R_i = p_f^w F_0^* + p_c^w C_0^*$. Thus he must be strictly better off since his new budget set is a strict subset of his old. ∎

This proof can also be done graphically. The heavy line is the old budget set, the light straight line is the new budget set, and the dashed line is the intermediate budget set. I have expanded the graph to make the difference between the lines clearer.
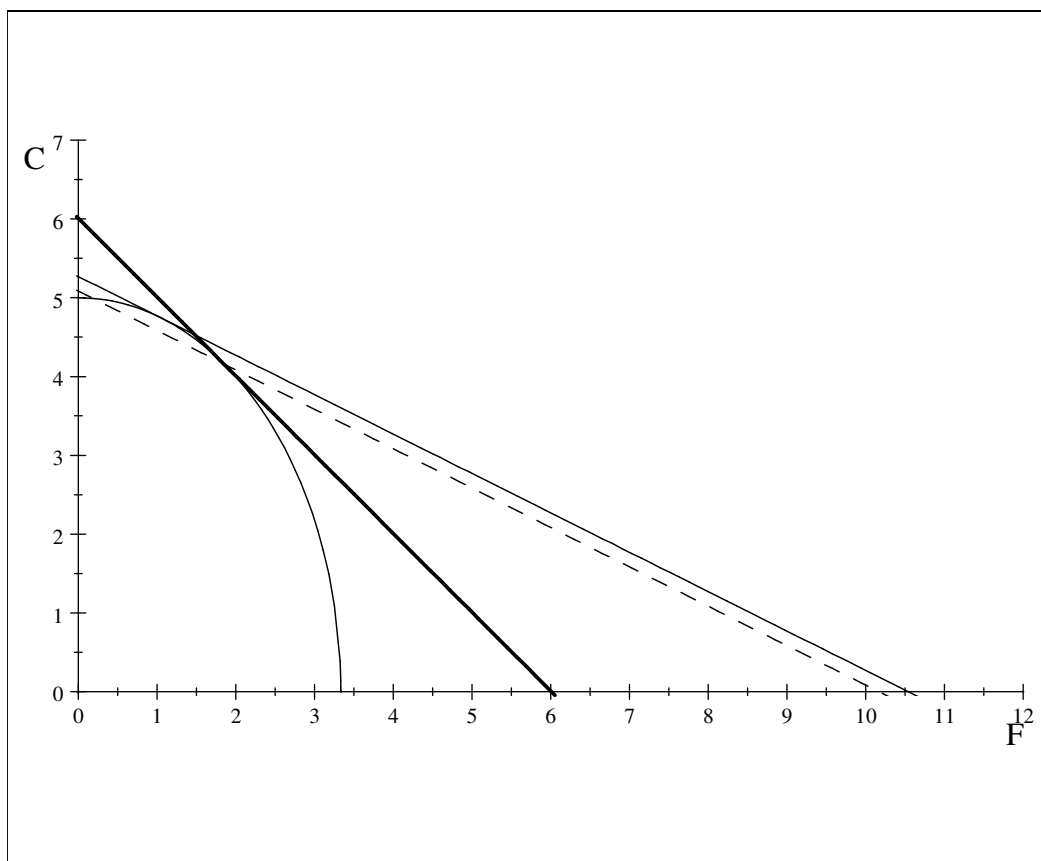
Let me point out the difficulty with going straight from the old to the new budget set. The critical difference is that there are points where he consumed a lot of C and only a little F in the old budget set that are no longer possible. I know, you say, but they're not feasible—he could have never consumed there without trade. That's not a big deal, they exist. So how

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can we be sure he's happier? Well what we need to prove is that the old consumption point is inside the new budget set, and proving that is easiest to do using the intermediate budget set.

Let me point out something very fundamental about this proof. The key step was showing that Robinson Crusoe's real income had strictly increased. At the new budget set everything he wanted to consume before was strictly inside the new budget set. This means that even if there were millions of people (a true economy) that everyone could be made better off. To be precise the total real income of the nation has increased. (Real income is basically the budget set, and what we've shown is that everything that is no longer feasible is also not wanted, so its lost is inconsequential.) Thus opening an economy to international trade can strictly increase every person's real income.

