Handout on the General Principle Agent Solution—Adverse Selection

Kevin Hasker

Every time I teach this material the students get confused because of the complexity of the algebraic tricks we do. But that's all it is, algebraic tricks. So in this handout I go over them in detail so that you can understand them.

We established in the last handout that one of the bounds of the set of viable contracts was:

$$\begin{aligned} w_1 &= c(e_1, \theta_1) + \bar{u} \\ w_{k+1} &= c(e_{k+1}, \theta_{k+1}) + \bar{u} + \sum_{j=2}^{k+1} \left(c(e_{j-1}, \theta_{j-1}) - c(e_{j-1}, \theta_j) \right) \end{aligned}$$

for $k \in \{2, 3, 4, ..., K-1\}$. It should be obvious that this is the cheapest contract, so this is the one the principle will use. Looking at w_2 we see that:

$$w_2 = c(e_2, \theta_2) + \bar{u} + (c(e_1, \theta_1) - c(e_1, \theta_2))$$

$(c(e_1, \theta_1) - c(e_1, \theta_2))$ is the "cost of revelation," or the amount we have to pay a type 2 to reveal that he is not a type 1. Given this contract, their objective will be:

$$\begin{aligned} &\max_{\{e_k\}_{k=1}^K} \sum_{k=1}^{K} p_k (\pi(e_k) - w_k) \\ &\max_{\{e_k\}_{k=1}^K} \sum_{k=1}^{K} p_k \left(\pi(e_k) - c(e_k, \theta_k) - \bar{u} - \sum_{j=2}^{k} \left(c(e_{j-1}, \theta_{j-1}) - c(e_{j-1}, \theta_j) \right) \right) \end{aligned}$$

where I define that for $k = 1$ $\sum_{j=2}^{k} (c(e_{j-1}, \theta_{j-1}) - c(e_{j-1}, \theta_j)) = 0$. By expanding this out we can get:

$$\max_{\{e_k\}_{k=1}^K} \sum_{k=1}^{K} p_k (\pi(e_k) - c(e_k, \theta_k)) - \bar{u} - \sum_{k=1}^{K} p_k \sum_{j=2}^{k} \left(c(e_{j-1}, \theta_{j-1}) - c(e_{j-1}, \theta_j) \right)$$

now we want to simplify this last double summation. This can be quite simply done by counting the number of times a particular term is in the summation. For example, how many times does

$$c(e_1, \theta_1) - c(e_1, \theta_2)$$

appear? It appears in all terms except for the first one since the summation always starts at $j = 2$. If $P_k = \Sigma_{j=1}^{k} p_k$ then the probability of this event is

$$1 - P_1 = 1 - p_1 = \Sigma_{j=2}^{K} p_k$$

. How many times does $c(e_2, \theta_2) - c(e_2, \theta_3)$ appear? Well this appears every time except for when $k \in \{1, 2\}$, and the probability of this event is $1 - P_3$. And so on. So:

$$\begin{aligned} \sum_{k=1}^{K} p_k \sum_{j=2}^{k} \left(c(e_{j-1}, \theta_{j-1}) - c(e_{j-1}, \theta_j) \right) &= \sum_{j=2}^{K} (1 - P_{j-1}) \left(c(e_{j-1}, \theta_{j-1}) - c(e_{j-1}, \theta_j) \right) \\ &= \sum_{j=1}^{K-1} (1 - P_j) \left(c(e_j, \theta_j) - c(e_j, \theta_{j+1}) \right) \end{aligned}$$

Where the last step is just a change of variables. To make this clearer let's write each term out explicitly for $K = 4$.

$$\begin{aligned}
& p_1 [(0)] \\
& + p_2 [(c(e_1,\theta_1) - c(e_1,\theta_2))] \\
& + p_3 [(c(e_1,\theta_1) - c(e_1,\theta_2)) + (c(e_2,\theta_2) - c(e_2,\theta_3))] \\
& + p_4 [(c(e_1,\theta_1) - c(e_1,\theta_2)) + (c(e_2,\theta_2) - c(e_2,\theta_3)) + (c(e_3,\theta_3) - c(e_3,\theta_4))] \\
= \; & (p_2 + p_3 + p_4)(c(e_1,\theta_1) - c(e_1,\theta_2)) \\
& + (p_3 + p_4)(c(e_2,\theta_2) - c(e_2,\theta_3)) \\
& + p_4 (c(e_3,\theta_3) - c(e_3,\theta_4)) \\
= \; & (1 - P_1)(c(e_1,\theta_1) - c(e_1,\theta_2)) \\
& + (1 - P_2)(c(e_2,\theta_2) - c(e_2,\theta_3)) \\
& + (1 - P_3)(c(e_3,\theta_3) - c(e_3,\theta_4))
\end{aligned}$$

So going back to the objective function:

$$\max_{\{e_k\}_{k=1}^K} \sum_{k=1}^K p_k (\pi(e_k) - w_k) = \max_{\{e_k\}_{k=1}^K} \sum_{k=1}^K p_k (\pi(e_k) - c(e_k,\theta_k)) - \sum_{j=1}^{K-1} (1 - P_j)(c(e_j,\theta_j) - c(e_j,\theta_{j+1}))$$

Now if we define $(c(e_K,\theta_K) - c(e_K,\theta_{K+1})) = \Delta$ (the exact value does not matter) then we can also rewrite this as:

$$\max_{\{e_k\}_{k=1}^K} \sum_{k=1}^K p_k \left(\pi(e_k) - c(e_k,\theta_k) - \frac{(1 - P_k)}{p_k} (c(e_k,\theta_k) - c(e_k,\theta_{k+1})) \right)$$

and $c(e_k,\theta_k) + \frac{(1-P_k)}{p_k}(c(e_k,\theta_k) - c(e_k,\theta_{k+1}))$ is sometimes referred to as the "virtual cost" of type $k \in \{1, 2, 3, ...K\}$. Analyzing this objective function gives us the following proposition:

Proposition 1 *Only the highest type gets the correct amount of education, all lower types get too little education. Only the lowest type gets paid the competitive wage.*

The former result is known as "no distortion at the top."