

# The Critical lessons in ECON 203

Kevin Hasker

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Every semester before every exam students always ask what the important lessons they just learned were. What a clever question! You think I'm going to tell you what's on the exam? I don't think so! But I can tell you the important lessons you will be learning, they are pretty much the same from year to year.

## 1 Optimization

The three characteristics of a maximum from the story of Nazerattin Hoca and the Ants.

1. If  $f'(x) > 0$  increase  $x$ , if  $f'(x) < 0$  decrease  $x$ . In words if the slope of the ground is rising go forward, if it's decreasing go back.
2. If  $f'(x) = 0$  then you might be at a maximum. In words it's always flat at the top of a hill.
3. If you are at a maximum then  $\frac{d}{dx}(f'(x)) \leq 0$ . I.e. the slope of the ground is decreasing at the top of a hill.

Then on a mechanical level you must learn the two optimization problems:

$$\begin{array}{ll} \max_{x,y} U(x,y) & \min_{l,k} wl + rk \\ \text{s.t. } p_x x + p_y y \leq I & \text{s.t. } f(l,k) \geq Q \end{array}$$

the solution to these two problems is mathematically identical. Precisely under standard assumptions one problem is the *dual* of the other. You won't need to exactly understand what duality is all about, but if you do it will help. The solution to both of these problems is a set of *demand functions*:

$$\begin{array}{ll} x(p_x, p_y, I) & l(w, r, Q) \\ y(p_x, p_y, I) & k(w, r, Q) \end{array}$$

As an example let me solve both problems using the Cobb-Douglas Function,  $f(x, y) = x^\alpha y^\beta$  where both  $\alpha > 0$  and  $\beta > 0$ .

$$\max_{x,y} \min_{\lambda} x^\alpha y^\beta - \lambda(p_x x + p_y y - I)$$

$$\begin{aligned}\alpha \frac{f}{x} - \lambda p_x &= 0 \\ \beta \frac{f}{y} - \lambda p_y &= 0 \\ p_x x + p_y y &= I\end{aligned}$$

$$\begin{aligned}\beta \frac{f}{y p_y} &= \lambda = \alpha \frac{f}{x p_x} \\ x &= \alpha y \frac{p_y}{\beta p_x}\end{aligned}$$

$$\begin{aligned}p_x \left( \alpha y \frac{p_y}{\beta p_x} \right) + p_y y &= I \\ y &= \frac{\beta}{\alpha + \beta} \frac{I}{p_y} \\ x &= \alpha \frac{p_y}{\beta p_x} \left( \frac{\beta}{\alpha + \beta} \frac{I}{p_y} \right) = \frac{\alpha}{\alpha + \beta} \frac{I}{p_x}\end{aligned}$$

and this is your solution. For the other problem:

$$\min_{x,y} \max_{\lambda} p_x x + p_y y - \lambda (x^\alpha y^\beta - q)$$

where  $q$  is output.

$$\begin{aligned}-\lambda \alpha \frac{q}{x} + p_x &= 0 \\ -\lambda \beta \frac{q}{y} + p_y &= 0 \\ x^\alpha y^\beta &= q\end{aligned}$$

$$\begin{aligned}\frac{p_y}{\beta q} y &= \lambda = \frac{p_x}{\alpha q} x \\ x &= \alpha y \frac{p_y}{\beta p_x}\end{aligned}$$

Notice this is the same solution as before. What differs is which function we substitute this solution into.