4.1.1 So how can There be Opposition to Free Trade?

The basic answer is interest groups. While free trade may increase society's real income that increase in wealth is never properly spread throughout the population. It's just not feasible. Let us, just to be conservative, say that you have a thousand people each of whom may have a negative direct impact from free trade. You know the total impact is positive, so you need to basically balance out the income gains of some groups with the income losses of others. Think you could do it? The only mechanism I know that can solve such a sophisticated problem is the competitive economy—and it won't solve this one. Even if we devoted every scientist in the world to it I doubt we could solve this problem. Especially—shudder—when we scale it up to the millions—no, billions—of people that are going to be affected.

Thus the bottom line is that in practice free trade is not Pareto improving. There are winners and losers. If the losers have more political clout, they win. Not too complex, is it?

To give a more specific example, assume there are two goods that do not have the same marginal rate of technical substitution. Assume there are three consumers, one owns both firms, the other owns all the capital, and the final consumer owns all the labor. Opening to free trade means altering the existing production of the two goods, and since the MRTS of the two production functions are not the same this means that either the wage or the rental rate on capital must fall (relatively speaking). Thus there will be a loser, and they will object to free trade. (I think the guy who owns the firms will always like free trade, but I'm not sure.) Sure, in this economy in an election the winners would vote for free trade friendly politicians, but (just for instance) say that the wage was going to fall and there wasn't one laborer but millions. You see the point?

On the other hand we do know that free trade is *potentially* Pareto Improving, or loosely speaking the *average* consumer is better off with free trade. Frankly that's enough for me, I'm all for free trade—as are most economists. And I'm pleased to say that it seems like in the modern day free trade is winning. Now let me be clear, no country fully embraces free trade. Everyone provides some protection to their local firms. Some of that is health and safety related—horse meat lasagna anyone? (http://edition.cnn.com/2013/02/08/world/europe/uk-horsemeat-probe/index.html) But a lot of it is simply protectionist. However in general more and more nations world wide are moving towards free trade, joining the world trade organization and etceteras. Quite frequently in the history of economic thought people have argued for (usually temporary) protectionist measures to help development, but economic history generally shows these theories were wrong. South Korea, for example, has grown amazingly despite being fairly free trade orientated. India used to be protectionist, and then recently they opened up to free trade. The result? Incredibly fast economic development.

The United States is now predicting that it will be one of the economically most powerful in the world by 2030.[7]

4.1.2   Comparative Advantage

There are two principle way to compare countries. The most obvious is *absolute advantage*. One country has absolute advantage over another if it can produce more of all goods and services. The naive would think this would mean these two countries would not trade. However this thinker is not considering the fundamental insight provided by relative prices. Does it matter if my salary is measured in billions or thousands? Was I upset when Turkey revalued their currency and therefore reduced my salary by a factor of a million? No, I didn't really care. I knew all prices were going to be reduced by an order of a million, so what? (By the way, when was the last time that someone asked you for 500,000 liras? It was fairly recent for me. I didn't stress, I just handed him 50 kurus and he was happy.) How does this apply to international trade? Well let's say that if I give you one cow you can give me back 10 ski jackets. Should we trade or not? Seems like a great deal to me (OK, perhaps we won't agree on the full 10 jackets, I'm still sure we can strike a bargain.) But wait, how can this be? How can you offer up to 10 ski jackets for one cow? I know that if I tried to produce ski jackets myself it would basically take two cows to produce one ski jacket. The simple answer is that the opportunity cost for you of producing ski jackets is much lower than for me, or technically speaking the slope of your production possibilities frontier (PPF) is lower. This is the concept of *comparative advantage*, first described by David Ricardo in his 1817 book: "On the Principles of Political Economy and Taxation."[8]

His example compared England and Portugal at the time. In Portugal it was possible to produce both wine and cloth with less labor than it would take to produce the same quantities in England. However the relative costs of producing those two goods were different in the two countries. In England it was very hard to produce wine, and only moderately difficult to produce cloth. In Portugal both were easy to produce. Therefore while it is cheaper to produce cloth in Portugal than England, it is cheaper still for Portugal to produce excess wine, and trade that for English cloth. Conversely England benefits from this trade because its cost for producing cloth has not changed but it can now get wine at a lower price, closer to the cost of cloth.

The simple theory this produces is based on linear production possibility frontiers—PPF's that look like budget constraints. Each nation should produce only the good that they have the highest comparative advantage in. This simple theory of course is modified if the PPF is strictly concave—as I explained it usually is. With strictly concave PPF's a nation should *export* only the good it has comparative advantage in *at equilibrium prices* and import all other goods. With a strictly concave PPF every nation should produce every good, but they will export only one good. To explain this intuition consider a nation that clearly doesn't have a comparative advantage in growing apples—like Norway. Now even though Norway is not a great place to produce apples people still do have apple trees. So in order to produce some apples all they have to do is go outside at the right time of the year and an apple will fall in their lap—literally.

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[7] In response to this I have to say "ha." I remember when similar predictions were made about Japan. We'll see, but I certainly hope that its true since it is the second most populated nation in the world. (China was also included in this prediction, and they're number one in population.)

[8] I extensively used Wikipedia (http://en.wikipedia.org/wiki/Comparative_advantage) to write this section.

Of course even this modified theory is incorrect. All countries export and import essentially the same goods all the time. For example I know that in the US you can buy Turkish Feta cheese (beyaz peynir). Not the good stuff, you Turks consume that yourselves (grrr) but some. At the same time I can go into any grocery store in Turkey and buy French cheese. How does that make sense? I know, you want to point out that those cheeses aren't exactly the same but I think you're splitting hairs. Turkey does export and import refrigerators as well—are you going to argue those are different too? Isn't easier just to accept there are some nuances of international trade that we don't understand?

Still this insight gives us great understanding about what drives international trade. Essentially every nation in the world is different, and they trade for what they can't produce locally as cheaply. Variety is the spice of life, it encourages trade within a nation and across national boundaries based on the same basic fundamental insight. We all produce what we have a comparative advantage in at equilibrium prices. (I, for example, would make a terrible farmer. So why don't you raise the cows and I make the ski jackets? Wait, I'm pretty bad at sewing too. How about I become a university professor and you send your kids to me so they can ignore what I'm teaching?)[9]

The last thing I want to do in this handout is prove the basic principle of comparative advantage. So let's assume economy 1 has the PPF of $F + 2C = 100$ and that economy 2 has the PPF of $2F + C = 10$. Notice economy 1 has an absolute advantage, but that economy 2 has a comparative advantage in C. Now assume that economy 1 consumes (C_1, F_1) and economy 2 consumes (C_2, F_2). Now I propose the following trade: Economy 1 produces one unit more food and gives it to economy 2. To do this they have to produce less clothing. Economy 2 consumes this food and produces more clothing. They consume what they did before $((C_2, F_2))$ give some clothing back to economy 1 so they can consume what they did before $((C_1, F_1))$ and I get the rest. If comparative advantage is not true then this I should not get anything, but we can see that is. First of all, how much will economy 1 have to give up in clothing to do this:

$$
\begin{aligned}
F_1 + 1 + 2\left(C_1 - \Delta C_1\right) &= F_1 + 2C_1 \\
1 - 2\Delta C_1 &= 0 \\
\Delta C_1 &= \frac{1}{2} \, .
\end{aligned}
$$

How much more clothing will country 2 be able to produce?

$$
\begin{aligned}
2\left(F_2 - 1\right) + C_2 + \Delta C_2 &= 2F_2 + C_2 \\
-2 + \Delta C_2 &= 0 \\
\Delta C_2 &= 2 \, .
\end{aligned}
$$

So I get 1.5 units of clothing! That's pretty good, let's do it again! Obviously this will work until the actual amount of food produced in country 2 goes to zero. And man, I'm I going to make out like a bandit. OK, OK, instead of me profiting maybe I should share the surplus between the two countries. Basically they're going to get free clothing for nothing! This means that both societies will be better off, a clear Pareto Improvement. Thus the basic insight of Comparative Advantage, each country should export the good they can produce at the lowest opportunity cost at equilibrium prices.

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[9] You get bonus points if you read to the point where you could get offended by what I've written. I'm just joking, sheesh, don't take me so seriously.