$$\begin{aligned}\left( \alpha y \frac{p_y}{\beta p_x} \right)^\alpha y^\beta &= q \\ y &= \left( \frac{\beta p_x}{\alpha p_y} \right)^{\frac{\alpha}{\alpha+\beta}} q^{\frac{1}{\alpha+\beta}} \\ x &= \alpha \left( q^{\frac{1}{\alpha+\beta}} \left( \frac{\beta p_x}{\alpha p_y} \right)^{\frac{\alpha}{\alpha+\beta}} \right) \frac{p_y}{\beta p_x} = \left( \frac{\alpha p_y}{\beta p_x} \right)^{\frac{\beta}{\alpha+\beta}} q^{\frac{1}{\alpha+\beta}}\end{aligned}$$

and in this case we sometimes want to find the cost function:

$$\begin{aligned}
 C(p_x, p_y, q) &= p_x x^* + p_y y^* \\
 &= p_x \left( \left( \frac{\beta p_x}{\alpha p_y} \right)^{\frac{\alpha}{\alpha+\beta}} q^{\frac{1}{\alpha+\beta}} \right) + p_y \left( \left( \frac{\alpha p_y}{\beta p_x} \right)^{\frac{\beta}{\alpha+\beta}} q^{\frac{1}{\alpha+\beta}} \right) \\
 &= \frac{\beta p_x^2 + \alpha p_y^2}{(\beta p_x)^{\frac{\beta}{\alpha+\beta}} (\alpha p_y)^{\frac{\alpha}{\alpha+\beta}}} q^{\frac{1}{\alpha+\beta}}
 \end{aligned}$$

Unfortunately the Cobb-Douglas Cost function is not very neat.

## 2 Utility Theory

Obviously maximizing utility given a budget constraint is part of this section, but the most important thing you will learn in this section is the *Income effect*. This explains why demand curves aren't downward sloping, why consumer surplus is an imprecise concept, and what the difference between gross and net substitutes/compliments are. An equation embodying this effect is the *slutsky equation*.

## 3 Cost Theory

There are three important lessons in this section, besides learning how to minimize costs (above).

1. The difference between *Accounting Costs* and *Economic Costs*.
2. The *four axioms of the cost function*.
3. The difference between *long run costs* and *short run costs*—the envelope theorem.

## 4 Solving Simultaneous Equations.

In general finding equilibrium (competitive, monopoly, or oligopoly) always boils down to solving two equations for two unknowns. This is solving the problem of simultaneous equations, and you need to be able to do this easily.

The easiest example is linear supply and demand:

$$\begin{aligned}
 Q_d &= D(P_d) = a - bP_d \\
 Q_s &= S(P_s) = c + dP_s
 \end{aligned}$$

Now in equilibrium we know that  $Q_d = Q_s = Q$  and  $P_d = P_s = P$  so we can use the former to solve for the latter.

$$\begin{aligned} Q_d &= Q_s \\ a - bP &= c + dP \\ a - c &= (b + d)P \\ \frac{a - c}{b + d} &= P \end{aligned}$$

$$\begin{aligned} Q &= a - bP_d = a - b\left(\frac{a - c}{b + d}\right) = \frac{ad + bc}{b + d} \\ Q &= c + dP = c + d\left(\frac{a - c}{b + d}\right) = \frac{ad + bc}{b + d} \end{aligned}$$

This is a very simple exercise and it is important that you know how to do it.

## 5 The Competitive Market

We just found competitive equilibrium above. I usually expect you to go one step beyond this and construct the short run supply curve. Assume there are  $J$  firms with costs

$$c(q) = cq + dq^2 + F$$

where  $F$  is all fixed start up costs. Then one firm's supply curve is:

$$s(P_s) = \begin{cases} \frac{P_s - c}{d} & P_s \geq c + 2\sqrt{dF} \\ 0 & P_s < c + 2\sqrt{dF} \end{cases}$$

and so the market supply curve is:

$$\begin{aligned} S(P_s) &= J s(P_s) = \begin{cases} J \frac{P_s - c}{d} & P_s > c + 2\sqrt{dF} \\ 0 & P_s < c + 2\sqrt{dF} \end{cases} \\ S(P_s) &= \frac{J}{d} P_s - \frac{J}{d} c \end{aligned}$$

Now we solve for the market equilibrium like before. Or in otherwords what was  $c$  before is now  $-\frac{J}{d}c$  and what was  $d$  is now  $\frac{J}{d}$ .